Fractional Moiré Strain Analysis Using Digital Imaging Techniques

by A.S. Voloshin, C.P. Burger, R.E. Rowlands and T.G. Richard

ABSTRACT—A new method for analyzing low-order moiré-fringe patterns of displacement fields is presented. This method adapts the techniques of half-fringe photoelasticity to moiré and extracts continuous displacement information in the regions between integral fringes. The effectiveness of the technique is illustrated with three examples: a uniform uniaxial field, a tapered specimen in tension, and a disk in diametral compression.

Introduction

Moiré patterns were first interpreted by Rayleigh more than a century ago in 1874. Developments since then have resulted in many applications of moiré fringes to static and dynamic displacement and strain analysis.[1,2] The conventional method for two-dimensional displacement measurement usually compares the distortions of either a stretchable grating bonded to a specimen, or a photographically reproduced grating on the surface of the specimen, with an undeformed reference grating. Under applied load the specimen grating deforms and optically interferes with the undeformed reference, 'master' or analyzing grating. Moiré fringes result from this interference. Each fringe corresponds to the locus of points of equal displacement in a direction normal to that of the reference ruling. If before deformation the two gratings had identical pitches, a displacement equal to the pitch produces one fringe. Thus, if there is no rigid-body displacement or rotation between the two gratings, a simple count of the obtained fringes gives the displacement

$$u = Np \tag{1}$$

where u = displacement normal to the moiré gratings, N = fringe count from any reference fringe, and p = pitch of the undeformed gratings.

If a Cartesian coordinate system is chosen such that u coincides with x, then consecutive differentials with respect to x result in the strain ϵ_x normal to the grating lines.

$$\epsilon_x = u_{,x} \tag{2}$$

A.S. Voloshin (SEM Member) is Associate Professor, Department of Mechanical Engineering and Mechanics, Lehigh University, Bethlehem, PA 18015. C.P. Burger (SEM Fellow) is Professor, Department of Engineering Science and Mechanics and Engineering Research Institute, Iowa State University, Ames, IA 50011. R.E. Rowlands (SEM Fellow) and T.G. Richard (SEM Member) are Professors, Department of Engineering Mechanics, University of Wisconsin-Madison, Madison, WI 53706.

Original manuscript submitted: July 31, 1984. Final manuscript received: March 25, 1986.

For a uniaxial field this means that the distance between successive fringes becomes the effective gage length and the pitch of the grating becomes the resolution. The value for ϵ_x is an average over the gage length. In the absence of rigid-body motions, this need for spatial rate of change of fringe density is the Achilles heel of the moiré method. For small strains it is not possible to obtain a sufficient number of moiré fringes for accurate differentiation.

Until now, attempts to reduce the seriousness of this limitation have concentrated on increasing the density of initial gratings or on increasing the number of obtained fringes from a specific grating by sharpening and/or multiplication of the moiré fringes.[1,3,4] Probably the most significant advance in this direction was Post's introduction of a practically useful method for generating and using very high density gratings (2400 ℓ/mm).[4,5] An alternative method which uses fractional moiré fringes was introduced and developed by Sciammarella in 1965.[6-9] The theory utilizes a light intensity versus displacement relationship and was applied to several examples by using a microdensitometer to record the light intensity. Unfortunately, this approach did not catch on, mainly because of the relative inconvenience of using the microdensitometer and the cumbersome way in which the data had to be processed. A renewed interest in using the fractional moiré system has been shown in recent work by Hunter.[10,11] Hunter, however, still utilizes fairly large numbers of full moiré fringes in the field.

Recent developments in the field of digital image analysis have led to significant progress in automated measurements of light intensity in photoelasticity over fairly large fields and with high accuracy.[12,13] When this general approach is modified to apply to moiré fields, it leads to a greatly improved capability for interpolating between relatively widely spaced moiré fringes. This paper describes how it can be done.

It can be shown[6] that for a grating with known pitch the relationship between displacement and light intensity is

$$I(x) = I_0 + I_1 \cos 2\pi\, \varrho(x) + I_2 \cos 6\pi\, \varrho(x) + \ldots \tag{3}$$

where [Fig. 1(b)]

I_0 = the average background intensity
I_1, I_2, etc. = the intensity amplitudes of corresponding harmonics [Figs. 1(a) and 1(b)]
$\varrho(x)$ = relative displacement of the two grids (master and deformed)

The relative displacement $\varrho(x)$ can be expressed through the displacement field $u(x)$. After substitution in eq (3) one gets

$$I(x) = I_0 + I_1 \cos \frac{2\pi u(x)}{p} + I_2 \cos \frac{6\pi u(x)}{p} + \ldots \tag{4}$$

where p is the pitch of the analyzing and original rulings.

This above analysis assumes an ideal optical system which is perfectly focused. Since such conditions are rarely met, the system's response to higher frequencies is significantly decreased. Therefore, if the aperture of the lens system is reduced and the system is slightly defocused, the higher harmonics of the moiré system are filtered out. It was shown that for gratings with a 20 lines per millimeter frequency, it is not necessary to take any precautions for the lens system.[7] Under commonly used conditions of observation of the moiré patterns, the condition of filtering the harmonics of order higher than one are usually met.[2,6,7] Consequently, the light-intensity distribution is related to the displacement field as

$$u(x) = \frac{p}{2\pi} \text{arc cos } [I(x) - I_0]/I_1 \tag{5}$$

The validity of eq (5) is extended to any displacement fields. But the equation is valid only over any one-half fringe of the moiré pattern, that is, any cycle of change in light intensity from dark to light or light to dark.

The authors have developed procedures that use a digital-image-analysis system to record and process fractional fringe orders in photoelastic models.[12,13] Those procedures have been modified to perform a similar task in moiré-fringe fields. The procedure utilizes eq (3) in a way analogous to the photoelastic method to determine fractional moiré-fringe orders. The result is equivalent to a moiré-fringe multiplication but is achieved without sophisticated and/or cumbersome optical setups. Thus, relatively coarse moiré rulings can be used to obtain high-resolution displacement data. The concept should be well suited to stress analysis of real problems in actual environments.

Method

A moiré pattern, which can be achieved by any conventional technique,[1,2] is viewed through the video camera of a digital-image-analysis sytem.[12] The image on the vidicon tube of the chosen field of view is divided spatially into 640 by 480 picture elements or pixels. The light intensity of each pixel is digitized to 8-bit resolution and is stored in the 'Z register' as a magnitude from 0 to 255. For one entire picture this process is completed in 33 μs. The characteristic response of the vidicon tube to light intensity is log linear rather than linear. The value, Z, is therefore related to the light intensity, I, in the following way:[13]

$$Z = K\, I^{\gamma} \tag{6}$$

where γ = the sensitivity of the tube, and K = a proportionality constant. Both γ and K are characteristics of the vidicon tube.

Information about displacement is usually required along particular lines rather than over an area. So, the required input data for a typical moiré analysis are the variations of light intensity along a specified line.

Since eq (5) requires light intensity values, the acquired Z values must first be transformed to corresponding intensity values according to eq (6) before displacements are calculated from eq (5). There is usually more than one half of an integer fringe value present in a moiré field such as Fig. 1.

A routine was therefore developed to account for increments involving more than just one half fringe. Since the minimum and maximum values of the recorded light intensity are located at positions defined by multiples of the half-fringe values (0.0, 0.5, 1.0, 1.5, 2.0, 2.5, etc.), each half-fringe region has to be treated separately. The resulting displacement is obtained by consecutively accumulating the partial displacements inside individual regions. Software has been developed to scan the recorded data until the first minimum is reached (which is in the black band). This position is assigned as fringe number zero, and the routine continues to scan until the next maximum is reached, which is assigned the fringe value of one half (light band). The displacements are calculated by using eq (5). Thus, the displacement at the zero fringe is zero and at the one half fringe is half pitch ($p/2$). The scan continues until the next minimum (black band) is reached; fringe value of one is assigned to this position. Substitution of the light-intensity data in this region into eq (5) results in a 'fictional' displacement field, which has to be shifted by one half of the pitch value. This procedure continues until the last maximum is reached. Thus, the information in the region from first minimum to last minimum is processed and is used for evaluation of the displacement field.

Results

To verify the proposed methodology, several cases with known displacement fields were studied. The first example was a pure polyethylene tensile specimen (250 ×

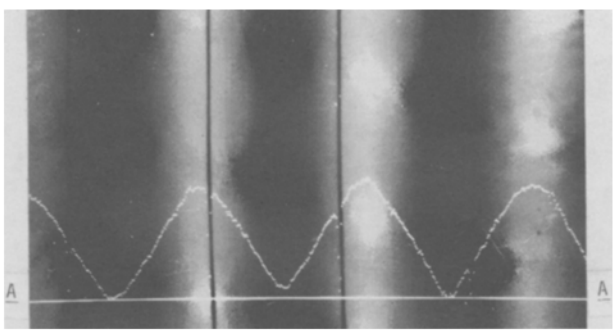


Fig. 1(a)—Moiré pattern for the pure tension (20 ℓ/mm). Black vertical lines are scale markers on the master grid. They are 6.4 mm apart. White wave pattern is a plot of Z values from intensity measurements along the horizontal line AA

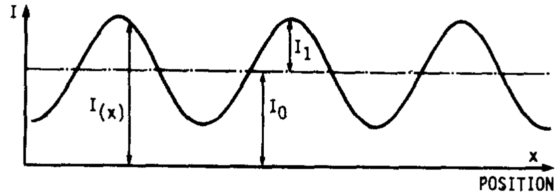


Fig. 1(b)—Definitions of intensity amplitudes

40 × 6 mm) which had a bondable moiré grid* with 20 lines per millimeter (500 ℓpi) applied to its surface. The specimen was loaded and the moiré-fringe pattern was observed through the vidicon videocamera. Figure 1(a) is a photograph of the image of the moiré pattern as it appeared on the video screen of the monitor. The dotted white curve is the plot of the light intensity along line AA. The solid white line is the baseline for the plot; that is, it is 'zero intensity', which is represented by the darkest point on the line AA. The vertical black lines are reference lines on the moiré grating 6.4 millimeters (0.25 in.) apart. The lines may be useful for scaling but were not used in the present approach. Since the monitor screen is curved, there is an apparent distortion of straight lines. This is a feature of the screen display and does not occur on the image as stored and manipulated in memory. Hard copies of the data are not distorted.

The traditional approach counts only three integer fringes on Fig. 1(a). They are spaced approximately 8.3 millimeters apart. This is insufficient if information is needed on the displacements in the regions between integer fringe positions.

The information acquired along AA was first smoothed by using a five-point sliding average routine. The smoothed data were converted to light intensity and were processed through eq (5) and the routine described above. This results in a plot of the real displacement field along any line of interest. The result is shown in Fig. 2, which is a photograph of the video screen of the image-analysis system. The 'flat spots' in the displacement curve are caused by the fact that the light intensity in these areas (extreme bright and dark) changes very little. The plot is a straight line for the displacement field along the AA baseline. The theoretical displacement for this case is, of course, a straight line.

As a second example, a tapered tensile specimen was prepared. A grating of 20 lines per millimeter (500 ℓpi) was photographically reproduced on its surface. The loading device and the specimen are shown in Fig. 3. This specimen was loaded axially to two different levels. The first load resulted in 10 full-fringe orders along the length of the specimen, while the second produced only three full fringes [Figs. 4(a) and (b)]. The apparent curvature in these figures is caused by the curvature of the video screen. The light-intensity distribution was acquired along the axis of symmetry of the specimen. The resultant light-intensity distribution is shown in Figs. 5(a) and (b). The solid white line is again the baseline.

The displacement field along the axis of symmetry of a tapered beam may be represented as

$$u = \frac{P}{(2Et)\tan\alpha} \cdot \ln(b + 2x\tan\alpha) \qquad (7)$$

where P = applied load, E = Young's modulus, t = thickness of the specimen, α = angle of the slope of the

**Type FTG 500 by Photolastic Div., Measurements Group, Inc., Raleigh, NC*

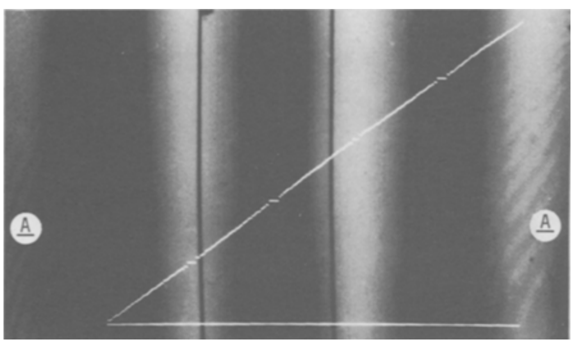


Fig. 2—Resulting displacement along line AA, as computed from Fig. 1

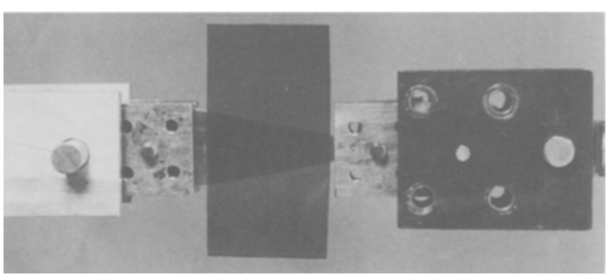
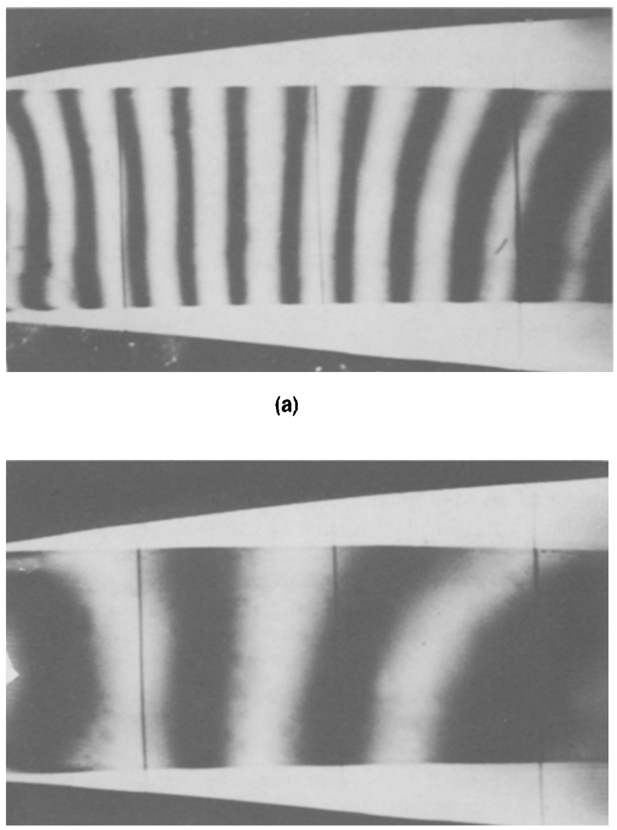


Fig. 3—Loading device and test setup for tapered specimen



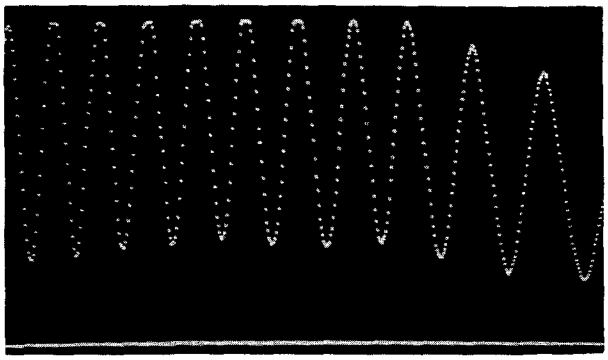
(a)

(b)

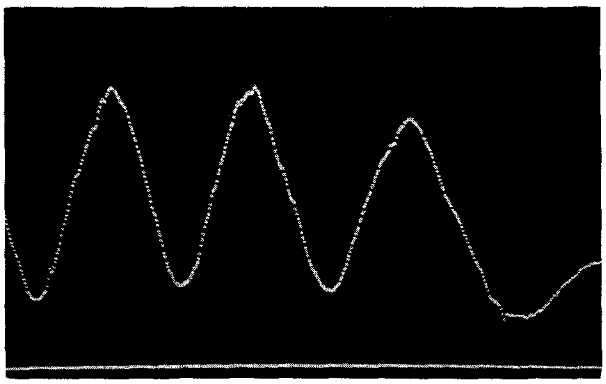
Fig. 4—Tapered tensile specimen at two different loads. The reference grid is a straight stock with parallel sides overlaid on the tapered specimen. Straight lines have apparent distortions on the curved screen of the video monitor. (a) Ten fringes. (b) Three fringes

specimen edge with respect to the axis of symmetry, b = tapered width of the specimen at the smallest cross section, and x = position along the axis of symmetry measured from b. The measured displacements for the two load levels are shown as photographs of the video screen in Figs. 6(a) and (b). When these results are redrawn and compared with the theoretical distribution, the match is within ten percent (Fig. 7).

The validity of the technique in biaxial fields was evaluated with a disk under diametral compression. The theoretical distribution of stress, strain, and displacement fields is well known.[14] The disk was prepared from polyethylene, and a grid of 20 lines per millimeter was applied by photographic transfer. The master grid had the same

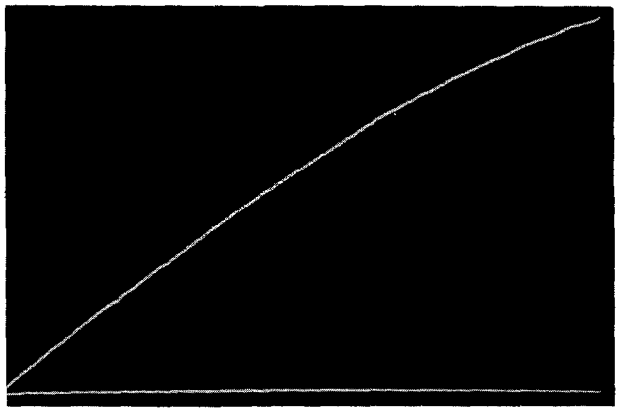


(a)

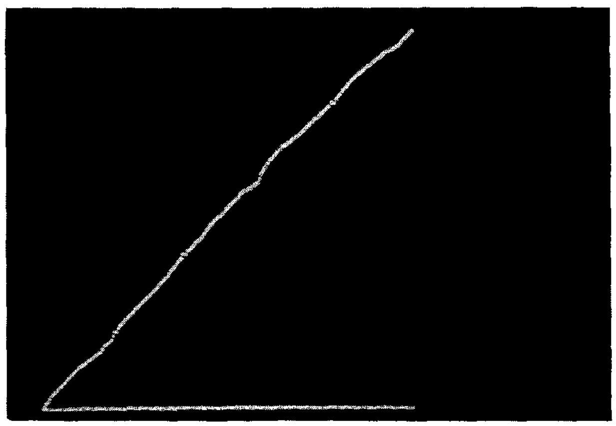


(b)

Fig. 5—Light intensity along the axis of symmetry from the tapered tensile specimen of Fig. 4. (a) Ten fringes in the field of view (high load). (b) Three fringes in the field of view (low load)



(a)



(b)

Fig. 6—Displacements for the tapered tensile specimen of Figs. 4 and 5. (a) Ten fringes. (b) Three fringes

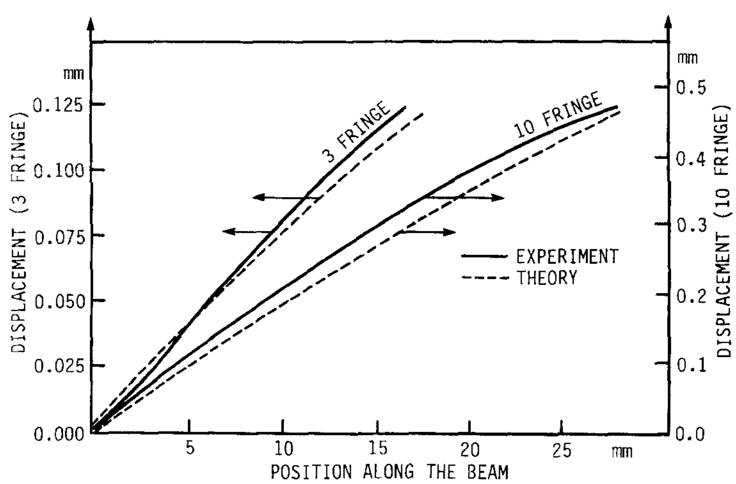


Fig. 7—Comparison between theoretical and measured distributions of displacement for the tapered specimen. (Left-hand scale is for three-fringe case; right-hand scale is for ten-fringe case)

density. The specimen was loaded in a direction normal to the grids; the resultant moiré-fringe pattern along the diameter is shown in Fig. 8. The information in the loading area (approximately 3 mm) was lost because the moiré grid peeled under the localized load. The light-intensity data of Fig. 8 were processed through the corresponding software to yield the displacement data shown as the solid line in Fig. 9. The theoretical displacement field is shown as the dotted line in Fig. 9.

Conclusions

These results show that digitally processed fractional moiré may be used effectively to evaluate displacement fields from low numbers of moiré fringes. This combination of half fringe with conventional moiré image analysis results in a simple, yet powerful tool for experimental stress analysis. Since the method does not need elaborate optical setups, it is applicable for use on actual structural components in a working environment. All that is needed on the site is a vidicon camera that can record the resulting moiré pattern on a video recorder. This information may be processed later as described in this paper.

Acknowledgments

This work was supported in part by equipment from NSF Grant No. MEA-8300376, by the Office of Research at Lehigh University, and by the Engineering Research Institute at Iowa State University.

References

1. *Durelli, A.J. and Parks, V.J., Moiré Analysis of Strain, Prentice-Hall, Inc., Englewood Cliffs, NJ (1970).*
2. *Theocaris, P.S., "Moiré Fringes in Strain Analysis," Pergamon Press, New York, 19-111 (1969).*
3. *Post, D., "Sharpening and Multiplication of Moiré Fringes,"* EXPERIMENTAL MECHANICS, *7 (4), 154-159 (1967).*
4. *Weissman, E.M. and Post, D., "Full Field Displacement and Strain Rosettes by Moiré Interferometry,"* EXPERIMENTAL MECHANICS, *22 (9), 324-328 (1982).*
5. *Post, D., "Full-Field Displacement and Strain Rosettes by Moiré Interferometry,"* EXPERIMENTAL MECHANICS, *22 (9), 324-328 (1982).*
6. *Sciammarella, C.A., "Basic Optical Law in the Interpretation of Moiré Patterns Applied to the Analysis of Strains—Part I,"* EXPERIMENTAL MECHANICS, *5 (5), 154-160 (1965).*
7. *Ross, B.E., Sciammarella, C.A. and Sturgen, D., "Basic Optical Law in the Interpretation of Moiré Patterns Applied to the Analysis of Strains—Part II,"* EXPERIMENTAL MECHANICS, *5 (6), 161-166 (1965).*
8. *Sciammarella, C.A., "Techniques of Fringe Interpolation in Moiré Patterns," Proc. 2nd Int. Cong. on Exp. Mech., Washington, D.C., 161-166 (1965).*
9. *Sciammarella, C.A. and Rowlands, R.E., "Numerical and Analog Techniques to Retrieve and Process Fringe Information," Proc. 5th Int. Cong. on Exp. Stress Analysis, Udine, Italy (1974).*
10. *Hunter, A.R. and Martinson, R.L., "Fringe-Intensity Interpretation Method for High Sensitivity Moiré Strain Analysis," Proc. 1982 Joint SESA-JSME Conf. on Exp. Mech., SEM, 52-57 (1982).*
11. *Hunter, A.R., Milly, T., Lapicz, D. and Bjeletich, J., "Application of Computerized Moiré Technique to Study Fracture on Filament Wound Composites," Proc. 1983 SESA Spring Conf. on Exp. Mech., SEM, 280-283 (1983).*
12. *Burger, C.P. and Voloshin, A.S., "A New Instrument for Whole Field Stress Analysis," ISA Trans., 22 (2), 85-95 (1983).*
13. *Voloshin, A.S. and Burger, C.P., "Half-Fringe Photoelasticity: A New Approach to Whole Field Stress Analysis,"* EXPERIMENTAL MECHANICS, *23 (3), 304-313 (1983).*
14. *Volterra, E. and Gaines, J., Advanced Strength of Materials, Prentice Hall, Inc., Englewood Cliffs, NJ, 177-186 (1971).*

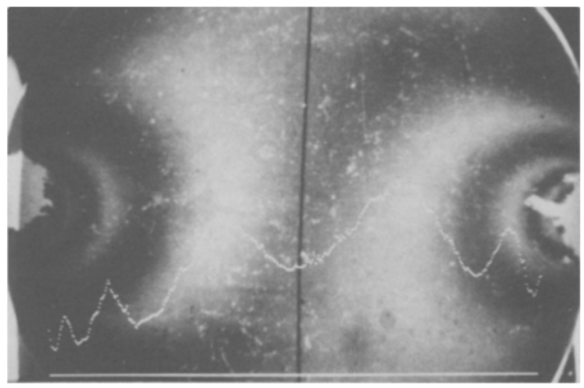


Fig. 8—The moiré pattern and superimposed light intensity along the horizontal diameter for a disk under axial load

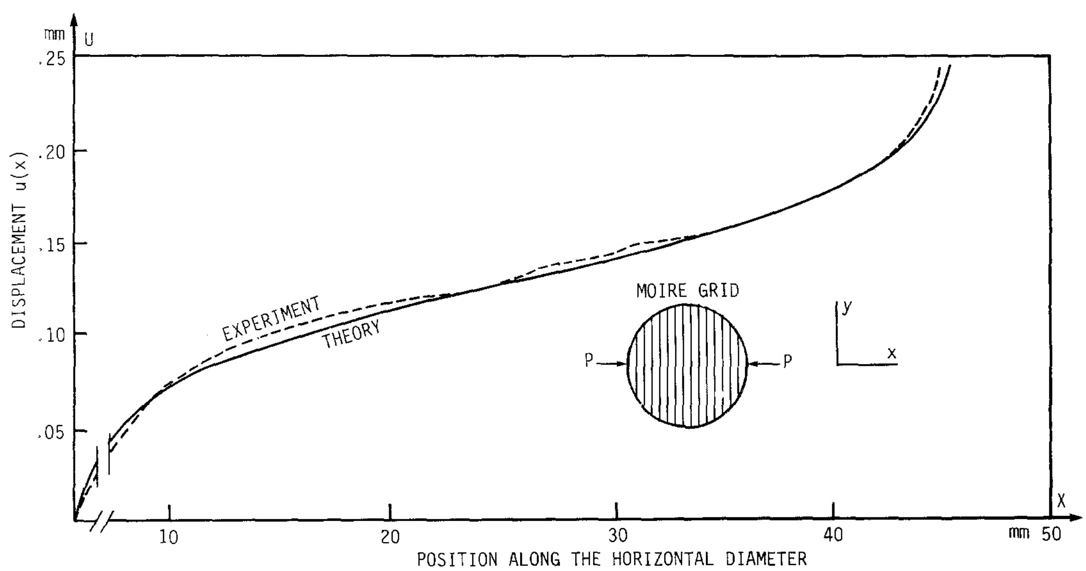


Fig. 9—Displacement field along the horizontal diameter