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Hyperintensional Logic

It is well known that it seems possible to have a situation in which there are two propositions p and q which are logically equivalent and yet are such that a person may believe the one but not the other. If we regard a proposition as a set of possible worlds then two logically equivalent propositions will be identical, and so if 'x believes that' is a genuine sentential functor, the situation described in the opening sentence could not arise. I call this the paradox of hyperintensional contexts. Hyperintensional contexts are simply contexts which do not respect logical equivalence.

There are a number of different replies one can make to the paradox of hyperintensional contexts. One is to say that propositions are not sets of worlds, although they determine sets of worlds. Another is to say that 'x believes that' is not a genuine sentential functor but, perhaps, is more like a predicate of sentences. In order to make use of what is of value in both these approaches I shall, in this paper, take an account of meaning advanced by David Lewis in [15] and apply it to the problem of hyperintensional contexts. Essentially Lewis' contention is that the meaning of a sentence is an entity which reflects the structure of the sentence which expresses it.

1. The indeterminacy problem

Suppose that we were to give up the idea that a proposition is a set of possible worlds. We might, if we wish, say that nevertheless a proposition *determines* a set of possible worlds. Let us call this set of worlds the *intension* of the proposition. We might have a function I, such that, where p is a proposition, $I(p)$ is its intension. The resolution of the paradox of hyperintensional contexts is immediate; for clearly we can have $I(p) = I(q)$ without $p = q$.

The problem about this solution is that the usual truth-conditional semantics for such functors as the truth functors and the modal functors do not determine a unique semantics for these operators in hyperintensional contexts. Consider, for example, negation. If propositions are sets of possible worlds then the semantics for negation is simple. Negation is represented by the operator $\omega_{\sim}$ such that, where $\mathbf{W}$ is the set of all possible worlds and where $a \subseteq \mathbf{W}$, then $\omega_{\sim}(a) = a'$. (i.e. the complement of a with respect to $\mathbf{W}$). However in hyperintensional contexts this will not do, since we may have a person so logically blind that he may believe p without believing $\sim\sim p$. (If this sounds far fetched we need only introduce other functors and produce more complicated logical equivalences.) The truth table for negation acts here as a constraint on possible meanings foi negation, in that $I(p) = (I(\sim p))'$, but it does not establish a unique function for $\sim$. [7, p. 55f]

A similar comment applies to the introduction of 'nonclassical worlds' as in [5] and [6]. In those articles worlds are divided into classical and non-classical. A proposition is a set of worlds, and where $\mathbf{C}$ is the set of classical worlds p and q are logically equivalent iff $p \cap \mathbf{C} = q \cap \mathbf{C}$, i.e. iff they are true in the same *classical* worlds.[1] We define negation in such a way that $p \cap \mathbf{C} = \sim\sim p \cap \mathbf{C}$ but otherwise impose no restrictions. (This way of doing it raises of course another problem. If an operator can be defined which behaves like a negation in the *classical* worlds why cannot it so behave in all worlds? But if it does then we are back where we began because in that case $\sim\sim p = p$.)

The fact is that in all these approaches the ordinary truth-conditional semantics is insufficient to determine the semantics of the operator in hyperintensional contexts. It may be, of course, that this indeterminacy is a desirable thing and that it shews the inadequacy of truth-conditional semantics. (Perhaps this is part of what Roman Suszko is speaking about in [23][2]). Perhaps we should base our semantics on something which has no direct connection with the notion of truth, perhaps on something more avowedly mental, like beliefs (or equivalence classes of beliefs under some kind of synonymy relation[3]). Instead of a connection

[1] In [7, p. 42] the non-classical worlds are called 'heavens' and an attempt is made to give them some intuitive content and relate them to the 'ordinary' possible worlds; but even that analysis does not solve the indeterminacy problem. Later chapters of [7] pretend that the possible worlds analysis is adequate and give truth conditional semantics for the symbols underlying many English words. The advantage of the solution to be proposed in this paper is that it enables the possible worlds semantics for any symbol to be immediately incorporated into hyperintensional contexts.

[2] Suszko is very concerned about 'intensional ghosts' and takes me to task [23, p. 46f] for advocating a possible worlds approach for languages in which material equivalence is not a propositional identity; but I do not wish to enter this controversy. My faith in possible worlds semantics rests not on formal grounds but on the success they have had in elucidating certain philosophically troublesome areas, e.g. the 'paradoxes of identity' (*vide*, e.g., [12] or [8]), and, perhaps more spectacularly, the analysis of the counterfactual conditional [22], [16].

[3] Something very like this has been argued by John Bigelow [3]. Martin Tweedale [24] has suggested that a proposition might stand to its intension in much the same way as a play stands to the situation it represents. It may be too that Frege's notion of sense [10] comes close to an account of this kind.

with truth we might then investigate a connection with linguistic behaviour.

The present work will not, however, take a mentalistic approach. This work will explore the analysis of propositions which assumes that they are *structured* entities, and that the clue to their structure is found in the sentences which express them. And we want to do this while preserving the highly desirable connection with the possible worlds approach to semantics. The most fully worked out account of structured meanings within a possible-worlds framework is that presented by David Lewis in [15, pp. 182-191], and in this paper I shall be adapting his account. The general idea[4] behind what Lewis says may be expressed somewhat as follows:

Given a sentence a the intension of a, $I(a)$, would be a set of possible worlds. Given a negation functor $\sim$ the intension of $\sim$, $I(\sim)$ would be the function ω such that for any set a of worlds $\omega(a) = a'$. A complex sentence like $\langle \sim, \langle \sim, a \rangle\rangle$ would have as its intension

$$I(\sim)\big(I(\sim), (I(a))\big)$$

which would be $(I(a)')'$ which is just $I(a)$. I.e. a and $\langle \sim, \langle \sim, a\rangle\rangle$ would have the same intension. (They would therefore be logically equivalent.) However they would have different meanings. If a is a simple sentence symbol then its *meaning*, $M(a)$, is $I(a)$, and if $\sim$ is a simple sentential functor then $M(\sim) = I(\sim)$. But

$$M(\langle \sim, \langle \sim, a\rangle\rangle) = \langle I(\sim), I(\sim), I(a)\rangle$$

A meaning is an entity made up from intensions (and therefore not language-dependent) but an entity which reveals the structure of the sentence. It is also clear how different tautologies, all with the same intension, can have different meanings. For if a and β have different intensions it can be seen that, e.g.

$$\langle I(a), I(\vee), \langle I(\sim), I(a)\rangle\rangle$$

will be different from

$$\langle I(\beta), I(\vee), \langle I(\sim), I(\beta)\rangle\rangle$$

and so $\langle a, \vee, \langle \sim, a\rangle\rangle$ will have a different meaning from $\langle \beta, \vee, \langle \sim, \beta\rangle\rangle$.

[4] This paper is not concerned with exegesis. Lewis' paper is eloquent enough to speak for itself and any differences of detail between his approach and my presentation of it do not affect any of the points made in the present paper. (The terminology of this paper is based on that of [7]; in particular, expressions are regarded as sequences and angle brackets are used to represent them.) Lewis' account has many similarities with the intensional isomorphism approach of Carnap, [4] but he has taken care to remove the language-dependence which is involved in Carnap's account. Another approach to the problem is taken by Hintikka in [12].

Unfortunately none of this is applicable as it stands to the logic of propositional attitudes, for the following reason. Consider a functor which is to mean 'x believes that'. (It is of course artificial to take it as a single functor but none of that affects the present discussion.) We can write this functor as 'B'. If B is a single symbol then (since it is a one-place sentential functor) its intension must be a function from $\mathscr{P}\mathbf{W}$ into $\mathscr{P}\mathbf{W}$. But this means that its arguments are sets of worlds, and that it operates on the intensions of sentences. Thus, where α and β are sentences, then for any $w \in \mathbf{W}$, if $I(\alpha) = I(\beta)$, then $I(\langle \beta, \alpha \rangle) = I(\langle B, \beta \rangle)$. I.e. if α and β are logically equivalent (have the same intension) then so are $\langle B, \alpha \rangle$ and $\langle B, \beta \rangle$, which is to say that x cannot believe a sentence without believing everything logically equivalent to that sentence.

Let us see where this puts us. What has been shewn is that if we have a system of meanings of the kind Lewis envisages, and if an operator representing a propositional attitude such as belief appears at the deepest level of 'semantic representation', then logically equivalent sentences are intersubstitutable in belief contexts *salva veritate.*

There are a number of courses one can follow having reached this point. One course is to accept, as some philosophers seem to, that logically equivalent sentences *are* so substitutable. If this course is taken then it seems to me that almost all the philosophical point of Lewis' distinction is removed. His distinction then has purely linguistic significance as a possible analysis of the deepest level postulated by a particular semantical theory.

Another course is to claim that although a belief operator takes a sentential argument (via a *that* clause) at the surface level yet it is transformed into something quite different at a deeper level, in particular it is transformed into an operator which does not operate on whole sentences.[5]

A third approach (popular with extensionalist logicians [11, pp. 48-51] and also with some intensionalists [17, p. 159f]) is to say that propositional attitudes are really attitudes to sentences, and that semantic analysis should bring out the fact that they are like quotational contexts. This paper is assuming that at least some propositional attitudes make genuine contexts, but I will not be arguing for that position. An argument for that position would first have to set up criteria for recognizing a context as quotational (particularly if the possibility of a context's being impli-

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[5] I take some analysis of this kind to be what Russell is hinting at in lecture IV of [20]. The view also seems to occur in Prior's [19, pp. 16–21], but he gives no clue as to what kind of formal semantics would be appropriate. A more explicit statement of the view is found in Ajdukiewicz' [2]. (*Vide* the review in JSL, Vol. 37 (1972) p. 179)

citly quotational is envisaged as in 'The predicate calculus is surprisingly complicated'), and then decide which contexts are and which are not.

I am not going to be interested here in adjudicating between alternatives to Lewis' approach. I am interested in modifying his account so that it will accommodate hyperintensional functors without abandoning the very considerable advantages it has. To this end I shall set out the approach in detail within the framework of the formal philosophy of language developed in [7].

2. Categorial Languages

We first recall the definition of a pure categorial language[6] given in [7, pp. 70-74]. The set Syn of syntactic categories is the smallest set such that, where Nat is the set of all natural numbers,

2.1 Nat $\subseteq$ Syn

2.2 If $\tau, \sigma_1, \ldots, \sigma_n \in$ Syn, then $\langle\tau, \sigma_2, \ldots, \sigma_n\rangle \in$ Syn

A pure categorial language $\mathscr{L}$ is a pair $\langle F, E\rangle$, where F is a function from Syn to finite (possibly empty) sets such that if $\sigma \neq \tau$ then $F_\sigma \cap F_\tau = \emptyset$, and E is the smallest function such that

2.3 $F_\sigma \subseteq E_\sigma$

2.4 If $\delta \in E_{\langle\tau,\sigma_1,\ldots,\sigma_n\rangle}$ and $a_1 \in E_{\sigma_1}, \ldots, a_n \in E_{\sigma_n}$, then $\langle\delta, a_1, \ldots, a_n\rangle \in E_\tau$.

$\langle\tau, \sigma_1, \ldots, \sigma_n\rangle$ is the category of a functor expression which makes an expression of category τ out of expressions of categories $\sigma_1, \ldots, \sigma_n$ respectively. 0 is intended as the category of *sentence* and 1 as the category of *name*. Although our definition allows all the natural numbers as basic syntactic categories, 0 and 1 are the only ones that we seem to need in practice. $\mathscr{L}$ is called *grounded* if where $\sigma \neq \tau$, $E_\sigma \cap E_\tau = \emptyset$. We assume that all languages we discuss are grounded.

A *system of (intensional) domains* for a pure categorial language is a function $\mathbf{D}$ from Syn such that where $\sigma = \langle\tau, \sigma_1, \ldots, \sigma_n\rangle$ then $\mathbf{D}_\sigma$ is a set of (partial) functions from $\mathbf{D}_{\sigma_1} \times \ldots \times \mathbf{D}_{\sigma_n}$ into $\mathbf{D}_\tau$. $\mathbf{D}_0$ is the domain of propositions and $\mathbf{D}_1$ the domain of things. These definitions do not commit us to any particular view of the nature of the entities in $\mathbf{D}_0$, but initially we would like them to be sets of possible worlds.[7] So we shall assume a set $\mathbf{W}$ of possible worlds and require that $\mathbf{D}_0 \subseteq \mathscr{P}\mathbf{W}$. Notice

[6] The notion of a categorial language goes back to Leśniewski (*vide* [1]). Some other references are given in [7, p. 71]

[7] We discuss context-dependence briefly in section 6. In context-dependent accounts, sentences have to be assigned something a little more complex.

that we are allowing the possibility that $\mathbf{D}_0$ may not be the whole of $\mathscr{P}\mathbf{W}$.[8] The members of $\mathbf{D}_0$ are intended to be the intensions of the sentences of $\mathscr{L}$ and serve to define logical relations, e.g. a and b are incompatible iff $a \cap b = \emptyset$, a entails b iff $a \subseteq b$ and so on.

Initially we suppose $\mathscr{L}$ to be an intensional language only. This means that we suppose it to contain no hyperintensional functors. Later we shall consider languages which do have such functors. Given a pure categorial intensional language $\mathscr{L}$ and a system $\mathbf{D}$ of domains we say that a function V is a value assignment to $\mathscr{L}$ iff for any $a \in F_\sigma$:

2.5 $\quad V(a) \in \mathbf{D}_\sigma$.[9]

Given such a function V we shew how it induces two functions I and M to all the expressions of $\mathscr{L}$. If $a \in E_\sigma$ then $I(a)$ is the *intension* of a and $M(a)$ the *meaning* of a. (For strictness our notation should indicate the dependence of I and M on V and we should write, say, I_V and M_V.)

If $a \in F_\sigma$ then

2.6 $\quad I(a) = M(a) = V(a)$

If a is $\langle \delta, a_1, \ldots, a_n \rangle$ then

2.7 $\quad I(a) = I(\delta)(I(a_1), \ldots, I(a_n))$

2.8 $\quad M(a) = \langle M(\delta), M(a_1), \ldots, M(a_n) \rangle$

The point is that the intension of a complex expression is obtained by allowing the intension of its functor to operate on the intensions of the arguments of the functor. The meaning however is simply the $n+1$-tuple consisting of the meaning of the functor together with the meanings of its arguments. It should be obvious that many different meanings can correspond to the same intension, though not *vice versa*.

What we have so far corresponds more or less with Lewis' account of meanings and intensions. But the language contains as yet no hyperintensional functors. This is not intended as a criticism of Lewis because he does not postulate meanings to account for the semantics of propositional attitudes and it may be that the had some other kind of analysis in mind for these functors. Nevertheless hyperintensional contexts do pose a problem and one which, as we saw at the end of the last section, Lewis' account is unable to deal with as it stands. So we proceed now to modify it.

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[8] $\mathbf{D}_0$ cannot be $\mathscr{P}\mathbf{W}$ if it is desired to have only a denumerable number of propositions. The possibility of restricting $\mathscr{P}\mathbf{W}$ so that not all sets of possible worlds are values of sentences answers, at least in part, one of Suszko's complaints in [23] against the possible worlds approach. It is perhaps desirable though that $\mathbf{D}_0$ be closed under the Boolean operations.

[9] If we regard a language as a free algebra then we can regard a value assignment as a homomorphism from this algebra to another algebra based on $\mathbf{D}$.

3. θ-categorial languages

One very simple repair suggests itself. Surely a belief functor does have an intension? Surely its intension is a function from meanings (not intensions) to sets of possible worlds? In this section and the next we consider how this modification can be accommodated.

Basically what we do is add to a categorial language a device, in the form of a logical symbol θ, which indicates that a functor is to operate on the meanings of its arguments rather than on their intensions. We shall first see how this device works before explaining the need for what at first sight may appear to be a gratuitous complication of the situation.

A θ-categorial language contains two classes of expressions in each syntactic category, functorial expressions and θ-expressions. We denote these by E^f_σ and E^θ_σ respectively, and let $E_\sigma = F_\sigma \cup E^f_\sigma \cup E^\theta_\sigma$. E^f and E^θ are defined simultaneously as the least functions such that

3.1 If $\delta \in E_{\langle\tau,\sigma_1,\ldots,\sigma_n\rangle}$ and $\alpha_1 \in E_{\sigma_1}, \ldots, \alpha_n \in E_{\sigma_n}$, then $\langle\delta, \alpha_1, \ldots, \alpha_n\rangle \in E^f_\sigma$

3.2 If $\alpha \in E^f_\sigma$ then $\langle\theta, \alpha\rangle \in E^\theta_\sigma$

(The point of these definitions is basically to weed out unwanted occurrences of θ.)

We suppose that we are given a system **D** of intensions for a categorial language. By a system **A** of *admissible meanings* based on **D** we mean a function such that

3.3 $\mathbf{D}_\sigma \subseteq \mathbf{A}_\sigma$

3.4 If $\sigma = \langle\tau, \sigma_1, \ldots, \sigma_n\rangle$ and $d \in \mathbf{A}_{\langle\tau,\sigma_1,\ldots,\sigma_n\rangle}$, $a_1 \in \mathbf{A}_{\sigma_1}, \ldots, a_n \in \mathbf{A}_{\sigma_n}$,

then $\langle d, a_1, \ldots, a_n\rangle \in \mathbf{A}_\tau$

In a hyperintensional language a value assignment is a slightly different thing from a value assigment in an ordinary intensional language. It is a function V as follows:

3.5 If σ is a basic category and $a \in F_\sigma$

then $V(a) \in \mathbf{D}_\sigma$

3.6 If $\sigma = \langle\tau, \sigma_1, \ldots, \sigma_n\rangle$ and $\delta \in F_\sigma$, then $V(\delta)$ is a partial function

from $\mathbf{A}_{\sigma_1} \times \ldots \times \mathbf{A}_{\sigma_n}$ into D_τ.

3.6 needs some explanation. If δ is an ordinary intensional functor then its arguments will be taken only from $\mathbf{D}_{\sigma_1} \times \ldots \times \mathbf{D}_{\sigma_n}$ (remember that $\mathbf{D}_\sigma \subseteq \mathbf{A}_\sigma$ for all $\sigma \in$ Syn). This makes the use of partial functions imperative. The use of partial functions means of course that an expression may lack a value. Notice that since all meanings are intensions, then a hyperintensional functor may well be defined for arguments which are intensions; indeed one would expect this.

3.6 allows the possibility that the value assigned to an intensional functor is not in the set of admissible meanings of that category. For obviously there is no reason to suppose that a function from $\mathbf{A}_{\sigma_1} \times \ldots \times \mathbf{A}_{\sigma_n}$ into $\mathbf{D}_\tau$ is in $\mathbf{A}_\sigma$ (where $\sigma = \langle \tau, \sigma_1, \ldots, \sigma_n \rangle$). It would be possible to extend the system of admissible meanings, but it is in fact unnecessary since by 'admissible' we merely mean 'admissible as the value of a function assigned to a hyperintensional functor'. The meanings assigned to expressions of a θ-categorial language may well include some which are not admissible, though our rule for evaluating θ-expressions will be designed to avoid any embarrassing consequences of this fact. Perhaps the most significant feature of 3.6 is that it requires the values of the functions assigned to hyperintensional functors to be intensions. This is in accord with the basic assumption of this paper, that a truth-conditional semantics is sufficient to determine meaning.

We now shew how the assignment V to the symbols of $\mathscr{L}$ induces both an intension $I(\alpha)$ and a meaning $M(\alpha)$ to every expression α.

3.7 If $\alpha \in F_\sigma$ then $I(\alpha) = M(\alpha) = V(\alpha)$[10]

Any complex expression α will either be in E^f_σ or in E^θ_σ; if the former then α is

$$\langle \delta, \alpha_1, \ldots, \alpha_n \rangle$$

for some appropriate expressions $\delta, \alpha_1, \ldots, \alpha_n$. In this case

3.8 $I(\alpha) = I(\delta)\big(I(\alpha_1), \ldots, I(\alpha_n)\big)$

3.9 $M(\alpha) = \langle M(\delta), M(\alpha_1), \ldots, M(\alpha_n) \rangle$

(This is as for ordinary intensional languages).

If α is a θ-expression then α must have the form $\langle \theta, \beta \rangle$ where β is a functorial expression, say

$$\langle \delta, \beta_1, \ldots, \beta_n \rangle$$

In this case,

3.10 $I(\alpha) = I(\delta)\big(M(\beta_1), \ldots, M(\beta_n)\big)$

[10] It is not absolutely essential to this approach that symbols should have simple meanings. We could suppose that each symbol is assigned both a meaning and an intension. (Or rather is assigned a meaning which determines also an intension.) Lewis imagines single lexical items replacing complexes by the transformations which relate deep and surface structure.

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3.11 $M(a) = I(a)$[11]

We shall in the next section examine 3.11 and shew why we have not chosen the seemingly more intuitive

3.12 $M(a) = \langle M(\delta), M(\beta_1), \ldots, M(\beta_n)\rangle$

Essentially the reason is that 3.11, unlike 3.12, ensures that the meanings of all θ-expressions are admissible. For when $a \in E^{\theta}_{\sigma}$, then $M(a) = I(a)$; and $I(a) \in \mathbf{D}_{\sigma} \subseteq \mathbf{A}_{\sigma}$.

4. Comments on θ-categorial languages

The rules of the last section are in fact little more than a formal definition of a theory of meaning of Lewis' kind but with the (vital) exception that the intensions of certain expressions involve the meanings rather than the intensions of their arguments. In particular both the meaning and the intension of every expression is uniquely determined given the initial assignment to the (finite number of) symbols of $\mathscr{L}$. In this way we have preserved Frege's principle.[12] We have also preserved the highly desirable requirement that every (defined) expression have an intension, and so we can have a truth-conditional semantics, which yet accommodates propositional attitudes. (For obviously a functor which takes account of meanings can distinguish between expressions which have different meanings but the same intension.)

The remainder of this section is concerned with the rule 3.11 and we must first shew why the alternative 3.12, which is at first sight the more intuitively natural choice, will not do.

Consider a belief functor. We shall again use 'B' but this time assume it to be a two place functor so that where x is a name and a a sentence

[11] It would be possible to split up the work done by θ into two parts. One could have a symbol θ^{+} which acts in such a way that the *intension* of $\langle\theta^{+}, a\rangle$ is the *meaning* of a, and then a symbol θ^{-} such that the *meaning* of $\langle\theta^{-}, a\rangle$ is the *intension* of a. $\langle\theta, \langle\delta, a_1, \ldots, a_n\rangle\rangle$ would then be replaced by $\langle\theta^{-}, \langle\delta, \langle\theta^{+}, a_1\rangle, \ldots, \langle\theta^{+}, a_n\rangle\rangle\rangle$. I have used the single θ for economy, because I can think of no cases where θ^{+} and θ^{-} would not occur together in the way indicated. At the other extreme we might try to dispense with θ altogether by classifying all expressions (and not merely the symbols) into intensional and hyperintensional, and suppose that any expression whose functor is hyperintensional is evaluated as if it were a θ-expression.

[12] 'Frege's Principle' was the name given in [7, p. 75] to the requirement that the meaning of all expressions be determined by the meanings assigned to the (finite number of) symbols of the language.

$\langle B, x, \alpha\rangle$ is intended to mean that x believes that α. Suppose that we try to formalize a sentence which involves nested belief functors, such as

(1) x believes that y believes that α

This comes out as

(2) $\langle\theta, \langle B, x, \langle\theta, \langle B, y, \alpha\rangle\rangle\rangle\rangle$

(Obviously the arguments of B must be treated as meanings rather than intensions if iterated hyperintensional functors are ever to be interesting. Without the θ, B would simply take intensions as arguments.)

If we had adopted 3.12 in place of 3.11 the meaning of $\langle\theta, \langle B, y, \alpha\rangle\rangle$ would be $\langle M(B), M(y), M(\alpha)\rangle$ and therefore by 3.10 the intension of (2) would be

(3) $I(B)(M(x), \langle M(B), M(y), M(\alpha)\rangle)$

Now B is a symbol of $\mathscr{L}$ and so $M(B) = I(B)$. And this means that in (3) $I(B)$ occurs as one of its own arguments. Since this is not set-theoretically possible (in most set theories) then (2) cannot have an intension. This does not lead to a contradiction, because we have allowed intensions to be partial functions, and have therefore permitted expressions to lack one. However to prevent (1) from having any meaning or intension, and more generally, to rule out any iteration or nesting of hyperintensional contexts, seems undesirable. Iteration of hyperintensional functors seems to me so desirable that 3.12 ought to be rejected even if so doing seems to involve a slight blurring of the distinction between meanings and intensions.

The modification proposed by 3.11 is quite simple, for it simply identifies the meaning of a θ-expression with its intension. What this means is that we regard the embedded 'y believes that α' in (1) as if it were a simple sentence symbol.

3.11 does entail that logically equivalent θ-expressions are synonymous.[13] It is therefore unwise to use them for all intensional functors. Consider, e.g., the modal operator L meaning 'it is necessary that'. If α and β are logically equivalent then so are $\langle L, \alpha\rangle$ and $\langle L, \beta\rangle$. This would make $\langle\theta, \langle L, \alpha\rangle\rangle$ and $\langle\theta, \langle L, \beta\rangle\rangle$ synonymous, yet obviously

'x believes that it is necessary that α'

may differ in truth value from

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[13] It ought to be clear that this does not entail that all doubly embedded expressions with the same intension are synonymous. All we need suppose is that a difference in *meaning* between α and β will be reflected in a difference in *intension* between sentences like 'x believes that α' and 'x believes that β'.

'α believes that it is necessary that β'

Thus θ ought not to precede the modal functor.

This last argument shews that the θ analysis is not plausible for many intensional functors; which in turn means that we cannot use it to dispense with a possible-worlds analysis. (At least not without intensive reworking; so much reworking in fact that the result could hardly be called a modification of Lewis' analysis, and would probably seem best regarded as a version of a quotational approach.)

5. θ-categorial logics

The present paper has treated θ-categorial languages from an entirely semantical point of view. Although the truth functors have been used on occasion as illustrations, there has been no systematic attempt to distinguish between 'logical constants' and other words. This is because the paper has been concerned with formal languages as the underlying structures of natural languages, and it is my belief that every word in a natural language is, in some sense, a constant. Although words like 'and', 'or', 'if' and so on are, in some vague way, more pervasive than others, yet no hard and fast line can be drawn between them and other words. It is also my belief that the words in natural languages which are regularly translated by the logical constants of formal languages may well not have quite the semantics they are given in those formal languages. This need not mean of course that they do not have a truth-conditional semantics, only that it is a more complicated one than the translations would suggest.[14]

Those who feel that without logical constants the word 'logic' is inappropriate may well take issue with the title of this paper. For my own part I consider that any study of possible worlds semantics has a claim to be in some sense dealing with 'logic', but I have no real objection if the title is denied me.

Obviously it would be possible to introduce, e.g., truth functors and modal operators, and provide an axiom system and completeness proof. Such a task may even shed illumination on the nature of θ-categorial languages. It is however a task that will be left to others.

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[14] The points of this section are elaborated in [6], [7] and [9]. Scepticism of the philosophical utility of a possible worlds analysis of the logical modalities has been expressed by Hintikka in [26 p. 52f] and elsewhere. In fact, as Karttunen notes in [14], modality in ordinary language is more of an epistemic one. It therefore takes account only of worlds fairly similar to the actual one.

One small illustration may suffice to shew the kinds of things that logical symbols might do in a θ-categorial language. Suppose that we have a two place propositional functor δ whose semantics are as follows:

5.1 $V(\delta)$ is the function ω such that for any $w \in \mathbf{W}$, and $a, b \in \mathbf{A}_0$, $w \in \omega(a, b)$ iff $a = b$.

δ is obviously a propositional identity operator. The interesting thing is however that in the absence of θ it reduces to strict equivalence. I.e. $\langle \delta, \alpha, \beta \rangle$ is true when α and β have the same intension, while $\langle \theta, \langle \delta, \alpha, \beta \rangle \rangle$ requires that α and β have the same meaning. This is easy to see because

$$I(\langle \delta, \alpha, \beta \rangle) = I(\delta)\big(I(\alpha), I(\beta)\big) = \omega(I(\alpha), I(\beta)$$

and if α and β are logically equivalent then $I(\alpha) = I(\beta)$, and so $w \in \omega(I(\alpha), I(\beta))$ for all $w \in \mathbf{W}$.

However

$$I(\langle \theta, \langle \delta, \alpha, \beta \rangle \rangle) = I(\delta)\big(M(\alpha), M(\beta)\big) = \omega\big(M(\alpha), M(\beta)\big)$$

and so $w \in I(\langle \theta, \langle \delta, \alpha, \beta \rangle \rangle)$ only if $M(\alpha) = M(\beta)$. What this shews is that in θ-categorial languages the very same operator can be more or less strict depending on the presence or absence of θ

6. Further developments

This paper has treated the intensions of sentences as sets of possible worlds. Thus our languages have made no room for context dependence. One way of treating context is to isolate a number of contextual 'indices' [17], [21], [15] and say that a context is made up of, e.g., a possible world, a time, a speaker ... and so on. Intensions then become somewhat more complex things, but their incorporation into a θ-categorial language would seem to pose no insuperable problems. In [7] a somewhat more complicated analysis of context-dependence was given, in which contexts were analysed as properties of utterances.

The θ-categorial languages discussed in this paper have made no use of variable-binding. This is a real limitation of their expressive power, and in addition has meant that we have had nothing to say on the very vexing problem of quantification into hyperintensional contexts.[15] One

[15] A very sensitive article on these problems from a linguistic point of view is Partee [18]. Partee is one of the few linguists who has shewn any awareness of the logical problems of hyperintensional contexts.

approach to variable-binding, that advocated by Lewis and Montague, supposes the values of free variables to be supplied by a contextual index. It may be that this approach will automatically carry over into θ-categorial languages but I am not entirely convinced that it is the best way. Variable-binding seems to me to be such a deep phenomenon that its problems should be tackled head on, and what I would like to be able to do would be to apply the analysis of abstraction developed in [7] to θ-categorial languages. I have a few ideas about how to do this but none I find really intuitive. [16]

The biggest philosophical test of θ-categorial languages will come of course when we try to formulate the truth conditions of specific hyperintensional functors, such as 'says that', 'knows that', 'believes that', 'deduces that' etc. And the plain fact of the matter seems to be that we are almost completely in the dark about the semantics of these notions. This is why I have not been concerned to argue that the θ-categorial approach must be superior to any other. I have myself in the past set out various alternative approaches to the semantics of hyperintensional contexts, and my present opinion is that the more approaches that are formally set out the more likely we are to discover which of them provides the most natural framework for the analysis of propositional attitudes. Until we are in that fortunate position there can be, I fear, no conclusive arguments in favour of any one approach; and that applies to the θ-categorial analysis as much as to any others.

[16] Part of the problem is that the applications of categorial languages to linguistic analysis in [7] relied heavily on the principles of λ-conversion. It is not at all clear to me whether and how these principles should be regarded as meaning-preserving. If they are not meaning-preserving in all contexts, then they need not be intension-preserving in contexts governed by θ. Yet on the ordinary analysis λ-conversion will be intension-preserving whatever entities we take as the values of sentences.

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Allatum est die 10 Julii 1974