Long-Term Wave Height Distributions at Seven Stations around the British Isles

By

J. A. Battjes

UDC 551.466.31; ANE North Sea, Irish Sea

Summary. Long-term instrumental measurements of significant wave height and mean zero-crossing period at 7 stations are analysed. The marginal distribution of significant heights is well described by a Weibull law. The long-term distribution of individual wave heights is calculated from the joint distribution of significant wave height and mean wave period. It is found to be nearly exponential.

Langfristige Verteilung der Wellenhöhen auf sieben Stationen rund um die Britischen Inseln (Zusammenfassung). Langfristige Messungen bedeutender Wellenhöhen und der mittleren Null-Durchgangsperiode, die mit Hilfe von Instrumenten auf sieben Stationen durchgeführt worden sind, werden analysiert. Die Randwertverteilung bedeutender Wellenhöhen wird in einem Weibull-Gesetz sehr gut dargestellt. Die langfristige Verteilung einzelner Wellenhöhen wird errechnet aus dem Zusammentreffen von bedeutenden Wellenhöhen und mittleren Wellenperioden. Es hat sich gezeigt, daß sie fast exponentiell ist.

Distributions de hauteurs de vagues pendant une longue durée en sept points situés autour des Iles Britanniques (Résumé). On analyse des mesures instrumentales, pendant une longue durée, de hauteurs de vagues signicatifs et de période moyenne voisine de zéro en 7 stations. La distribution marginale de hauteurs significatives correspond bien à une loi de Weibull. La distribution sur une longue durée de hauteurs de vagues individuelles, est calculée à partir de la distribution conjointe de hauteur de vague significative et de période de vague moyenne. On trouve qu'elle est approximativement exponentielle.

Introduction. The N.I.O. (National Institute of Oceanography, Wormley, England) has carried out wave measurements at a number of locations. The measurements generally cover a one-year period, and the results may serve as a basis for estimating extreme wave conditions in the respective areas. In making such estimates, the one-year data must be extrapolated. To this end the data are considered to be the result of random sampling from a population, the distribution of which is to be estimated. Once a distribution is found which gives a sufficiently close fit to the data, then extrapolation beyond the original range of the measurements can be made. The confidence which one has in the extrapolation increases with increasing goodness of fit of the distribution on which it is based. Some authors, following N. H. Jasper [1956], have stated that the logarithm of the significant wave height would be Gaussian distributed. This distribution function was found not to give a fully satisfactory fit to the N.I.O. data, the measured wave heights in the upper range tending to fall below the line of best fit for a given probability of exceedance. N. Nordenstrøm [1969] analysed distributions of the significant wave height obtained from visual and instrumental data at locations in the North Atlantic. He found that the Weibull distribution fitted the data well. It was deemed desirable to extend his analysis to data which were available from the Irish Sea and the North Sea. Some results are given in the following. In addition, long-term distributions of individual wave heights are calculated.

Wave data used. The majority of long-term wave data presently available is based on visual observations. Instrumental data are far fewer both in number of locations and in time. It was nevertheless decided to use only instrumental data in the present study because of their greater reliability. There exists a systematic difference between the two sets for relatively large wave heights. L. Draper and M. J. Tucker [1971] report that at Ocean Weather Ship station "India" the significant height exceeds 10 m in 1.5 % of the instrumental measurements, and in only 0.02 % of the visual observations.

The instrumental data chosen for analysis have been obtained by the N.I.O. from measurements with shipborne wave recorders. Table 1 contains pertinent information about the wave data. "India" and "Juliett" are Ocean Weather Ship stations, and the others are Light Vessel stations.

The original data generally consist of records of 12 minutes duration, taken every 3 hours during one year, for a total of 2920 records. For purposes of analysis each record is regarded as a (short) sample from a stationary random Gaussian process. The work of S. O. Rice [1944], M. S. Longuet-Higgins [1952], D. E. Cartwright and M. S. Longuet-Higgins [1956] and D. E. Cartwright [1958] provides the theoretical basis for the subsequent analysis, a convenient procedure for which has been described by M. J. Tucker [1961]. Each record yields, among others, an estimate of the significant height $H_{1/3}$ and of the mean zero-crossing period T_z appropriate to the random process of which the record is a sample. The publications referred to in Table 1 give, among others, scatter diagrams with the fractions of the observations, expressed in parts per thousand, for which $H_{1/3}$ and T_z simultaneously fall in certain ranges. Only these scatter diagrams for $H_{1/3}$ and T_z will be used in this paper. An example is given in Fig. 1.

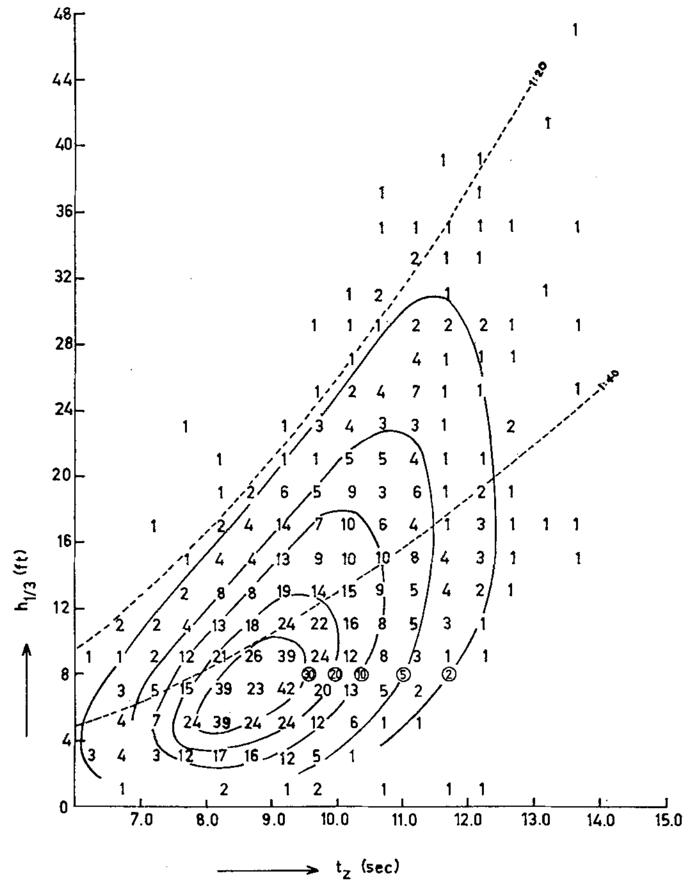

Fig. 1. Frequencies of occurrence (in ‰) of $H_{1/3}$ and T_z at station "India"

It is to be noted that the scatter diagrams used herein represent the lumped data for a one-year period. This implies that neither seasonal variations nor variations between years (H. Walden [1969]) can be dealt with.

Variables considered. It is necessary to distinguish statistics obtained from a single record, with a duration on the order of 15 minutes, and statistics obtained from a collection of records, covering e.g. several years. The former are conveniently called short-term statistics, the latter long-term statistics. The short-term probability structure can to some extent be deduced theoretically assuming that one is dealing with a random process which is approximately stationary and Gaussian. The long-term probability structure is a reflection of local and distant climatological features and cannot be dealt with by deductive methods.

The only wave parameters which will be considered herein are:

- the wave height H: the difference between maximum and minimum water surface elevation between two adjacent zero up-crossings (also referred to as "individual wave heights").
- the significant wave height $H_{1/3}$: the short-term mean of the highest one third of the wave heights.
- the zero-crossing period T_z: the short-term mean value of the time intervals between adjacent zero up-crossings.

Probability distributions. In this section several probability functions will be defined and some relationships between them will be given. Stochastic variables are denoted by capital letters, and particular values which they may assume by the corresponding lower-case letters. Probability densities (abbreviated: p.d.) will be written as p and cumulative probabilities as P.

Joint distribution of $H_{1/3}$ and T_z. The joint p.d. of $H_{1/3}$ and T_z is written as $p(h_{1/3}, t_z)$.

Marginal distribution of $H_{1/3}$. The marginal p.d. of $H_{1/3}$ is given by

$$p(h_{1/3}) = \int p(h_{1/3}, t_z)\, dt_z \tag{1}$$

(Unless otherwise stated, the integrations are over all possible values of the variables).

Table 1

Station	Location	Depth	Dates of observations	Total number of observations	Reference
		fathoms			
OWS Station "India"	59°N 19°W	–	'52–'64 (intermittently)	2400	Draper and Squire [1967]
OWS Station "Juliett"	52°30'N 20°W	–	'52–'64 (intermittently)	1440	Draper and Whitaker [1965]
"Seven-stones"	20 mi. S.W. of Land's End	33	Jan. '62–'63	2920	Draper and Fricker [1965]
"Morecambe Bay"	15 mi. W. of Fleetwood (Irish Sea)	12	Nov. '56–'57	2920	Draper [1968]
"Mersey Bar"	3 mi. W. of buoyed channel to the Mersey (Irish Sea)	9.6	Sept. '65–'66	2920	Draper and Blakey [1969]
"Varne"	Dover Strait	15	Feb. '65–'66	2920	Draper and Graves [1968]
"Smith's Knoll"	22 mi. E.N.E. of Great Yarmouth	27	Mar. '59–'60	2920	Draper [1968]

Conditional distribution of H. The short-term p.d. of individual wave heights H is the conditional p.d. of H for given $H_{1/3}$ and T_z, formally written as $p(h \mid h_{1/3}, t_z)$. It is approximately given by the Rayleigh p.d.:

$$p(h \mid h_{1/3}, t_z) = \frac{4h}{h_{1/3}^2} \mathrm{e}^{-2(h/h_{1/3})^2} \tag{2}$$

The cumulative probability is

$$P(h \mid h_{1/3}, t_z) = 1 - \mathrm{e}^{-2(h/h_{1/3})^2} \tag{3}$$

The validity of (2) and (3) will be assumed here without further inquiry. Reference may be made to G. D. Hess, G. M. Hidy and E. J. Plate [1969] for a recent survey of empirical evidence in support of the Rayleigh distribution.

Marginal distribution of H. The marginal (long-term) p.d. of individual wave heights, $p(h)$, can be derived as a weighted sum of Rayleigh probability densities. The weight factor should not only include the variability of $H_{1/3}$ but that of T_z as well, despite the fact that the Rayleigh p.d. does not contain the wave period as a parameter. The reason for this is that probabilities of occurrence of certain $H_{1/3}$-values, expressed as fractions of time, are transformed into probabilities of occurrence of certain H-values, expressed as fractions of a number of waves. At some stage in the transformation one is converting time intervals into number of waves; in other words, one must divide by the wave period. It follows that the long-term p.d. of H can be found as a sum of the conditional (short-term) p. densities, weighted with T_z^{-1} (i.e., with the short-term mean number of waves per unit time) and with the probability that $H_{1/3}$ and T_z simultaneously fall in certain ranges:

$$p(h) = \frac{\iint p(h \mid h_{1/3}, t_z)\, t_z^{-1}\, p(h_{1/3}, t_z)\, \mathrm{d}h_{1/3}\, \mathrm{d}t_z}{\iint t_z^{-1}\, p(h_{1/3}, t_z)\, \mathrm{d}h_{1/3}\, \mathrm{d}t_z} \tag{4}$$

The denumerator in this expression is equal to $\overline{T_z^{-1}}$, the long-term average number of waves per unit time. A more detailed derivation of this equation has been given elsewhere (J. A. Battjes [1970]).

Integration of (4) with respect to h gives the cumulative probability of H:

$$P(h) = \frac{\iint P(h \mid h_{1/3}, t_z)\, t_z^{-1}\, p(h_{1/3}, t_z)\, \mathrm{d}h_{1/3}\, \mathrm{d}t_z}{\overline{T_z^{-1}}} \tag{5}$$

Substitution of the Rayleigh law for $P(h \mid h_{1/3}, t_z)$, given by Eq. 3, and rearranging gives

$$Prob\ [H > h] = 1 - P(h) = \frac{\iint \mathrm{e}^{-2(h/h_{1/3})^2}\, t_z^{-1}\, p(h_{1/3}, t_z)\, \mathrm{d}h_{1/3}\, \mathrm{d}t_z}{\overline{T_z^{-1}}} \tag{6}$$

This equation differs from the corresponding expression usually given (N. H. Jasper [1956]; International Ship Structure Congress [1964]; E. V. Lewis [1967]; N. Nordenstrøm [1969]), in which the effect on $P(h)$ of the variability of T_z is not mentioned at all:

$$1 - P(h) = \int \mathrm{e}^{-2(h/h_{1/3})^2}\, p(h_{1/3})\, \mathrm{d}h_{1/3} \tag{7}$$

The effect of this omission depends on the degree of correlation which exists between $H_{1/3}$ and T_z. If these are stochastically independent then both approaches yield identical results. Generally, however, there is a positive correlation between $H_{1/3}$ and T_z (see Fig. 1 for example). This means that neglecting the effects of variations of T_z results in overestimating H, because the number of large waves occurring in a given length of time will on the average be less than the number of small waves. A comparison of the results from both methods will be given later.

Return period. In engineering applications of probability distributions it is customary to introduce the return period, which is equal to the average time interval between occurrences of the event being considered. Let the result of a random experiment be X. Successive trials are assumed independent; in other words, the prob $[X \leq x] = P(x)$ at each trial, independent of the outcome of the other trials. If n^{-1} is the fraction of (a great number of) trials for which $X > x_n$ then n is the dimensionless return period corresponding to exceedances of x_n:

$$n = \{Prob\ [X > x_n]\}^{-1} = \{1 - P(x_n)\}^{-1} \tag{8}$$

If the trial is repeated every τ time units then the dimensional return period y would be

$$y = n\tau = \tau\ \{1 - P(x_n)\}^{-1} \tag{9}$$

This will be applied to individual wave heights. The long-term expected number of waves per unit time is $\overline{T_z^{-1}}$, and the expected number of waves during the return period y is therefore $n = y\overline{T_z^{-1}}$, from which it follows that

$$Prob\ [H > h_n] = 1 - P(h_n) = (y\overline{T_z^{-1}})^{-1}. \tag{10}$$

N. Nordenstrøm [1969] uses $\bar{T}_z$ as the time unit for converting probability of exceedance into return period. This does not seem to be correct, because the return period is based on the expected number of occurrences, which is $y\overline{T_z^{-1}}$ and not $y(\bar{T}_z)^{-1}$. However, the differences between the two were found to be very minor for all the stations analyzed herein.

The idea of return period cannot very well be applied to $H_{1/3}$ because this variable is defined (has a value) at each instant of time. Thus one cannot speak of the number of occurrences that $H_{1/3}$ has a given value. The notion of return period can perhaps be fruitfully applied to $H_{1/3}$ by considering the maximum value reached each year. This variate should have the Fisher-Tippett double exponential distribution of extremes because, as will be shown later, the underlying or parent distribution is approximately given by the Weibull distribution, which is of the exponential type (E. J. Gumbel [1958]). However, many years of wave measurements would be required for such an analysis. The data treated herein cover a one-year period only.

The Weibull distribution. This distribution will be used in the following sections. It is defined by

$$\begin{aligned} Prob\ [X \leq x] = P(x) &= 1 - e^{-\left(\frac{x-A}{B}\right)^C} \quad \text{for} \quad x \geq A \\ &= 0 \quad \text{for} \quad x < A \end{aligned} \tag{11}$$

A is a lower limit of X. B is a scale parameter ($B > 0$). C is a shape parameter ($C > 0$). For $C = 1$ the Weibull distribution reduces to the exponential distribution of the variate $(X - A)$, while for $C = 2$ it reduces to the Rayleigh distribution. From Eq. (11) it follows that

$$\ln \ln\ \{1 - P(x)\}^{-1} = C \ln (x - A) - C \ln B \tag{12}$$

so that a plot of the Weibull distribution is a straight line on paper with $\ln \ln\ \{1 - P(x)\}^{-1}$ as one coordinate and $\ln (x - A)$ as the other.

Analysis of the data. Joint distribution of $H_{1/3}$ and T_z. The scatter diagrams referred to previously, of which Fig. 1 is an example, provide estimates of $p(h_{1/3}, t_z)$, the joint probability density of $H_{1/3}$ and T_z. No attempt has been made to find analytical approximations to these measurements.

It was noted by Draper et al. (References in Table 1) that all the scatter diagrams showed a cutoff at some upper limit of $H_{1/3}/T_z^2$. The limiting steepness $s_{\max}$ was found to range from 1:16 to 1:20, with most values near 1:18, where the steepness s is defined as the ratio of significant height to the deep-water wave length based on mean zero-crossing period:

$$s = \frac{2\pi H_{1/3}}{gT_z^2} \tag{13}$$

The value of 1:18 for s_{max} has sometimes been compared with the theoretical limiting steepness of irrotational, periodic, progressive, two-dimensional gravity waves in deep water, which is 1:7. Sea waves depart too much from waves of this category for the comparison to be satisfactory. Particularly the assumption that the waves are periodic is unrealistic. This assumption is not made in the calculation which is outlined in the following, and which is believed to provide a more meaningful basis for comparison with the measurements.

The elevation of the sea surface in a fixed point above its mean value is considered as a stationary random process in time with a variance density spectrum $S(\omega)$. If the moments of $S(\omega)$ are given by

$$m_j = \int_0^\infty \omega^j S(\omega)\,\mathrm{d}\omega\,, \tag{14}$$

then

$$H_{1/3} = 4\sqrt{m_0}, \tag{15}$$

(M. S. Longuet-Higgins [1952]) and

$$T_z = 2\pi\sqrt{\frac{m_0}{m_2}}\,, \tag{16}$$

(S. O. Rice [1944, 1945]) so that

$$s = \frac{2}{\pi g}\,\frac{m_2}{\sqrt{m_0}}\,. \tag{17}$$

If it is supposed that $S(\omega)$ has the shape of a Pierson-Moskowitz-Bretschneider spectrum, then

$$S(\omega) = \alpha g^2 \omega^{-5}\,\mathrm{e}^{-\beta(\omega/\omega_0)^{-4}} \tag{18}$$

which gives

$$s = \sqrt{\frac{\alpha}{\pi}}\,. \tag{19}$$

The values of α, determined from equilibrium ranges in the spectra of wind-driven waves, vary from (0.8 to 1.4) 10^{-2} (O. M. Phillips [1966]). This gives maximum values of s ranging from 1:20 to 1:15, in very close agreement with the observed range of 1:20 to 1:16.

Marginal distribution of $H_{1/3}$. As stated in the introduction, the distribution of $\log H_{1/3}$ was found to be clearly non-Gaussian in the upper ranges.

Three examples of the measurements of $\log H_{1/3}$ plotted on Gaussian paper are given in Fig. 2. The coordinates of the plotted data points are the upper limit of the class interval, and the fraction of the observations for which $\log H_{1/3}$ is less than this upper limit. This plotting rule has been used throughout.

The examples given in Fig. 2 from stations "Juliett" and "Smith's Knoll" were chosen because they seemed to represent the best and the worst fit of the Gaussian distribution for $\log H_{1/3}$. (The data from "India" are almost identical with those from "Juliett" and could equally well have been chosen for this purpose.)

The poor fit of the log-normal distribution to measurements of $P(h_{1/3})$ has been noted by N. Nordenstrøm [1969], who proposes to use the Weibull distribution for the description of long-term instrumental wave data at "India" and "Juliett" and visual data at these and other stations in the North Atlantic. The application of the Weibull distribution to wind wave problems had previously been suggested by C. L. Bretschneider [1965] for a description of the short-term statistics.

The data for the 7 stations considered herein have been plotted on Weibull paper, both for $A = 0$ and, where necessary, for $A \neq 0$ such that the best fit was obtained, as judged by eye. Examples are given in Fig. 3. The parameters B and C have been estimated from the best-fitting straight lines so obtained. The results are tabulated below:

Table 2
Parameters of fitted Weibull distributions of $H_{1/3}$

Station	A	B	C	Area
	m	m		
"India"	0.80	2.70	1.22	Atlantic Ocean
"Juliett"	0.90	2.70	1.24	
"Sevenstones"	0.60	1.67	1.21	
"Morecambe Bay"	0.00	0.78	1.05	Irish Sea
"Mersey Bar"	0.00	0.69	1.01	
"Varne"	0.00	1.05	1.30	North Sea
"Smith's Knoll"	0.08	0.89	1.28	

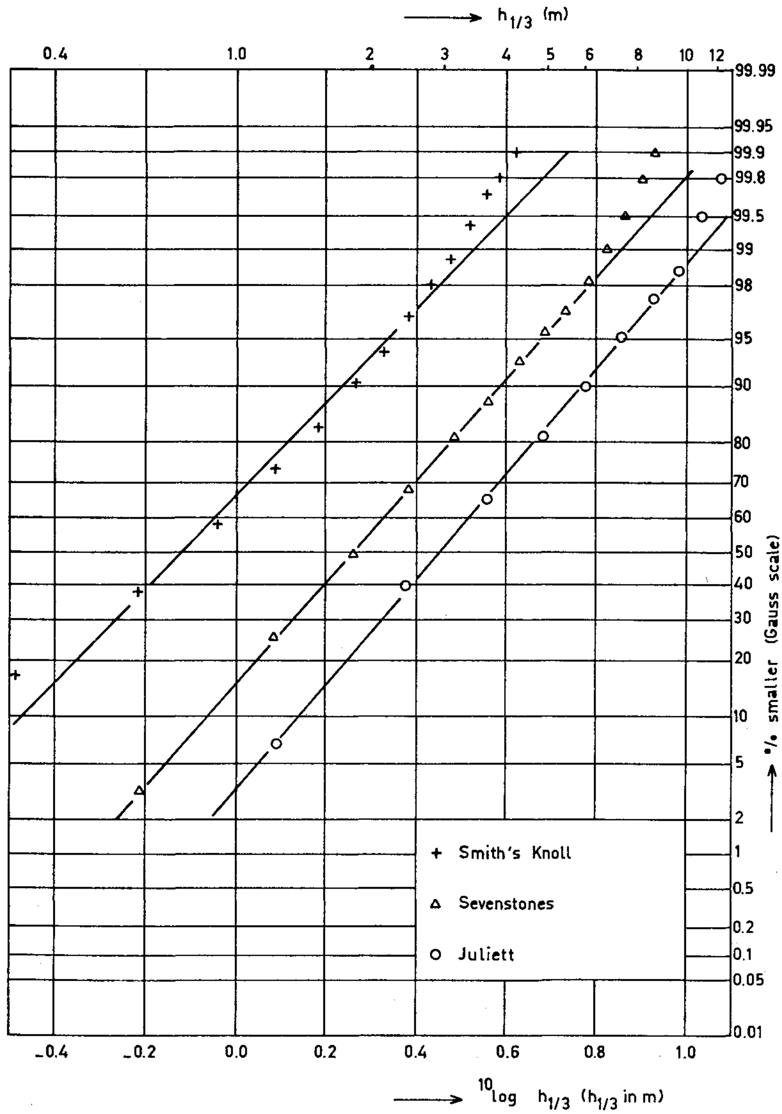

Fig. 2. Marginal distributions of $H_{1/3}$ plotted on log-normal paper

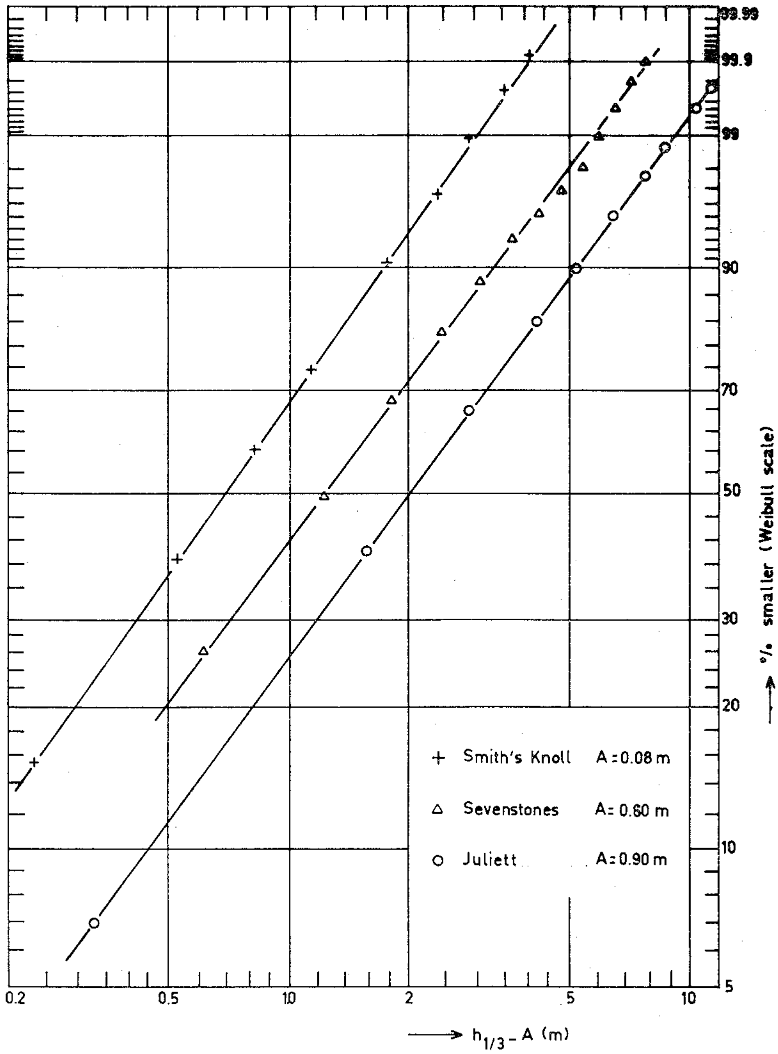

Fig. 3. Marginal distributions of $H_{1/3}$ plotted on Weibull paper

The seven stations where the data were obtained can be broadly grouped into three areas, as indicated in the last column of Table 2. It is noteworthy that the shape parameter C does not vary much between stations from one area, although it varies appreciably between areas. The parameter A, which can be loosely described as an indication of "background noise" (such as might be due to swells) appears to be positively correlated with the degree of exposure of the locations.

It was found that at all the stations the Weibull distribution fitted the data far better than the log-normal distribution. For three stations this can be seen from a comparison of Fig. 2 with Fig. 3.

Marginal distribution of H. The cumulative probability distributions of H were calculated on the basis of Eq. 6 and the scatter diagrams of which Fig. 1 is an example. Values of $P(h)$ were obtained for $h = 0$ ft., 4 ft., 8 ft., etc; up to a value of twice the maximum significant height measured at the station. This upper limit was chosen because it is fairly representative of the upper range of the measurements, inasmuch as for these data the most probable maximum wave height in 3 hours, as well as its expected value, is approximately twice the significant height. The results have been plotted in a coordinate system in which the Weibull distribution is represented by a straight line. The figures showed that a two-parameter Weibull distribution, with $A = 0$, fitted the computed values quite well, except for the lower range ($h < 8$ ft. appr.) at "India" and "Juliett". Examples are given in Fig. 4.

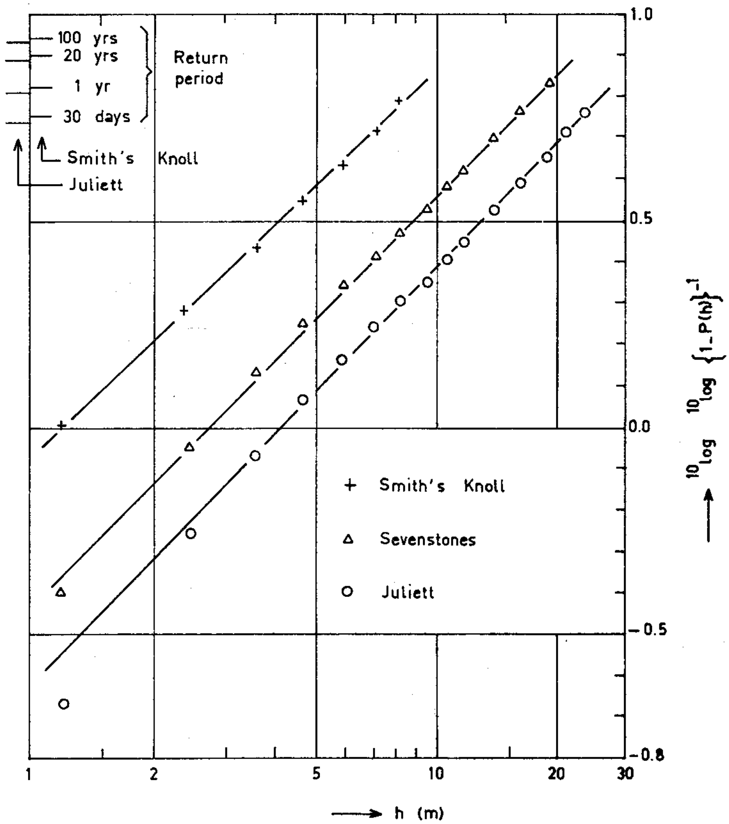

Fig. 4. Long-term distributions of H plotted on Weibull paper

The values of the scale – and shape parameters B and C were estimated from the straight lines drawn through the points by eye. They are given in Table 3 for the respective stations. The shape parameter C is fairly close to 1 in all cases but one ("Morecambe Bay"), which implies that the long-term distribution of individual wave height is nearly exponential. This type of distribution has previously been found to apply to wave-induced stress "heights" in a drilling rig (A. O. Bell and R. C. Walker [1971]) and in ship's hulls (N. Nordenstrøm [1965]).

Table 3
Parameters of long-term distribution of H

Station	B	C	F	$(\overline{T_z^{-1}})^{-1}$	$\{(1-F)\,\overline{T_z^{-1}}\}^{-1}$
	m			s	s
"India"	1.89	0.97	0	9.26	9.26
"Juliett"	1.91	0.99	0	9.34	9.34
"Sevenstones"	1.23	0.97	0	7.80	7.80
"Morecambe Bay"	0.50	0.85	0.159	4.98	5.92
"Mersey Bar"	0.62	1.06	0.517	4.82	9.98
"Varne"	0.76	1.03	0.065	5.25	5.61
"Smith's Knoll"	0.55	0.93	0	5.96	5.96

Three stations ("Morecambe Bay", "Mersey Bar" and "Varne") require special consideration because calms are reported there during a given fraction of time. The "calm" conditions are not defined explicitly in the original reports from which the scatter diagrams of $H_{1/3}$ and T_z were taken. The values have here been accepted at face value. In the calculation of $P(h)$ the integrations in Eq. 6 only extended over the values of $p(h_{1/3}, t_z)$ for the non-calm conditions. The resulting values of $P(h)$ must therefore be interpreted as the expected ratio between the number of waves for which $H \leq h$, and the total number of waves occurring. By definition, no waves occur during calms.

The occurrence of calms necessitates a slight modification of the relationship between return period y and cumulative probability $P(h)$. The expected number of waves per unit time, given that it is not calm, is $\overline{T_z^{-1}}$. If the fraction of time during which calms occur is F, then the expected number of waves in the return period is

$$n = y(1-F)\,\overline{T_z^{-1}} = \{1 - P(h_n)\}^{-1}. \tag{20}$$

Values of F and of the reciprocals of $\overline{T_z^{-1}}$ and $(1-F)\,\overline{T_z^{-1}}$ are given in Table 3. With these data, values of $P(h_n)$ can be computed for various values of the return period y. The corresponding values of h_n can be calculated from the Weibull distribution with $A = 0$, and B and C as given in Table 3. For $y > 1$ year the measured distributions have to be extrapolated. This extrapolation is hazardous for $y \gg 1$ year, even though the Weibull distribution gives a very good fit to the 1-year data, because of the considerable variation in wave intensity which can occur between years (H. Walden [1969]).

The marginal distributions of H were not only calculated according to Eq. 6, but also according to Eq. 7, in which the variability of T_z is ignored. Table 4 gives the results from both methods for station "India", for $h = 0$ (16) 96 ft. The effect of not taking the period variability into account is to over-estimate the probabilities of exceedance of individual wave heights. This is to be expected in view of the positive correlation between $H_{1/3}$ and T_z, as noted previously. The magnitude of the relative error increases with h. At all the 7 stations it was approximately 50% for the height with a return period of about 1 year (see the last line of Table 4 for the example of station "India").

Table 4

Probabilities of exceedance of individual wave heights at station "India"

h	$1 - P(h)$	
	acc. to Eq. 6	acc. to Eq. 7
ft		
0	1.0000	1.0000
16	0.6526×10^{-1}	0.7520×10^{-1}
32	0.5435×10^{-2}	0.6813×10^{-2}
48	0.5077×10^{-3}	0.6679×10^{-3}
64	0.4194×10^{-4}	0.5779×10^{-4}
80	0.3045×10^{-5}	0.4371×10^{-5}
96	0.1844×10^{-6}	0.2708×10^{-6}

Conclusions

1. The upper envelope bounding the observed values of the significant wave height $H_{1/3}$, for given values of the short-term mean zero crossing period T_z, has been shown to be consistent with current knowledge of energy spectra of wind-driven waves.
2. The statement that the logarithm of $H_{1/3}$ is Gaussian distributed does not apply to the data analysed herein.
3. The measured marginal significant wave height distributions can be well approximated by the Weibull function. This statement is based on visual inspection, rather than statistical tests of goodness-of-fit.

4. Long-term distributions of individual wave heights H have been calculated from the measured joint distributions of $H_{1/3}$ and T_z. The results are well described by a Weibull function with an exponent close to 1.
5. The long-term distribution of H is conventionally calculated from the marginal distribution of $H_{1/3}$, disregarding the effect of period variability. This leads to overestimates of the probabilities of exceedance of H.

Acknowledgements. The work reported herein was carried out during a visit to the National Institute of Oceanography, Wormley England, in June and July 1970. The author would like to express his thanks to the "Fundatie van de Vrijvrouwe van Renswoude" from the Hague, Holland, for awarding him a prize which made this visit financially possible; to Professor W. C. Bischoff van Heemskerck of the Delft University of Technology, for giving his permission for the author's temporary leave from the University; to Dr. G. E. R. Deacon, F.R.S., Director of the National Institute of Oceanography, for giving his permission for the author's stay at the National Institute of Oceanography; and to Mr. L. Draper of the National Institute of Oceanography for suggesting the subject of research and for his helpfulness in many other ways.

References

Battjes, J. A., 1970: Long-term wave height distributions at seven stations around the British Isles. Internal Rep., Nat. Inst. Oceanogr., Wormley No. A 44.

Bell, A. O. and R. C. Walker, 1971: Dynamic stresses in an offshore mobile drilling unit. Proc. Conf. on Dynamic Waves in Civil Engineering, Swansea. London: John Wiley and Sons, Ltd.

Bretschneider, C. L., 1965: Generation of waves by wind, state of the art. Nat. Eng. Science Co. Report SN–134–6, 96 pp.

Cartwright, D. E. and M. S. Longuet-Higgins, 1956: The statistical distribution of the maxima of a random function. Proc. R. Soc. (A) **237,** 212.

Cartwright, D. E., 1958: On estimating the mean energy of sea waves from the highest waves in a record. Proc. R. Soc. (A) **247,** 24.

Draper, L. and H. S. Fricker, 1965: Waves off Land's End. J. Inst. Navig. Lond. **18,** 180.

Draper, L. and M. A. B. Whitaker, 1965: Waves at Ocean Weather Ship Station "Juliett". Dt. hydrogr. Z. **18,** 25.

Draper, L. and E. M. Squire, 1967: Waves at Ocean Weather Ship Station India. Quart. Trans. R. Instn. nav. Architects **109,** 85.

Draper, L., 1968: Waves at Morecambe Bay Light Vessel, Irish Sea. Internal Rep., Nat. Inst. Oceanogr., Wormley No. A 32.

Draper, L., 1968: Waves at Smith's Knoll, North Sea. Internal Rep., Nat. Inst. Oceanogr., Wormley No. A 33.

Draper, L. and R. Graves, 1968: Waves at Varne Light Vessel, Dover Strait. Internal Rep., Nat. Inst. Oceanogr., Wormley No. A 34.

Draper, L. and A. Blakey, 1969: Waves at the Mersey Bar Light Vessel. Internal Rep., Nat. Inst. Oceanogr., Wormley No. A 37.

Draper, L. and M. J. Tucker, 1971: The determination of wave conditions for marine Engineering. Proc. Conf. on Dynamic Waves in Civil Engineering, Swansea. London: John Wiley and Sons, Ltd.

Gumbel, E. J., 1958: Statistics of extremes. Columbia Univ. Pr. 375 pp.

Hess, G. D., G. M. Hidy and E. J. Plate, 1969: Comparison between wind waves at sea and in the laboratory. J. mar. Res. **27,** 216.

International Ship Structure Congress, 1964: Report of Committee 1. Delft.

Jasper, N. H., 1956: Statistical distribution patterns of ocean waves and of wave-induced ship stresses and motions, with engineering applications. Prepr. Soc. Nav. Architects Mar. Engn. No. 6 (New York Meeting).

Lewis, E. V., 1967: Predicting long-term distributions of wave-induced bending moment on ship hulls. New York: Soc. Nav. Architects Mar. Engn. (Advance copy of pap. to be presented at Spring Meeting, Montreal).

Longuet-Higgins, M. S., 1952: On the statistical distribution of the heights of sea waves. J. mar Res., **11,** 245.

Nordenstrøm, N., 1965: Further analysis of full scale measurements of midship bending moments. Division of Ship Design, Chalmers Univ. of Technology., Gothenburg.

Nordenstrøm, N., 1969: Long term distribution of wave heights and periods. Oslo: Det Norske Veritas, Rep. No. 69–21–S, 27 pages.

Phillips, O. M., 1966: The dynamics of the upper ocean. Cambridge Univ. Press, 261 pages.

Rice, S. O., 1944, 1945: The mathematical analysis of random noise. Bell Syst. Techn. J. **23,** 282, and **24,** 46.

Tucker, M. J., 1961: Simple measurement of wave records. Dock Harbour Author. **42,** 231.

Walden, H., 1969: Zur Variabilität des Seegangs innerhalb langer Zeiträume. Dt. hydrogr. Z. **22,** 66.

Eingegangen im März 1972

Anschrift des Verfassers: J. A. Battjes, Technische Hogeschool Delft, Afdeling der Weg- en Waterbouwkunde, Delft, Oostplantsoen 25, The Netherlands