Broken Beams

Tearing and shear failures in explosively loaded clamped beams

by S. B. Menkes and H. J. Opat

ABSTRACT—A series of experiments has been conducted, utilizing sheet explosive applied to clamped aluminum beams, with a neoprene buffer. As the load is monotonically increased, three damage modes are identified, which respectively are major inelastic deformation, tearing at the extreme fiber, and transverse shear at the support.

Satisfactory correlation is reported for the extent of inelastic deformation using a lumped parameter, finite-difference code; thresholds for tearing and shear failure based on empirical criteria are presented. Using a Timoshenko beam theory, the shear threshold appears to be dependent on the section velocity, rather than upon the shear stress.

List of Symbols

c_2 = propagation speed for shear waves (in./s)
G = shear modulus (ksi)
h = beam thickness (in.)
H.E. = high explosive (as abbreviation)
I = impulse intensity (ktaps)*
$I_{0\epsilon}$ = reference impulse, arbitrary strain, eq (1) (ktaps)†
I_{05} = reference impulse, 5 percent strain, eq (1) (ktaps)†
i = subscript indicating material layer
K_ϵ = defined by eq (3) (in./s)
L = beam length (in.)
r = radius of gyration = $\sqrt{h^2/12}$ (in.)
$\bar{t}$ = time (s)
$t = \bar{t}/t_R$ (dimensionless)
$t_R = L/c_2$ (s)
$\bar{x}$ = distance along beam (in.)
$x = \bar{x}/L$ (dimensionless)
$\bar{v}_o$ = initial average beam velocity (in./s)
$v_o = \bar{v}_o/c_2$ (dimensionless)
Δ = residual central deflection of beam (in.)
ϵ = strain (in./in.)
λ = slenderness ratio = L/r (dimensionless)
μ = defined by eq (3) (lb_f-s^2/in.^{3})
ρ = mass density (lb_f-s^2/in.^{4})
σ = uniaxial tensile stress (ksi)
$\bar{\tau}$ = shear stress (ksi)
$\tau = \bar{\tau}/G$ (dimensionless)

Objective and Scope

This paper describes an experimental–theoretical study of the dynamic response of clamped beams to very high transverse pressures of extremely short duration. The following objectives are delineated:

(1) As the loads become progressively more severe, to identify the different damage mechanisms.
(2) For each damage mode, to distinguish the controlling variables.
(3) To seek means by which the incidence of such damage modes (and where appropriate the extent of the damage) may be predicted.

Only gross physical damage is considered, involving large deformations and/or rupture. Superimposed buckling patterns are not treated.

Introduction and Background

Genesis of Problem

The actual structure of current interest is a reentry vehicle, shown schematically in Fig. 1, of monocoque construction with internal rigid-ring supports.

Under certain conditions, the vehicle may be suddenly subjected to high-intensity, asymmetric, short-duration external pressures. A quantitative prediction of the shell response is difficult; it may generally be expected to deform radially inward in the sections between the rib stiffeners. If the pressures are sufficiently high, the skin may be torn at or ruptured over the supports.

As one approach (which aids in understanding the phenomenology), an axial section is taken through the vehicle, disclosing a continuous beam subjected to a uniform transverse pressure. The beam supports are the circular reinforcing ribs. One span of this continuous beam is selected for study. It is assumed that the supports are ideally rigid. The single span is then modeled as a clamped beam; the clamp both acts to prevent rotation and to provide axial restraint.

Some Theoretical Considerations

In the real problem, the pressures are in the order of kilobars, and the pulse length in the order of microseconds. The loading conditions may be simulated by sheet explosive (see Fig. 2). The transient stress history through the section depends on wave propagation, and is dominated by the discontinuity in the rear-face reflection conditions at points A and B in Fig. 2.

Ultimately, an understanding of the very early

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* *1 ktap = 1000 taps 1 tap = 1 dyne-s/cm^2*

† *Equation (1) yields the impulse intensity in lb_f-s/in^2. A conversion factor is necessary to convert to ktaps; 1 lb_f-s/in^2 = 69.5 ktaps.*

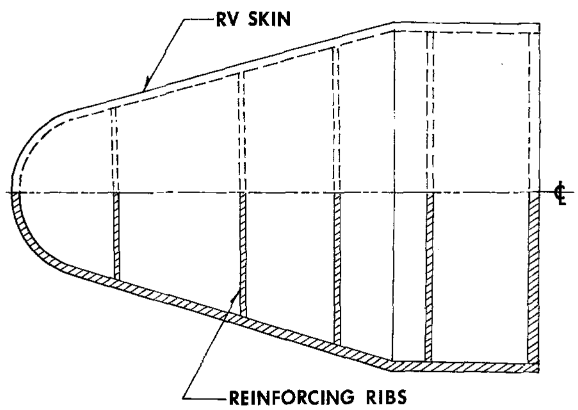

Fig. 1—Schematic reentry vehicle

material response will probably require a two-dimensional analysis using finite-element methods. The approach is described by Costantino.[1] In its present form, however, his code SLAM is not yet suitable. It must be modified to include a finite-strain model, appropriate criteria for the detection of local ruptures, and a treatment for post-failure motion.

If the rear-surface discontinuity can be accommodated by sufficient inelastic deformation without rupture, the beam soon acquires a uniform average velocity, and the problem reduces to one of response to an impulsive load.

Review of Literature

A number of investigators[2-7] have treated the question of the response of beams to transverse impulsive loads sufficient to cause finite inelastic deformation. They have, however, avoided loads so severe as to cause tearing or rupture at the support points. The lumped parameter, finite-difference numerical methods described by Witmer[6] offer the most versatile possibilities, reasonable correlation and low operational cost. These techniques cannot, however, provide an accurate, detailed representation of stress or strain distribution through the beam thickness near the points of support.

It will be found (in this study) that there are several reasons to conclude that, if a transverse shear failure occurs, it will take place at extremely early times—long before there is any opportunity for significant plastic deformation to occur. This suggests that it may be possible to develop a correlation between an experimentally observed shear threshold and a shear-stress resultant obtained by a pseudo-elastic analysis.

The available approximate models for describing elastic beam behavior are either the Euler-Bernouilli or the Timoshenko theories. The former is fundamentally inadequate to the analysis of impact conditions, because it yields the nonphysical result that waves of infinitesimal wavelength (and, therefore, discontinuities) propagate with infinite velocity.[8] Prescott shows[9] that this theory yields satisfactory results only for the lower modes. It can be shown that the shear-stress series for the Euler theory is nonconverging.

While the Timoshenko bending mechanism retains the one-dimensional nature of the more elementary theory, it does include the effects of transverse shear deformation and of rotatory inertia. Several investigators[10-17] have been concerned with various phases of the problem of an infinite or semi-infinite Timoshenko beam subjected to impulsive loads. Normal-mode solutions were developed by Anderson[18] for finite spans, and then applied to a simply supported beam subjected to a concentrated load at midspan. Both Anderson and Thomson[19] provide the orthogonality conditions. Garrelick[20] studied finite, simply supported beams. The probable incidence of shear failure at very high initial velocities was suggested by Bleich and Shaw[21] and by Karunes and Onat.[22]

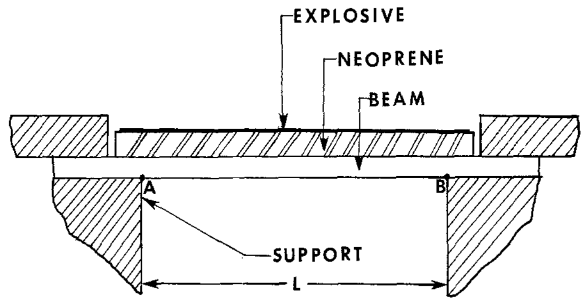

Fig. 2—Fixture for sheet-explosive tests on beams

Experimental Plan

Figure 2 shows the experimental configuration. The high pressure is provided by sheet explosive, which is cemented to a neoprene buffer which, in turn, is bonded to the top surface of a clamped aluminum beam. It should be noted that the explosive extends well over the point of support.

All tests were conducted on 6061-T6 aluminum. Three thicknesses (.187 in., .250 in. and .375 in.), considered to be representative, were employed. The hardpoint separation distances were arbitrary: two (8.0 in. and 4.0 in.) were used to detect the possible influence of length. All beams were 1.0-in. wide.

To prevent rear-surface spallation, a .250-in.-thick neoprene buffer was used in all cases. The high explosive was Du Pont Detasheet D, and the adhesive was Dupont 4684 cement, thinned 1:1 with acetone. In order to vary the impulse loads in reasonable increments, four thicknesses of H.E. (10, 15, 25 and 50 mils) were utilized in different combinations. The latter three are standard stock, with $\pm$ 5 percent tolerance. The 10-mil sheet is especially rolled, and the tolerance on it is $\pm$ 20 percent.

The uppermost layer of explosive was extended for 4.0 in. beyond one end of the beams, serving as the detonation source. With single-ended detonation, one could be concerned lest the resultant deformation be asymmetrical. The detonation velocity of the H.E. is 0.72 cm/μs, so that the time to traverse a loaded 8-in. beam is 35.3 μs. The shock transit time through the beams is of the order of 1-2 μs. Thus, there is ample time for the end of the beam nearest the detonation point to start deforming before the detonation wave reaches the other end of the beam. Fortunately, however, there is no significant evidence of deformation asymmetry in the experimental results. Presumably, even though one end of the beam may start to deform before the other, the extent of the deformation is so small as not to affect the final result. A limited number of X-ray photographs support this presumption.

There was no active instrumentation; special techniques are required and must be developed. Central deformation (Δ) thus describes a permanent or residual state, rather than a maximum transient deformation.

Results of Experiments

General Comments

For any one set of beams, the impulse intensity was gradually increased by using thicker H. E. assemblies; Fig. 3 shows a typical result. Characteristically, for all beams examined, as I increases, three distinctly different damage modes may be noted:

I. Large inelastic deformation.
II. Tearing (tensile failure) in outer fibers, at or over the support.
III. Transverse shear failure at the support.

For Mode I, the extent of damage is described by the amount of residual central deflection (Δ). The threshold for Mode II is taken as that impulse intensity which first causes tearing. As the load increases, Modes II and III overlap. A pure, well defined shear failure is characterized by *no significant deformation* in the severed central section.

Normalization of Results

Sliter[23] has suggested the use of a normalizing parameter $I_{0\epsilon}$ in connection with the study of damage caused by impulse. This parameter is the uniform radial impulse intensity required to effect a plastic strain ϵ in a rigid–plastic ring.

For a single-layer material, $I_{0\epsilon}$ may be found from the expression:

$$I_{0\epsilon} = \sqrt{2\,\mu}\,K_\epsilon \tag{1}$$

while for a two layered material,

$$I_{0\epsilon} = \sqrt{2\mu}\,\sqrt{\mu_1(K_{\epsilon 1})^2 + \mu_2(K_{\epsilon 2})^2} \tag{2}$$

wherein:

$$\mu_i = \rho_i h_i \quad K^2_{\epsilon i} = \frac{1}{\rho_i}\int_0^{\epsilon_i} \sigma d\epsilon \quad \mu = \mu_1 + \mu_2 \tag{3}$$

Sliter and others at Stanford Research Institute have used the specific strain of 5 percent to compute the parameter I_{05} in connection with several attempts to classify experimental data. We treat the beams as two layers (neoprene and aluminum); the parameter I_{05} for each beam is shown in Table 1. The values of K_ϵ and ρ necessary to define I_{05} were taken either from Ref. 23 or 24.

Empirical Correlations

Analysis of the data suggests that, for Mode I damage, the central deflection is related to, and generally proportional to, the length of the beam. An attempt to correlate the dimensionless ratios (Δ/L) and (I/I_{05}), however, *discloses only a weakly defined pattern* with considerable scatter.

Alternatively, however, it appears that the Mode II and Mode III thresholds (for a cross section composed of neoprene and aluminum) do *not* depend on the length of the beam, but only on the thickness. This may be seen from Fig. 4, which includes all test data in the region of the thresholds.* Admittedly, the selection of the individual tearing and shear thresholds is highly subjective. But it is quite plain that there is no significant difference in these quantities imposed by the length. Furthermore, the linear relationship between thickness and impulse intensity is quite reasonable.

TABLE 1—MODE II AND III THRESHOLDS EXPERIMENTAL CORRELATION

Mode	Beam Thickness (Inches)	I_{05} (ktaps)	Impulse Intensity (ktaps)	I/I_{05}	Initial Velocity (inches/sec)
II	.187	19.4	26.0	1.34	8000
III			40.0	2.06	12300
II	.250	23.5	32.0	1.36	7350
III			48.0	2.04	11000
II	.375	33.0	45.0	1.34	6900
III			65.0	1.96	10000

Table 1 displays these conclusions in somewhat more useful form, including the appropriate I_{05} parameter, the normalized impulse intensity I/I_{05}, and the initial section velocity [see eq (4)]. The nominal impulse intensity (I) to be expected from an assembly of H.E. sheets has been computed by simply summing Clark's[25] experimentally obtained values for the individual sheets.

Discussion of Experimental Error

It may be noted (Fig. 4 and Table 1) that the results are critically dependent on the initial velocity actually acquired by the aluminum. This velocity is

* *(On Fig. 4, T = tearing, S = shear, TS = tearing and shear, and bracketed numbers are Δ).*

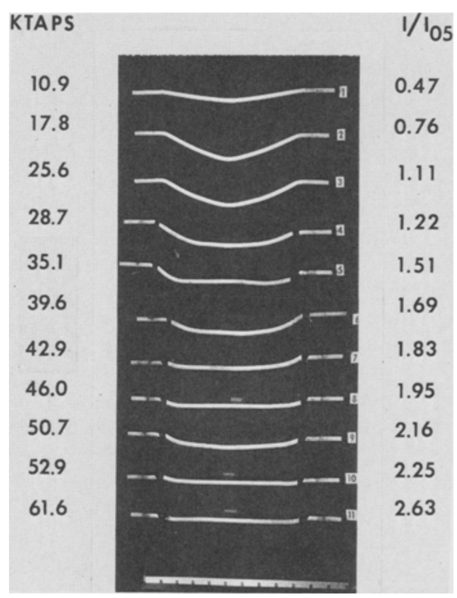

Fig. 3—Results for series of 6061-T6 beams (.250 × 1 × 8-in. beam)

computed by the formula:

$$\overline{v}_o = I/\rho h \qquad (4)$$

in which the impulse intensity I is not so well defined as the density ρ or the thickness h.

The value of I depends on two factors difficult to control:

(1) The amount of impulse delivered to the neoprene surface by the sheet explosive.
(2) The manner in which this incident impulse is partitioned between the neoprene and the aluminum.

In any one case, for example, the assembled sheet explosive might consist of as many as two to five layers. Even if it is assumed that there is no 10-mil sheet, so that each of the layers is made to a $\pm$ 5 percent tolerance, the expected variation in the thickness of a four-layer assembly would be $\pm$ 10 percent. The impulse intensity delivered to the neoprene surface is approximately proportional to the assembled H.E. thickness, and may thus be expected to vary by as much as $\pm$ 10 percent from a nominal value.

It is known that the neoprene and aluminum layers separate at a very early time. The partition of the incident impulse between the neoprene and the aluminum is determined by the actual pressure–time curve, and is a function of the complex shock-wave behavior within the materials and at the interfaces. It has been assumed that the momentum transferred to the aluminum is, in fact, exactly the same as that originally delivered to the neoprene. The theoretical partition of momentum has been predicted for several cases from the material response code PUFF;[26] from these results it is estimated that the assumption of complete momentum transfer does not err by more than $\pm$ 5 percent.

Combining these two factors, the uncertainty limits for the initial velocity value computed for each nominal loading are about $\pm$ 12 percent. To obtain better data, it would be desirable actually to measure the aluminum velocity by means of a high-speed motion picture, break rods, or a ballistic pendulum. This was not done in the current series because the chief focus has been on the disclosure of the damage modes. The correlation studies have been made primarily to test the feasibility of predicting the onset of different damage modes by purely empirical rules. Similar damage modes have been observed in clamped cylindrical shells; it is hoped that any empirical methods would have ultimate application to this geometry. It is evident that the accuracy should be improved.

Mode I Correlation with DEPROSS

Calculations have been made using the DEPROSS code.[7] For these runs, the beam is represented as if it were 20 lumped masses connected by massless deformable links. The aluminum is taken to be elastic–perfectly plastic. Table 2 lists all the beams which suffered inelastic deformation; a comparison is drawn between the estimated permanent central deflection, as predicted numerically by DEPROSS, and the experimental result. These beams do not exhibit either of the other damage modes.

Although neither data nor curves are shown to describe the way in which the central deflection changes with time, it may be noted that the maximum values are reached at about 0.3 ms for the 8-in. beam and at about 0.15 ms for the 4-in. beam. These are in agreement with typical structural response times.[6]

Figure 5 shows a comparison between the residual central deflection as found by DEPROSS and as found by experiment, for the .375 $\times$ 1 $\times$ 8-in. beam. The asymptotic behavior of the experimental curve appears to be related to the onset of Mode II damage.

The correlation between theory and experiment is quite good. Some adjustments are possible in the DEPROSS calculation as, e.g., the use of more mass points, and slight changes in the constitutive relations. But this type of manipulation is not considered to be useful, until an improvement in the calculations is effected by a direct measurement of the initial velocity.

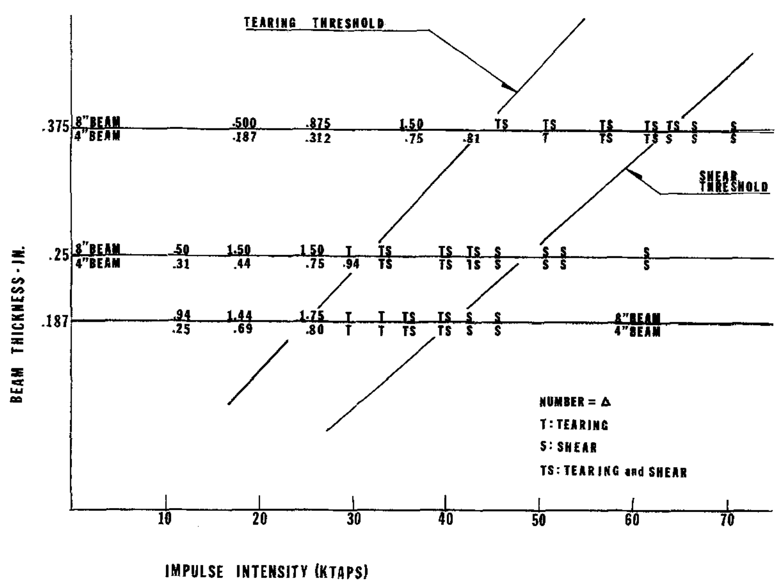

Fig. 4—Results of beam experiments

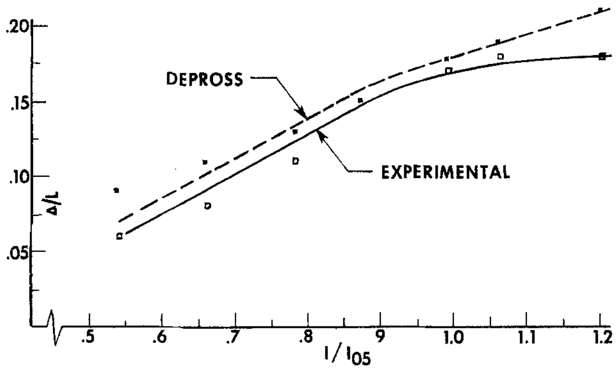

Fig. 5—Residual central deflection: DEPROSS vs. experiment (.375 × 1 × 8-in. beam)

Mode II Threshold, as Predicted by DEPROSS

An attempt was made to predict Mode II thresholds with DEPROSS by incorporating a tensile failure criterion, based either on maximum tensile stress or on maximum total strain. This effort was unsuccessful; if, however, DEPROSS were to be modified to include transverse shear deformation and rotatory inertia, the technique should be satisfactory.

Mode III Threshold, as Predicted by DEPROSS

DEPROSS has been considered as a possible tool for the prediction of the transverse shear threshold.

TABLE 2—DEPROSS THEORY AND EXPERIMENT

Impulse Intensity (ktaps)	Initial Velocity (in./s)	Beam Length (in.)	Beam Thickness (in.)	Central Deflection DEPROSS (in.)	Central Deflection Experiment (in.)
10.9	3354	8	.187	0.99	0.94
17.8	5478			1.56	1.44
25.6	7879			2.28	1.75
10.9	3354	4	.187	0.44	0.25
17.8	5478			0.77	0.69
25.6	7879			1.17	0.80
10.9	2509	8	.250	0.54	0.50
17.8	4098			1.07	1.50
25.6	5894			1.64	1.50
10.9	2509	4	.250	0.32	0.31
17.8	4098			0.53	0.44
25.6	5894			0.81	0.75
17.8	2732	8	.375	0.72	0.50
21.8	3346			0.89	0.75
25.6	3929			1.06	0.88
28.7	4405			1.20	1.25
33.4	5126			1.42	1.31
35.1	5387			1.51	1.50
39.6	6078			1.71	1.50
42.9	6584			1.88	1.44
17.8	2732	4	.375	0.48	0.18
21.8	3346			0.57	0.31
25.6	3929			0.65	0.31
28.7	4405			0.71	0.38
33.4	5126			0.77	0.56
35.1	5387			0.83	0.81
39.6	6078			0.88	0.75
42.9	6584			0.94	0.81

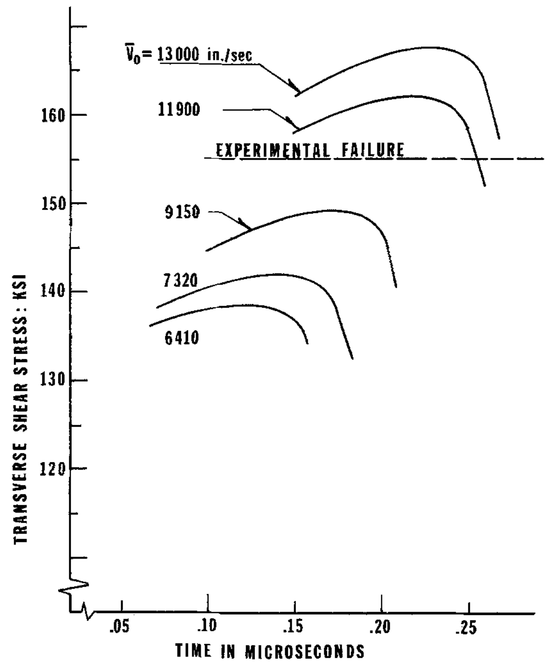

Fig. 6—Transverse shear stress, as computed by DEPROSS (.250 × 1 × 8-in. beam)

For the .250 × 1 × 8-in. beam, a detailed early time history is shown in Fig. 6. (The velocities shown do not correspond to the actual tests, but they span the same range). From Table 1, the shear threshold was estimated at 48.0 ktaps, corresponding to an initial velocity of 11000 in./s and (from Fig. 6) a shear stress of 155 ksi.

Other runs, not shown, indicate that DEPROSS predicts a transverse shear stress inversely proportional to beam length. This contradicts the experimental evidence that the shear threshold does *not* depend on length.

One of two conclusions is inescapable:

(1) *Either* DEPROSS does not predict the shear-stress resultant accurately near the point of support, *or*
(2) The threshold for Mode III damage depends *not* on the maximum shear stress, but *only on the initial velocity.*

Judgement is reserved until the results from the Timoshenko beam calculations are examined (below). For the moment, however, it is noted that:

(1) The peak shear stress occurs at very early times. (In this case at 0.2 μs).
(2) The dynamic shear strength appears to be much higher than the static shear strength.
(3) For any one beam, the shear stress is linearly proportional to the initial velocity.

Mode III Thresholds, from Timoshenko Theory

Because this is an elastic theory, it cannot be used to predict Mode I damage or the Mode II threshold. Using a pseudoelastic approach, however, it remains a possible tool for the prediction of Mode III thresholds.

Two closely related studies are contained in Refs. 10 and 20. Leonard and Budiansky[10] analyzed the response of a cantilever beam subjected to a step velocity input at the root; Garrelick[20] studied finite, simply supported beams subjected to transverse dynamic loads.

Both investigators show results for shear stress which disclose very severe discontinuities at those times when reflections occur, but do indicate that the curve is well behaved near zero time. This observation prompts consideration of an ordinary eigenfunction analysis for the Timoshenko beam, in order to detect a shear threshold, even if the stress levels predicted are fictitious. In support of this decision, it should be noted that:

(1) The eigenfunction expansions in Refs. 10 and 20 are well behaved at early times.
(2) The actual experimental results show that no inelastic deformation precedes pure transverse shear failure.
(3) DEPROSS calculations indicate that high transverse-shear stresses occur at less than 1.0 μs.

The differential equations are given in Refs. 18 and 19; the orthogonality conditions are demonstrated in Ref. 18. In the development as applied to a clamped beam, a difficulty is encountered in that a single characteristic equation cannot be directly solved for the natural frequency. Instead a trial and error approach originally advanced by Flugge,[27] and more recently described by Forsberg[28] and by Fisher and Menkes,[29] must be used. This involves the use of a trial value of natural frequency, computation of the wave-shape parameters, the formulation of a boundary-condition determinant, and an iteration of the frequency until a required value of the determinant (usually zero) is obtained.

It is necessary to decide how many terms should be used in the shear-stress summation. Figure 7 shows a typical plot for the dimensionless shear stress (τ) vs. dimensionless time (t), for a beam with $\lambda^2 = 12288$. (This corresponds to the .250 $\times$ 1 $\times$ 8-in. beam). Alternative summations for 11, 19 and 25 modes are displayed. The stress is taken at the dimensionless location (x) of .010, and does not vary appreciably (not shown) from the result at .000.

The stress at the *first peak* is higher than those that follow. The actual value of τ attained depends on how many modes are used in the summation. In order to compare properly the shear stresses in the different beams, it is necessary to identify the *wavelengths* for each frequency. For different beams, the shortest periods (corresponding to the highest mode used) should be similar.

In Fig. 7, the shear stress is for a dimensionless initial velocity of 1.0. Using the criterion of similar periods, Fig. 8 shows the maximum shear stress obtained for beams of different values of λ^2 in one case including all modes down to a period of 0.400 and, in another, all modes down to a period of 0.150.

Noting from the formula (not shown) that the shear stress is linearly proportional to the velocity, Fig. 9 is constructed, showing theoretical distribution of maximum shear stress for velocities lower than 1.0. Also shown on Fig. 9 are the experimentally obtained best estimates (from Fig. 4) of the

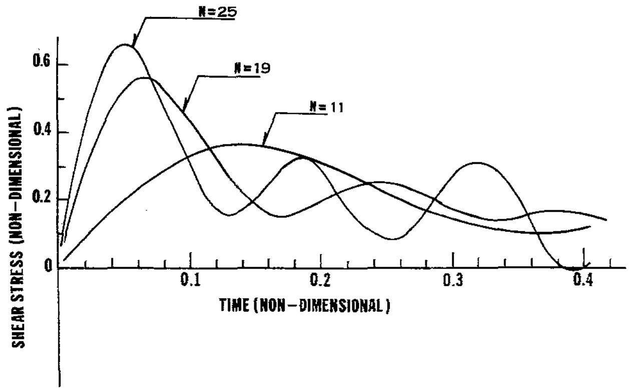

Fig. 7—Dimensionless plot of shear stress (τ) vs. time (t); $\lambda^2 = 12288$, x = .010

transverse-shear threshold. The data support the following comments:

(1) The transverse shear threshold will occur for any beam at about the same value of velocity; i.e., at about 0.1, dimensionless, corresponding to 12,000 in./s.
(2) The magnitude of the shear stress actually reached depends on λ^2, ranging here from .03 to .055 (or dimensionally, from 120 ksi to 220 ksi).

Conclusions

Three objectives were delineated. These are repeated below, for clarity, together with the appropriate conclusions.

1. To identify the damage mechanisms

Three damage modes, involving inelastic deformation (I), tearing over the support (II) and transverse shear failure at the support (III) are characteristic of the response of clamped aluminum beams to high-intensity short-duration transverse pressures.

2. For each damage mode, to distinguish the controlling variables

(a) For MODE I, the severity of the damage may be described by the residual central deflection. This depends on the impulse intensity, the density and constitutive relations of the materials, the manner in which the impulse is partitioned (determining the initial velocity in the substrate), and the beam thicknesses and length.
(b) For MODE II, the threshold depends on the same variables cited in the paragraph above, with the exception of the length. It is characterized by a small tear in the top fiber.
(c) For MODE III, the threshold depends on the same variables as for Mode II, occurring at higher values of the impulse intensity. It is characterized by *no* appreciable plastic deformation. The initial velocity appears to be critical.

3. To seek means for prediction

(a) Deformation in Mode I can be correlated with the numerical code DEPROSS.
(b) Thresholds for Mode II and III can be experimentally correlated as occurring at about 1.36 and

2.0 times the parameter I_{05} respectively.

(c) On the basis of both DEPROSS and Timoshenko beam theory, Mode III threshold depends on the initial velocity; it occurs at the dimensionless value of 0.1; this corresponds to different shear stresses, depending on beam proportions.

Other experiments are now in progress on clamped aluminum cylindrical shells; cylinder thicknesses are taken the same as beam thicknesses, and distance between supports the same as beam lengths. It may be noted that similar damage modes are encountered at similar impulse-intensity levels. This suggests the tentative conclusion that empirically based methods for prediction of damage in beams may have application in the prediction of damage to reentry vehicles.

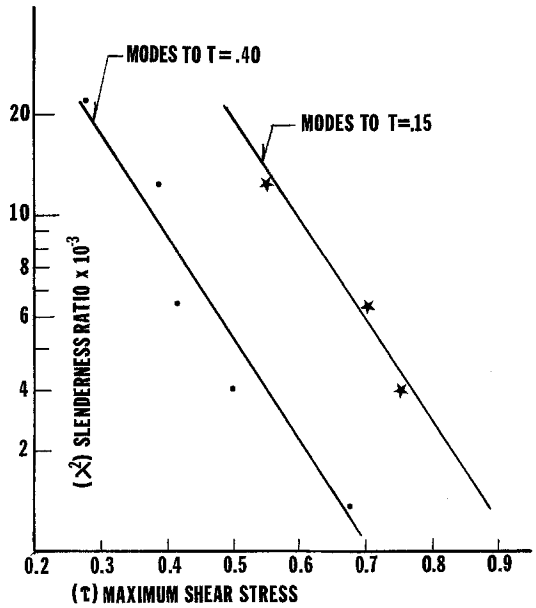

Fig. 8—Maximum shear stress (τ) for all beams; velocity (v_0) = 1.0

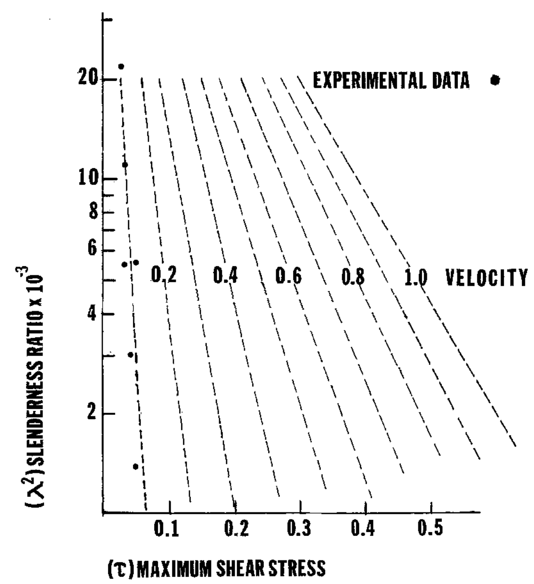

Fig. 9—Maximum shear stress (τ); fractional initial velocities

Acknowledgment

This work was funded by SAFSCOM for the U. S. Army, through a grant to the City College Research Foundation from Picatinny Arsenal.

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