Dependence of Crack Acceleration on the Dynamic Stress-Intensity Factor in Polymers

by K. Takahashi and K. Arakawa

ABSTRACT—The caustics method in combination with high-speed photography was employed to study velocity effect on the dynamic-stress-intensity factor of fast cracks in polymethyl methacrylate and in Araldite D. The specimen geometry was so determined that both the accelerating and decelerating crack propagation occurred noticeably in one fracture event. Instantaneous crack velocity as well as its acceleration were expressed as a function of the crack length by using polynomials of the ninth order which were given on the basis of the least-square method. The results show that the dynamic-stress-intensity factor depends not only on the crack velocity but also on crack acceleration, and that the accelerating crack has a smaller value stress-intensity factor than the decelerating crack at the same velocity.

Introduction

The dependence of crack velocity ($\dot{a}$) on fracture toughness, or dynamic-stress-intensity factor (K_d), has been a subject in experimental[1-7] and theoretical[8-10] works on fracture studies. In the experimental works, the method of dynamic photoelasticity was used for the measurement of K_d for Homalite 100[1,3,4] and KTE epoxy.[1] By using the method of caustics[11,12] the measurement was conducted on Araldite B[2,7] and steel[6] with optical alignments in transmission and in reflection, respectively.

An array of strain gages on a specimen surface was also utilized for measurement in metals.[5] In all previous measurements the dependence of $\dot{a}$ on K_d has shown that K_d is an increasing function of $\dot{a}$. The result is confirmed theoretically,[8,10] although at a constant value of $\dot{a}$. Results of the experiments stated above, however, suggest that K_d is not an explicit function of $\dot{a}$. Dependence of the specimen configuration on the $\dot{a}$–K_d relation was reported with Araldite B.[7] However in the experiments on Homalite 100, little influence of the specimen configuration on the $\dot{a}$–K_d was shown.[1,4] Temperature naturally has an influence on the dependency.[5] As was suggested with KTE epoxy[1] and Araldite B,[2] fracture acceleration or deceleration should also produce an effect on K_d. As far as the dependence of acceleration is concerned, however, experimental results which explicitly indicate the effect in one fracture procedure have hardly been verified conclusively. This problem may be due to the difficulty involved in the experiment. The present study particularly focuses on the dependence of acceleration on the value of K_d in glassy polymers. Polymethyl methacrylate (PMMA) and an epoxy resin (Araldite D) were used as representatives of a thermoplastic and a thermosetting polymer, respectively.

Experimental Study

Experiments were performed on single-edge-cracked, pin-loading, tensile specimens of PMMA (Acrylite S-001) and epoxy (room-temperature hardened Araldite D) at room temperature. Their dimensions were 120 mm in length, 150 mm in width and 5 mm in thickness. The specimen geometry, shown in Fig. 1, was so determined that both acceleration and deceleration stages occurred noticeably during each fracture event. The axis of pin loading was chosen at the location which is 10- to 30-mm apart internally from the precrack-tip position. By varying this distance, values of the fracture velocity in the initial course and at its maximum were varied. The single-edge-crack was carefully generated by a momentum-controlled chisel impact onto a premachined saw cut on a specimen edge. The specimen was loaded in a tensile machine with a cross-head speed of 1 mm/min under room temperature. A silver paste line on the specimen surface was used to trigger the spark-light circuit. The experimental setup is illustrated in Fig. 2(a). Figure 2(b) shows an in-laboratory constructed Cranz-Schardin camera.[13] The camera has been modified in the frame number from the original 24 to the present 30. The light duration of each spark shot is about 450 ns at its half-maximum of light intensity. The framing rate manually set on a digital-circuit console ranges from 1 to 10^6 frames/s. The precise identification of the flashing time at successive spark shots was performed with a storage-type oscilloscope which received electric pulses from a PIN-Si photodiode. The camera takes photographs with excellent spatial linearity.[13] The photograph film under a photomagnifier gives caustics diameter and crack-tip position for evaluation of K_d[11,12] and the crack length a at respective instances. The value

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of K_d was obtained from the equation

$$[2(2\pi)^{1/2} / 3z_0 dc\mu^{3/2}](\phi_\mu/3.17)^{5/2}$$

where ϕ_μ is the caustic diameter transverse to the crack direction, z_0 the distance between the specimen and the image plane, d the specimen thickness, μ a convergency factor for incident light and c a stress-optical constant. Dynamic-stress-optical constants of 0.49×10^{-10} m²/N and 0.70×10^{-10} m²/N were used for PMMA and Araldite D, respectively. The former was cited from literature.[11] The latter, on the other hand, was obtained through an experiment. SEN-type Araldite D specimens of the same thickness were uniaxially elongated at a strain rate of 1.1×10^{-2}/s in the caustics optical system. On the assumption that values of K_I given by both the conventional fracture-mechanics formula[14] and the caustics method are equal, the value of the optical constant was obtained. It is supposed, therefore, that this value is kept constant over the higher strain-rate range involved in the present fast fracture.

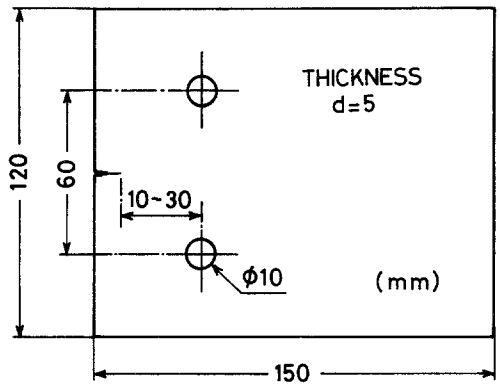
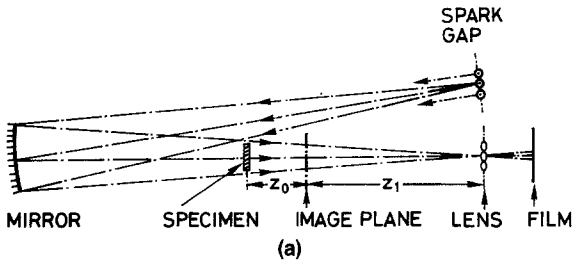


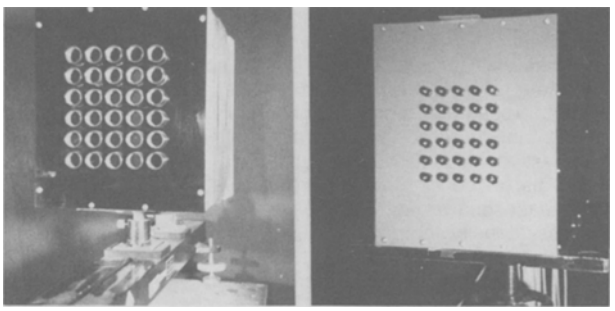
Fig. 1—Geometry of the employed SEN specimen

Evaluation of K_d, $\dot{a}$ and $\ddot{a}$ as a Function of a

If one obtains crack velocity $\dot{a}$ and acceleration $\ddot{a}$ simply from the first and second time derivatives of Δa, which is obtained from successive pictures taken on a film, considerable scatter in data is inevitably caused. The scatter is due mostly to errors arising from visual identifica-



(a)



(b)

Fig. 2—(a) Experimental set-up. (z_o = 1.5 m, z_1 = 1.2 m). (b) In-laboratory constructed Cranz-Schardin camera

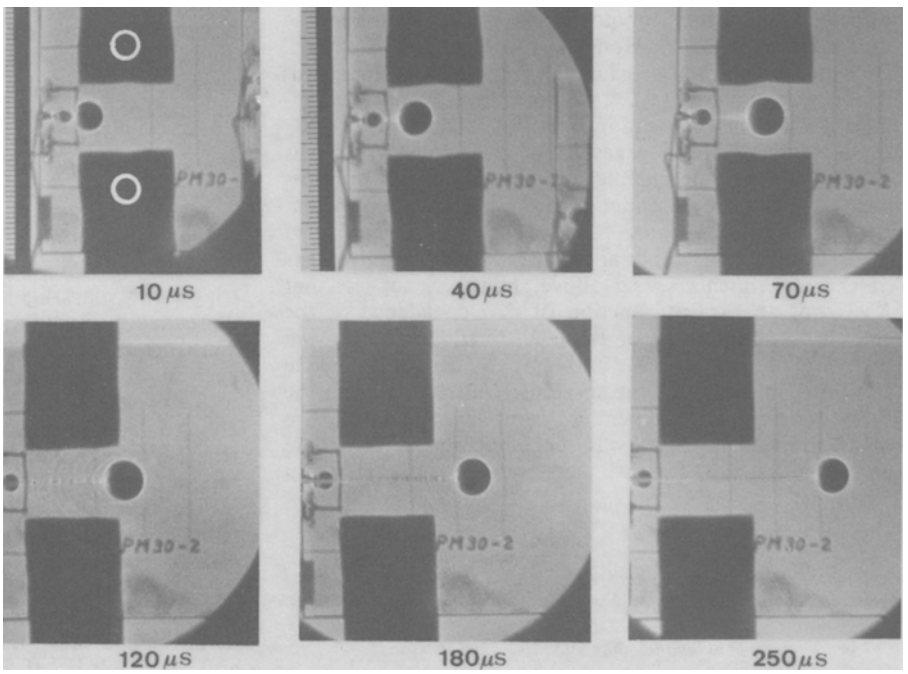


Fig. 3—Typical caustic patterns obtained from a propagating crack in PMMA

tion of the crack-tip position on film. If the number of pictures taken is not sufficient, moreover, the scatter becomes larger, which makes it difficult to see the effect of $\dot{a}$ and even more so of $\ddot{a}$. It seems that this is the reason why the effect of acceleration on K_d has been little studied experimentally. In order to solve this problem two attempts were made in the present experiment. First the maximum number of frames for photography was increased to 30 as stated above. Second a method of polynomial approximation was introduced for evaluation of K_d, $\dot{a}$ and $\ddot{a}$ as a function of a. Values of a and K_d obtained successively from high-speed photography were expressed with polynomials of the ninth order: $\sum_{n=0}^{9} a_n t^n$ and $\sum_{n=0}^{9} a'_n t^n$, respectively. On the basis of the least-square method, computer calculations gave values of the coefficients a_n and a'_n. The first and second time-derivatives of $a(t)$ thus obtained yielded $\dot{a}(t)$ and $\ddot{a}(t)$, respectively. The $a(t)$, $\dot{a}(t)$, $\ddot{a}(t)$ and $K_d(t)$ were combined to give $\dot{a}(a)$, $\ddot{a}(a)$ and $K_d(a)$ by computer calculations, which elucidated the effect of acceleration on K_d as stated below.

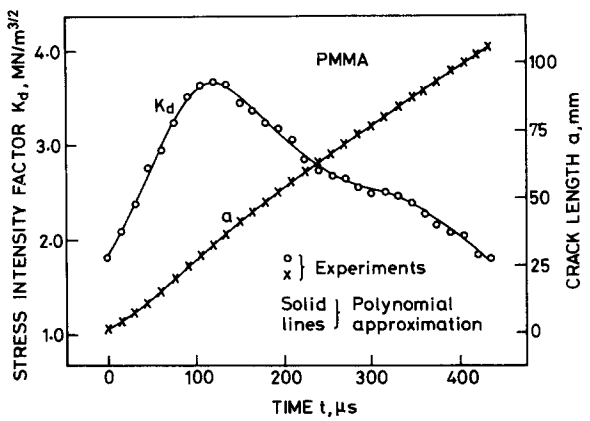


Fig. 4—Stress-intensity factor K_d and crack length a for PMMA as a function of time t

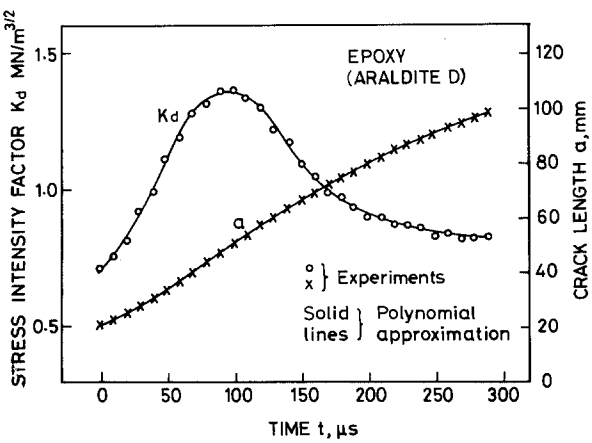


Fig. 5—Stress-intensity factor K_d and crack length a for epoxy as a function of time t

Results and Discussion

Examples of caustics pictures selected from each series of 30 frames for PMMA are shown in Fig. 3. It should be noted that the size of caustics showed an increase in the initial course and then decreased, indicating the similar change of K_d. Figures 4 and 5 show values of K_d and a experimentally obtained as a function of time for PMMA and epoxy, respectively. From the change of the crack length a with time, t, one may see that acceleration and deceleration were included in the initial and later stages. Solid curves in both figures represent the fitted curves obtained through the polynomial approximation. The agreement between the experimental data and the fitted curves is satisfactory. By using the equations of $K_d(t)$ and $a(t)$, computer calculations yielded the equations of K_d, $\dot{a}$ and $\ddot{a}$ as a function of a as shown in Figs. 6 and 7. It should be noted that in both figures values of a giving the maximum of $\dot{a}(a)$ and of $K_d(a)$ differ considerably. The maximum of $\dot{a}(a)$ gave rise earlier than that of $K_d(a)$. This means that $K_d(a)$ was still increasing when $\dot{a}(a)$ was decreasing. In Fig. 6 considerably different values of K_d (A′ and B′) were obtained for the same crack velocities (see points A and B) in the acceleration (A″) and deceleration (B″) stage, respectively. The difference was about 30 percent in this case. Results of Figs. 6 and 7 indicate that K_d is not a unique function of $\dot{a}$. The dependence of $\dot{a}$ on K_d is seen more distinctly in Figs. 8 and 9, where

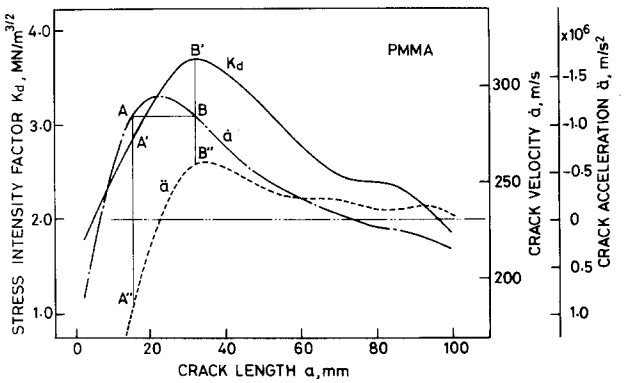


Fig. 6—Stress-intensity factor K_d, crack velocity $\dot{a}$ and crack acceleration $\ddot{a}$ for PMMA as a function of crack length a

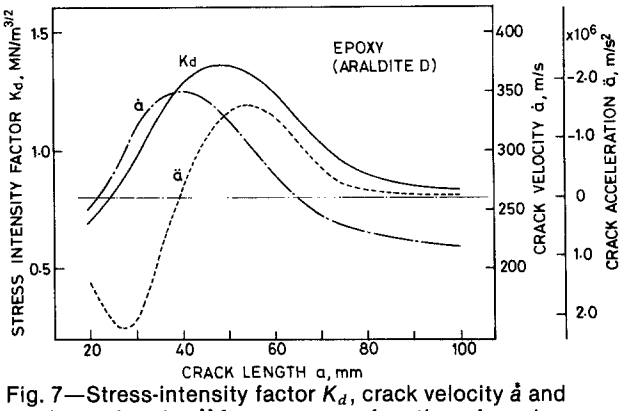


Fig. 7—Stress-intensity factor K_d, crack velocity $\dot{a}$ and crack acceleration $\ddot{a}$ for epoxy as a function of crack length a

results of specimens with various peak velocities are presented. The directions of the arrows in the figures correspond to the path of the fracture event in each fracture experiment. One can draw several conclusions from the results shown in Figs. 8 and 9. First, the decelerating crack has larger values of K_d than the accelerating one for the same velocities, as stated above, which results in an 'inverse hysteresis' curve on the $\dot{a}$–K_d diagram. Second, the increase of K_d in the deceleration area is larger for samples with higher peak velocities. This tendency is more clearly seen in the results for the epoxy specimens shown in Fig. 9, where the highest peak velocity shown is very close to the maximum velocity attainable. Upon reaching a peak velocity of about 380 m/s, as shown in Fig. 9, considerable K_d increase, followed by crack branching, occurred. The critical value of K_d at the branching was about 4.0 MN/m$^{3/2}$, which is about six times the value at crack arrest, 0.64 MN/m$^{3/2}$. These results suggest that crack velocity is not the only influential parameter for branching and that K_d may be the controling factor. Similar results were obtained by Kobayashi and Dally[1] in Homalite 100 which exhibited crack branching in the course of the K_d increase with the crack velocity staying almost unchanged. Third, the points corresponding to the peak velocities, indicated by open circles in Figs. 8 and 9, form the curve $K_d(\ddot{a} = 0) - \dot{a}$ in both figures. Each of these curves may be regarded as a characteristic one for each material in the sense that no acceleration effect is associated. There is a considerable difference between the characteristic curves in Figs. 8 and 9. While the K_d–$\dot{a}$ relation for PMMA could be represented by a nearly straight line, that for epoxy exhibited a somewhat parabolic curve, i.e., K_d increased faster than before towards the maximum of $\dot{a}$. We believe this difference mainly arises from the fact that PMMA specimens were not tested at about 700 m/s, the crack velocities high enough to yield an enhanced increase in K_d in the velocity range finally limited by the branching of cracks.

Other 'equi-acceleration curves', formed by connecting $\dot{a}$–K_d data at equal crack accelerations, are shown in Figs. 10 and 11. All curves are described plainly. The larger the acceleration or the deceleration, the further the corresponding curve lies from the characteristic curve. These features indicate that the offset of the value of K_d from the characteristic curve (ΔK_d) is a function of $\ddot{a}$, although the relationship is not simply linear and depends on $\dot{a}$, particularly for Araldite D.

Some dynamic factors should be discussed concerning the measurement of K_d in the present experiment. First, the stress-optical constant c depends on the strain rate $\dot{\epsilon}$ in general. In a reported measurement on PMMA, for example, the value of c varied by 0.067×10^{-10} m^2/N for one decade change of $\dot{\epsilon}$.[16] However, since the crack-velocity change involved in Fig. 8 is only about 0.5 decade, we may neglect the variation of c in the experiment as a first approximation. Similar measurement on $c(\dot{\epsilon})$ in Araldite D showed that the stress-optical constant of this material is less strain-rate dependent than that of PMMA. Second, a dynamic correction based on the dynamic-stress-distribution change at the propagating crack tip should be made for K_d, which has been evaluated by the static caustic formula. According to Beinert and Kalthoff,[17] a dynamic correction factor of 0.9 is needed for K_d at a crack velocity of 500 m/s in PMMA. This dynamic modification, however, only results in a slight decrease in the inclination of the curves in Fig. 8 and also in Fig. 9, and has no significant decisive effect on the behavior of K_d in the $\dot{a}$–K_d diagram.

Kalthoff, Beinert and Winkler[7] report on the effect of specimen geometry on the $\dot{a}$–K_d relationship for Araldite B. They show that values of K_d for DCB (double-cantilever-beam) specimens are larger than those for SEN (single-edge-notched) specimens at the same crack velocity. An explanation of the effect of specimen geometry may be made in the light of the present experimental results as follows. Usually specimens of SEN type show higher crack acceleration than those of DCB or CT type.[18] Figure 12 illustrates typical curves of $\dot{a}/\dot{a}_0$ for brittle polymers, where $\dot{a}_0$ denotes the initial crack velocity, i.e., the velocity immediately after a transition occurred from a low to a fast mode of crack propagation.[18] If the averaged crack acceleration is greater for SEN-type than for DCB- or CT-type specimens, it is more probable for the DCB- and CT-type specimens that measurements have led to comparatively large values of K_d.

As far as the dissimilarity of K_d for the accelerating and decelerating crack is concerned, Kobayashi and Dally[1] report on formation of two branches in the K_d–$\dot{a}$ relation of KTE epoxy. According to Kobayashi, the lower energy branch is associated with decelerating cracks, and the higher energy one is associated with both accelerating and decelerating cracks. It should be noted that the dependence of acceleration obtained in the present experiments are opposite to those of Kobayashi. Molecular structure may

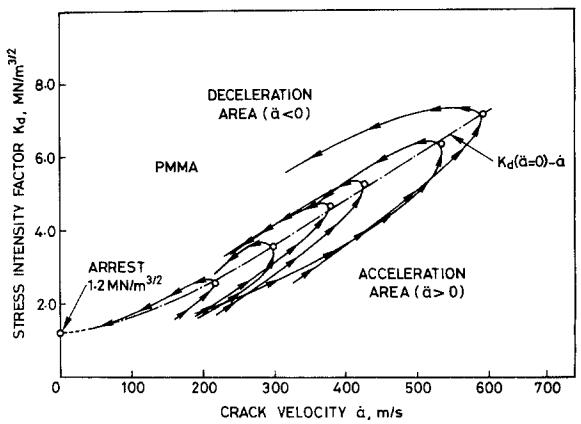


Fig. 8—Stress-intensity factor K_d for PMMA as a function of $\dot{a}$

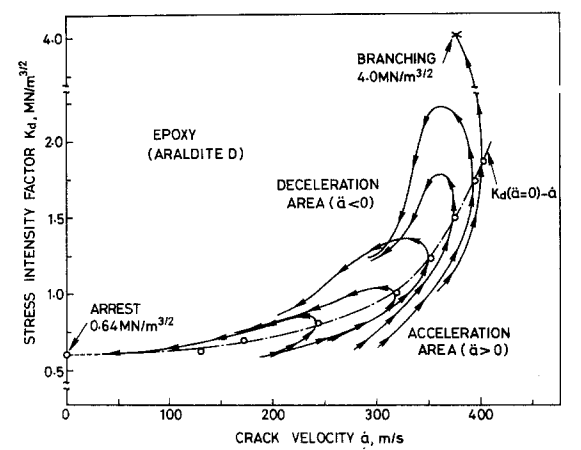


Fig. 9—Stress-intensity factor K_d for epoxy as a function of $\dot{a}$

have little relation with the cause of the dependence in the light that similar results were obtained with both thermoplastic and thermosetting polymers. In a slow-crack region the dependency of $\dot{a}$ on K_d is attributed to the dependence of the modulus E on time t involved in the fracture phenomena.[15] It is hard to apply this concept to the present fast fracture, because the change of K_d obtained is extraordinarily large as compared with the small change of $\dot{a}$.

It has been assumed by many investigators that an accelerating crack should have a larger value of K_d than a decelerating crack, because cracks require more energy during the acceleration than during the deceleration stage. However, the present experiments indicate that this speculation is not necessarily plausible. In order to elucidate the physical meaning of the present results, further research programs are necessary. One should be a fractographic study, where features on the fracture surfaces may be correlated with $\dot{a}$, K_d and other parameters. Such a study now being carried out has provided findings which support the present results. Another research study to be done is a theoretical one based on the dynamic energy balance at the tip of the propagating crack. It is hoped that this study will provide a clear understanding of the physical meaning.

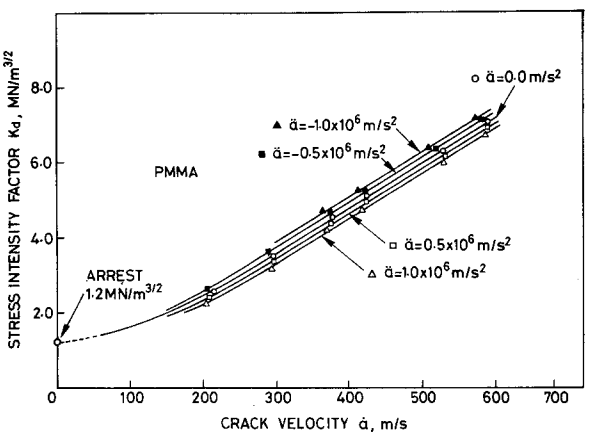


Fig. 10—Equi-acceleration curves for PMMA as a function of $\dot{a}$

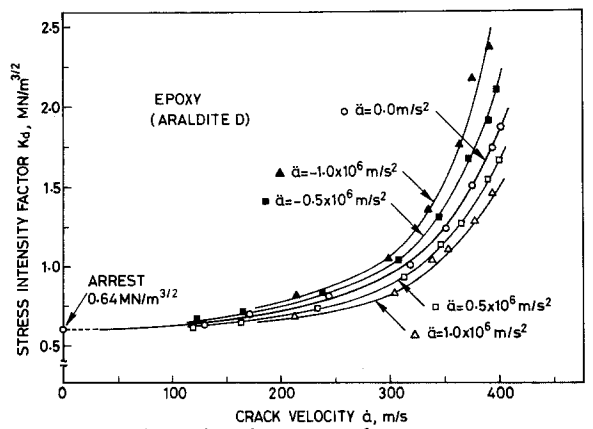


Fig. 11—Equi-acceleration curves for epoxy as a function of $\dot{a}$

Conclusions

(1) Eccentric-pin-loading, single-edge-cracked specimens have both acceleration and deceleration states in their fracture-propagation procedure. (2) Use of polynomial expressions makes it possible to see the effect of acceleration on K_d. (3) Decelerating cracks in PMMA as well as in epoxy have a larger value of K_d than those of accelerating cracks at the same crack velocity. The difference becomes much larger as the peak velocity approaches the branching velocity.

Acknowledgments

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References

1. Kobayashi, T. and Dally, J.W., "Relation between Crack Velocity and the Stress Intensity Factor in Birefringent Polymers," Fast Fracture and Crack Arrest, ASTM STP 627, 257-273 (July 1977).

2. Kalthoff, J.F., Beinert, J. and Winkler, S., "Measurements of Dynamic Stress Intensity Factors for Fast Running and Arresting Cracks in Double-Cantilever-Beam Specimens," Fast Fracture and Crack Arrest, ASTM STP 627, 161-175 (July 1977).

3. Kobayashi, A.S. and Mall, S., "Dynamic Fracture Toughness of Homalite-100," EXPERIMENTAL MECHANICS, **18** *(1), 11-18 (Jan. 1978).*

4. Irwin, G.R., Dally, J.W., Kobayashi, T., Fourney, W.L., Etheridge, M.J. and Rossmanith, H.P., "On the Determination of the ȧ-K Relationship for Birefringent Polymers," EXPERIMENTAL MECHANICS, **19** *(4), 121-128 (April 1979).*

5. Kanazawa, T., Machida, S., Teramoto, T. and Yoshinari, H., "Study on Fast Fracture and Crack Arrest," EXPERIMENTAL MECHANICS, **21** *(2), 78-88 (Feb. 1981).*

6. Shockey, D.A., Kalthoff, J.F., Klemm, W. and Winkler, S., "Simultaneous Measurements of Stress Intensity for Fast-running Cracks in Steel," EXPERIMENTAL MECHANICS, **23** *(2), 140-145 (June 1983).*

7. Kalthoff, J.K., Beinert, J. and Winkler, S., "Analysis of Fast Running and Arresting Cracks by the Shadow Optical Method of Caustics," Proc. IUTAM Symp. on Opt. Meth. in Mech. of Solids, ed. A. Lagarde, Sijthoff & Noordhoff, 497-508 (1981).

8. Sih, G.C., "Inelastic Behavior of Solids," ed. M.F. Kanninen, F.A. William, A.R. Rosenfield and I.J. Robert, McGraw-Hill, 607-639 (1970).

9. Freund, L.B., "Crack Propagation in an Elastic Solid Subjected to General Loading - II Non-Uniform Rate of Extension," J. Mech. Phys. Solids, **20**, *141-152 (1972).*

10. Tsai, B.V., "Propagation of a Brittle Crack at Constant and Accelerating Speeds," Int. J. Solids Struct., **9**, *625-642 (1973).*

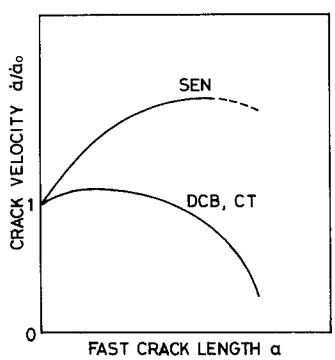

Fig. 12—Typical change of $\dot{a}/\dot{a}_o$ as a function of a for the SEN-, DCB- and CT-type specimens

11. Manogg, P., "Andwendung der Schattenoptik zur Untersuchung des Zerreissvorgangs von Platten," Diss., Univ. Freiburg, Germany (1964).
12. Theocaris, P.S., "Local Yielding around a Crack-Tip in Plexiglas," J. Appl. Mech., **37**, *409-415 (1970).*
13. Takahashi, K., "In-Laboratory Construction of a Pulse-Control Type Multi-Spark Camera," J. Jap. Soc. Aero. and Space Sci., **32** *(363), 447-452 (1981).*
14. Brown, W.F., Jr. and Srawley, J.E., "Plane Strain Crack Toughness Testing of High Strength Metallic Materials," ASTM STP 410 (1966).
15. Marshall, G.P., Coutts, L.H. and Williams, J.G., "Temperature Effects in the Fracture of PMMA," J. Mat. Sci., **9**, *1409-1419 (1974).*
16. Takahashi, K. and Sakurada, Y., unpublished data.
17. Beinert, J. and Kalthoff, J.F., "Experimental Determination of Dynamic Stress Intensity Factors by Shadow Patterns," Mechanics of Fracture, VII, ed. G.C. Sih, Nijhoff Publishers, Hague (1981).
18. Takahashi, K., Matsushige, K. and Sakurada, Y., "Precise Evaluation of Fracture Velocities at the Slow-to-Fast Transition," J. Mat. Sci., **18**, *4026-4034 (1984).*

Discussion

by Takao Kobayashi

The authors are congratulated for their excellent experimental work. Their results, together with the results obtained by Kalthoff *et al.* reveal clearly a very interesting aspect of a caustic method of stress-intensity-factor determination.

This discussant believes that the geometric effect on the caustic method of stress-intensity-factor determination is real and does not totally agree with the authors' view on the affect of crack acceleration and deceleration on the stress-intensity factor. The authors explain the increase in the stress-intensity factor during a crack propagation in the latter part of their specimens from the viewpoint of crack deceleration. Furthermore, they attempt to explain the finding of Kalthoff *et al.* relating to the difference in the relationships of the stress-intensity factor (K) and crack velocity ($\dot{a}$) for a single-edge-notch (SEN) specimen and for a double-cantilever-beam (DCB) specimen or a compact-tension (CT) specimen from the same viewpoint of crack acceleration and deceleration. However, these results can also be explained consistently from the geometric effect on the caustic method of stress-intensity-factor determination.

First of all, it should be noted that the specimen employed by the authors has SEN/DCB combined characteristics. The specimen has strong SEN characteristics (i.e., a rising stress-intensity field) during the first part of crack propagation. However, as the crack passes beyond the loading pin line, it takes on DCB-specimen characteristics. Thus the increase in the stress-intensity factor observed by the authors in the latter part of propagation may represent the effect of specimen geometry on the stress-intensity factor determined by the caustic method.

Secondly, the results obtained by Kalthoff *et al.*, indicating a lower K versus $\dot{a}$ curve for SEN specimens than that for RDCB specimens, were generated during decelerating crack propagation. The decelerating crack propagation in the SEN specimens was produced by longitudinal wedge loading. Kalthoff *et al.* also report a result from a DCB/SEN combination specimen which shows that, when the crack was propagating in the DCB section, the stress-intensity factor followed a higher stress intensity K-$\dot{a}$ curve for RDCB specimens; however, as the crack moved into the SEN section, the stress-intensity factor dropped to follow the SEN K-$\dot{a}$ curve. In this specimen the crack was monotonically decelerating from the start. These results obtained by Kalthoff *et al.* thus contradict the conclusion drawn by the authors.

This discussant believes that the results obtained by the authors and also by Kalthoff *et al.* indicate consistently the geometric effect on the caustic method of stress-intensity determination. It is, thus, of particular interest to investigate why the caustic method shows such strong dependence on the geometries of the specimen.

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Authors' Closure

According to our view, the geometric effect is no more than a phenomenological term, i.e., the term seems to lack a physically definite meaning. If the effect has some connection with a dynamic change in stress distribution or in strain rate at a crack tip, stress-intensity determination should be influenced irrespectively of the methods employed, i.e., the caustic method cannot be exceptional. We may agree with Dr. Kobayashi if the geometric effect operates solely in the case when the caustic method is used for determining stress-intensity factors. At present, however, we are still unaware of any physical grounds for this presumption.

We have not been informed in detail about the crack-velocity change observed by Kalthoff *et al.* Hence it is hard for us to discuss the present problem further. However we believe that the following two points are worthy of attention. First, it is suggested from Figs. 10 and 11 that, even if decelerating crack propagation is dominant in the determination of K_d for specimens of SEN type, values of K_d determined should be on the average lower for SEN specimens than for DCB- or CT-type specimens, as far as the magnitude of deceleration is on the average lower for SEN specimens than for DCB or CT specimens. Second, Figs. 8 and 9 are in qualitative agreement with the results of an examination carried out to study fracture-surface roughness of PMMA and epoxy; the surface was observed to be more coarse in crack-deceleration than in crack-acceleration regions for the same crack velocity.

In this paper we attempt to explain one aspect of the K_d-$\dot{a}$ relationship, particularly taking the $\ddot{a}$ dependence of K_d into consideration. For the purpose the specific specimen geometry was employed. We agree with Dr. Kobayashi in the point that further studies are needed to reach a clear understanding of the geometric effect. Also, we are indebted to him for providing us with this opportunity for discussion.