Measurement of Apparent Young's Modulus in the Bending of Cantilever Beam by Heterodyne Holographic Interferometry

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ABSTRACT—Detection of micromechanical phenomena in material requires a sufficiently high-accuracy measurement. This paper shows the possibility of applying heterodyne holographic interferometry to such experimental verification. The effects of the ratios of thickness to grain size on the apparent Young's modulus are evaluated based on the deflection patterns of the cantilever beam. The characteristic length ℓ of couple-stress theory is calculated by applying the analytical results of Koiter to our experimental results. This value is about one fifth of the grain size.

Introduction

Recently in the development of material and the study of fracture toughness, concepts of micromechanics, such as couple-stress theory[1] and micropolar theory[2] have become very important and have received many theoretical considerations. Some problems have been solved by several authors with these theories. They point out that the effects of couple stress in materials become significant when some physical dimension of the body approaches a characteristic length.

Few experimental efforts have been made to determine the order of magnitude of the effect of couple stress, however. Schijve[3] (1966) conducted bending tests on aluminum sheets and was unable to detect any couple-stress effect. In his paper, he predicts that a characteristic length (couple-stress constant) ℓ is considerably smaller than 1 mm. Ellis and Smith[4] (1967) conducted cylindrical bending tests on pure aluminum and low-carbon steel thin plates. But their specimens behaved according to classical theory. They indicate that ℓ is as large as or a little smaller than the grain size d. Gauthier and Jahsman[5] (1975) conducted torsion tests on a composite model to evaluate the elastic constant of micropolar theory, which is close to the classical theory of elasticity compared to couple-stress theory. They were also unable to detect any micropolar effect. We believe that in all of these experiments the measurement accuracy required was not carefully investigated and that the selection of model material was not suitable.

Yang and Lakes[6] (1982) used bone, a natural fibrous composite in place of metal as the model of Cosserat continuum in their experiments. The results of quasistatic bending experiments on human compact bone obeyed micropolar theory and the characteristic length was comparable to the size of structural elements, osteons.

According to these researchers,[3,4,6] in such materials with microstructure, the predicted characteristic length is generally as large as—or a little smaller than—the order of the size of the structural elements. Therefore for its experimental verification, slight differences of micromechanical behavior must be detected. The success of the experiment depends on whether or not the adopted experimental technique is sufficiently accurate.

In this paper we report on an experiment to measure deflection distribution in the cantilever beam using heterodyne holographic interferometry. The cantilever beams were made of high-purity aluminum and had various sizes of grains. Their apparent Young's moduli were evaluated based on the measurement of deflection patterns of the cantilever beams. The aim of the experiment is to show the possibility of applying heterodyne holographic interferometry to this kind of experimental verification.

In contrast to the strain-gage method, heterodyne holographic interferometry allows us to record the field of deformation on the whole surface of an object and to collect the displacement data at any desired position of the object. The loading condition of the specimen can be judged easily from the reconstructed fringe pattern. The accuracy of displacement measurement is better than about 0.9×10^{-3} μm and the spatial resolution is about 0.15 mm. These properties may allow us to adopt simple equipment for small magnitudes of load.

Experimental Apparatus and Procedure

In heterodyne holographic interferometry,[7] a small frequency shift is given between the optical frequencies of the two interfering light fields before and after deformation. As a consequence, optical phase difference is converted to the phase of the electrical beat signal and the phase is detected electronically.

The experimental optical system is shown schematically in Fig. 1. Two direct beams from a 50-mW He-Ne laser are used as the reference beams to obtain bright images on image planes. They are separated by an angle larger than the angle subtended by the width of the cantilever beams

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Final manuscript received: July 8, 1985.

at the hologram. Therefore, the desired self-reconstruction images do not overlap with the undesired cross-reconstruction images. The position O_1 of an object before deformation is recorded with the reference beam R_1. The position O_2 of an object after deformation is recorded with reference beam R_2. The shutter S is used to control the exposure time. The normal of the hologram plane is made to agree with the bisector of the two reference beams so that the detected phase is insensitive to hologram mispositioning.[8]

The recorded hologram is processed and returned to the original position where it was taken. It is illuminated with the two reference beams. Two acousto-optical modulators (AOM) with driving frequencies of ω_2 = 40.000 MHz and ω_1 = 40.001 MHz are used to produce a frequency shift of 1 kHz for the reference beams R_1. We use the beam which is diffracted in the positive first order $+\omega_1$ in AOM 1 and in the negative first order $-\omega_2$ in AOM 2. The frequency shift of reference beam R_1 becomes $\omega_1 - \omega_2$. An image lens is used to form the image of the object. It is placed as close to the hologram as possible to reduce the error of phase difference in the image.[7] The phase difference of the two reconstructed waves is detected with two photodetectors on the image plane.

High-purity aluminum is selected as the material of the test specimens. Table 1 shows the chemical composition of the material. All test specimens produced by machining are annealed at 250°C for three hours to remove the residual stress and the unisotropy of the specimens. Table 2 shows their mechanical properties. Test specimens with various ratios of thickness to grain size are made by adjusting conditions of the heat treatment to control recrystallization. Their dimensions and heat treatments are shown in Table 3. In order to determine the size of grains, some representative specimens are cut, polished, etched, and examined metallographically.

Figure 2 shows the cantilever beam, together with clamping and loading configuration. One end of the specimen is fixed to the steel block. The cantilever beam is loaded at its free end by the extension of a spring connected to a screw device. The bending load is calculated by multiplying the spring constant $K = 17.5 \times 10^{-6}$ N/μm by the elongation of the spring measured using the electrical micrometer. An initial load of $W_o = (34.20 \pm 0.01) \times 10^{-3}$ N is applied to the free end of the specimen. The initial state is recorded on the hologram with the reference beam R_1. After recording, another additional load of $W = (29.40 \pm 0.01) \times 10^{-3}$ N is applied to it. Then, the displacement at the free end is about 1 μm. The second deformation state is recorded with the reference beam R_2.

TABLE 1—CHEMICAL COMPOSITION

	Al	Cu	Fe	Si
Wt %	99.99	0.004	0.001	0.001

TABLE 2—MECHANICAL PROPERTIES

Al 99.99	Young's Modulus E (GPa)	Poisson's Ratio ν	Yield Stress σ_y (MPa)	Micro-Vickers Hv (MPa)
Wt %	61.7	0.33	58.8	265

TABLE 3—SPECIMENS

t/d	Grain Size d (mm)	Thickness t (mm)	Width b (mm)	Heat Treatment °C	Heat Treatment hours
62.80	0.05	3.14	20.02	250	6
11.63	0.27	3.14	20.02	350	6
8.05	0.39	3.14	20.03	365	6
5.71	0.55	3.14	20.03	380	6
4.91	0.64	3.14	20.02	390	6
2.97	1.06	3.15	20.02	400	6
2.47	1.27	3.14	20.01	450	6

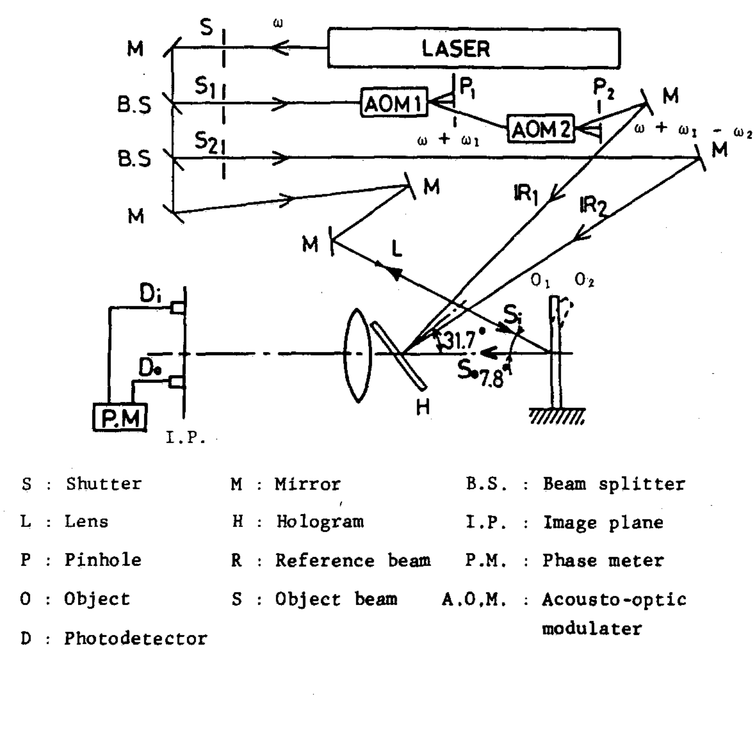

Fig. 1—Optical apparatus for heterodyne holographic interferometry

During recording, both modulators are driven at the same frequency so that the frequency shift is zero.

Figure 3 shows the schematic representation of interference fringes on the cantilever beam and the corresponding beat frequency signals obtained from the scanning photodetector Di and the fixed photodetector D_o. The outputs of both photodetectors oscillate at 1 kHz but their phases differ from each other. This phase difference

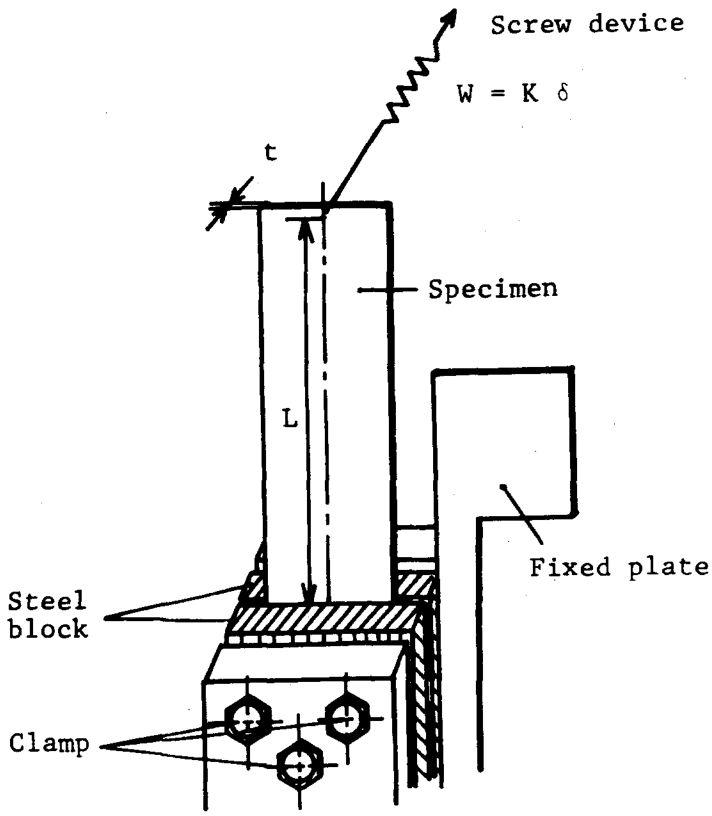

Fig. 2—Test setup for bending

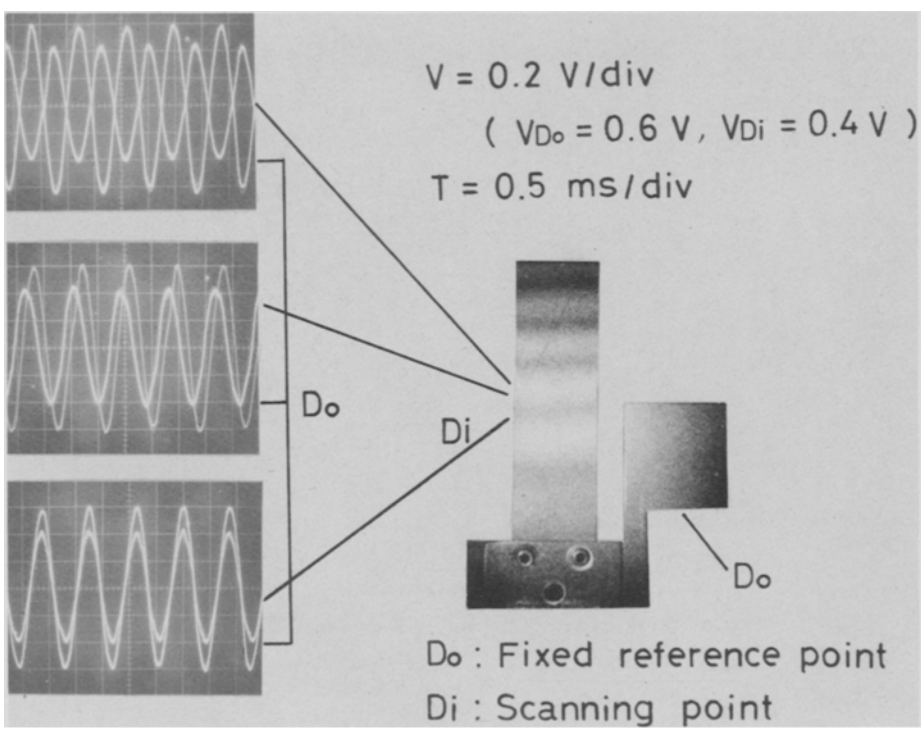

Fig. 3—Interference fringes on the cantilever and the corresponding beat frequency signals

is equal to the optical phase shift. This measuring method enables us to measure displacement with an accuracy independent of the measuring position in the reconstructed image. The phase difference of the two signals is measured by adjusting the phase shifter to give a straight line of the Lissajous's figures. To measure the distribution of displacement, photodetector Di is moved sequentially at intervals of 1 ±0.001 mm along the center line of the cantilever beam.

The measured phase difference $\Delta\phi$ is proportional to the displacement U of the object surface:

$$\Delta\phi = \frac{2}{\lambda} U (S_i - S_o) \tag{1}$$

where S_i, S_o are unit vectors in the illuminating and observing directions respectively and λ is the wavelength of the laser light. Since fringe-reading error is estimated within 1/360 of a fringe, the sensitivity of displacement is about 0.9×10^{-3} μm in this experimental optical system.

The optical resolution of the measurement is determined by the size of the slit aperture of the photodetector. In this experiment, the photodetector Di has a slit aperture of 5-mm length × 0.15-mm width parallel to the interference fringe in the image of unit magnification. Thus, the optical resolution of measurement is 0.15 mm along the long axis of the cantilever beam.

Experimental Results and Discussion

Measurements of the Apparent Young's Modulus by Heterodyne Holographic Interferometry

In the coordinate system with the origin at a loading point as shown in Fig. 4, apparent Young's modulus can be calculated from the following equation,[9]

$$E = \frac{W}{6IY}(-X^3 + 3L^2 X) \tag{2}$$

where I is the moment of inertia of cross section, W is the applied force, L is the length of beam and $Y = Y_o - Ya$ is defined as the deflection difference of the cantilever beam at a distance X from the loading point as shown in Fig. 4. In the neighborhood of the fixed end and loading end, nonuniform deformation is likely to be produced across the cantilever beam. For avoiding the error due to this effect, we use measured values far from both ends of the cantilever beam. The apparent Young's modulus is determined as the average of Young's modulus calculated by eq (2) at many points along the center line of the cantilever beam.

Experimental values of deflection for one loading of the cantilever beam are plotted in Fig. 5. The solid line shows the theoretical values calculated using the obtained apparent Young's modulus. The results agree quite well with the theoretical values.

The error of the calculated apparent Young's modulus is influenced by the measuring error of the W, I, Y, X and L. If we apply the law of error propagation to eq (2), the standard deviation SE of the error of apparent Young's modulus E is given in terms of the standard deviation SW, SI, SY, SX and SL as follows:[10]

$$\left(\frac{SE}{E}\right)^2 = \left(\frac{SW}{W}\right)^2 + \left(\frac{SI}{I}\right)^2 + \left(\frac{-3X^3+3L^2X}{-X^3+3L^2X}\right)^2 \left(\frac{SX}{X}\right)^2 + \left(\frac{6L^2X}{-X^3+3L^2X}\right)^2 \left(\frac{SL}{L}\right)^2 + \left(\frac{SY}{Y}\right)^2 \tag{3}$$

In the present experiment, the maximum error of measured values on the apparent Young's modulus SE/E can be estimated at about two percent by eq (3).

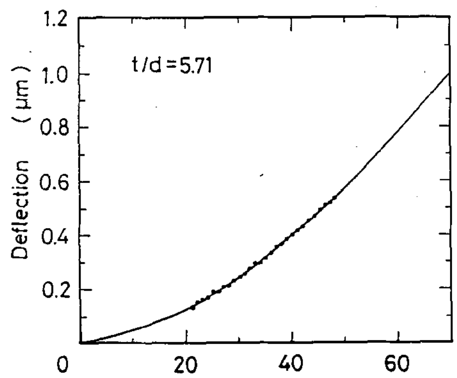

Fig. 5—An example of measured values of the deflection of a cantilever beam

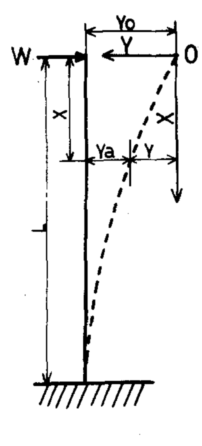

Fig. 4—Coordinate system

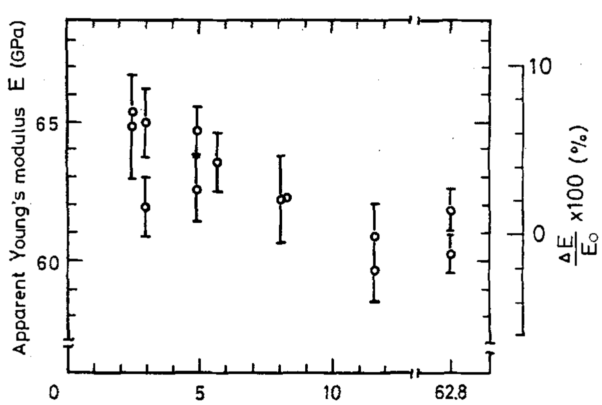

Fig. 6—Effect of t/d on the apparent Young's modulus E, $\Delta E/E_0$

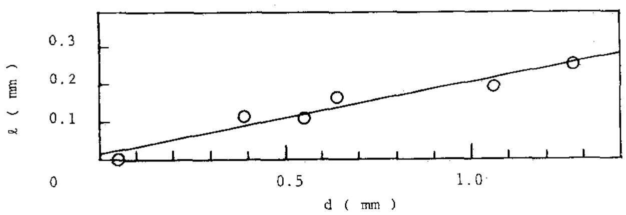

Fig. 7—Influence of grain size d on characteristic length from couple-stress theory

Dependence of the Apparent Young's Modulus on the Ratios of Thickness to Grain Size

Figure 6 shows the experimental results, which give the dependence of apparent Young's modulus on the ratios of thickness to grain size t/d. In this figure, the open circles indicate the average values measured at about 30 points along the cantilever beam and the vertical bars indicate the standard deviation σ of the experimental values. The vertical scale on the right of the graph indicates the percentage increase $\Delta E/E_o$ of the apparent Young's modulus E compared with that for the standard specimen annealed at 250°C for six hours, which is denoted by E_o. At the same time, the Young's modulus measured by the tension test with the same kinds of specimens as those for the bending test is always equal to E_o within the range of experimental error, independent of t/d. The apparent Young's modulus increases gradually when t/d is smaller than 10. The percentage increase goes up to almost seven percent for $t/d = 2.5$. The differences observed between average values of E for specimens with the same t/d are large for $t/d = 3$ and 5. This seems to be due to the effect of difference in shapes and orientations of grains for the specimens with large grain size. In the case of the small value of t/d, the standard deviation σ of the measured Young's modulus is relatively large compared with the standard specimen. This seems to be due to the fact that a small number of grains are included within the view field of the photodetector for the specimens with a small t/d.

Characteristic Length of Material in Couple-Stress Theory

Characteristic length is estimated by using the analytical results[11] of Koiter on the effect of beam thickness on flexural rigidity, based on the couple-stress theory. According to the Koiter theory, the percentage increase $\Delta E/E_o$ of the apparent Young's modulus can be written as follows:

$$\frac{\Delta E}{E_o} = \frac{24(1-\nu)\ell^2}{t^2} \tag{4}$$

where ν is the Poisson's ratio, t is the plate thickness and ℓ is the characteristic length of material. The characteristic length ℓ is calculated by eq (4) from the average value of the data of $\Delta E/E_o$ for each t/d shown in Fig. 6. The influence of grain size d on characteristic length ℓ from the couple-stress theory is shown in Fig. 7. In this figure, the experimental results are approximated by a straight line increasing by an incline of 1 in 5, using a least-squares method. Accordingly, we obtain a factor $\ell/d = 2$ from the straight line. This result is not consistent with the expectation that the characteristic length ℓ would be as large as or a little smaller than d.[3,4] This may be caused by the assumption of such constraint in the couple-stress theory that macrorotations and microrotations coincide.[12] It may be interesting to calculate ℓ by the theory of micropolar elasticity, which does not assume such constraint of rotational degrees of freedom of a rigid body or is close to the classical theory of elasticity.[13] However, we cannot calculate ℓ because an analytical micropolar elasticity solution of the bending of a cantilever beam with rectangular cross section has not been published.

Conclusion

Heterodyne holographic interferometry has been used to evaluate the effect of the grain size on the apparent Young's modulus by using the bending of a cantilever beam. It forms a part of the experimental verification of micromechanical theories of solids.

The distribution of deformation along the center line of the cantilever beam is measured with an optical resolution 0.15 mm, and measurement sensitivity of 0.9×10^{-3} μm by using heterodyne holographic interferometry. The apparent Young's modulus is found to increase by almost seven percent for the specimen with $t/d = 2.5$ compared to those with large t/d. From these results, heterodyne holographic interferometry has been proven to have the potential to become a useful experimental method for micromechanics.

The characteristic length ℓ is calculated by applying the analytical results of Koiter to the experimental results. However, the order of magnitude of ℓ is about one fifth of the grain size d.

Acknowledgment

We would like to thank Dr. K. Seo of Himeji Institute of Technology for valuable discussions.

References

1. *Mindlin, R.D. and Tiersten, H.F., "Effects of Couple Stress in Linear Elasticity," Arch. Rational Mech. Anal.,* **11**, *415-448 (1962).*
2. *Eringen, A.C., "Theory of Micropolar Elasticity," Fracture,* **11**, *621-729, ed. H. Liebowitz, Academic Press, New York and London (1968).*
3. *Shijive, J., "Note on Couple Stresses," J. Mech. and Phys. of Sol.,* **14**, *113-120 (1966).*
4. *Ellis, R.W. and Smith, C.W., "A Thin-plate Analysis and Experimental Evaluation of Couple Stress Effects,"* EXPERIMENTAL MECHANICS, *7, 372-380 (1967).*
5. *Gauthier, R.D. and Jahsman, W.E., "A Quest for Micropolar Elastic Constants," J. Appl. Mech.,* **42**, *369-374 (1975).*
6. *Yang, J.F.C, and Lakes, R.S., "Experimental Study of Micropolar and Couple Stress Elasticity in Compact Bone in Bending," J. Biomech.,* **18** *(2), 91-98 (1982).*
7. *Dändliker, R., "Heterodyne Holographic Interferometry," Progress in Optics, 1-84, ed. E. Wolf, North-Holland, Amsterdam (1980).*
8. *Kakunai, S., Iwata, K. and Nagata, R., "Experimental Conditions in Heterodyne Holographic Interferometry," under preparation for Opt. Com.*
9. *Timoshenko, S.P., "Strength of Materials," 147-150, D. Van Nostrand Company, Inc. (1951).*
10. *Ernest, O. Doebelin, "Measurement Systems," 59-68, Mc-Graw-Hill KoGakusha, LTD, Tokyo (1975).*
11. *Koiter, W.T., "Couple-Stresses in the Theory of Elasticity. II," Proc. Roy. Netherlands Academy of Sci., Series B, 30-44 (1964).*
12. *Eringen, A.C., "Mechanics of Micromorphic Continua," Mechanics of Generalized Continua, Proc. IUTAM, ed. E. Kröner, Freudenstadt, Stuttgart, Germany, 18-35, Springer-Verlag, Berlin (1968).*
13. *Cowin, S.C., "Stress Functions for Cosserat Elasticity," Int. J. Sol. Struc.,* **6**, *389-398 (1970).*