Neural Processing Letters, Vol. 1, No. 1, 6-8, 1994

Fractal variation of attractors in complex-valued neural networks

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Abstract. Fractal variation of dynamical attractors is observed in complex-valued neural networks where a negative-resistance nonlinearity is introduced as the neuron nonlinear function. When a parameter of the negative-resistance nonlinearity is continuously changed, it is found that the network attractors present a kind of fractal variation in a certain parameter range between deterministic and non-deterministic attractor ranges. The fractal pattern has a convergence point, which is also a critical point where deterministic attractors change into chaotic attractors. This result suggests that the complex-valued neural networks having negative-resistance nonlinearity present the dynamics complexity at the so-called edge of chaos.

1. Introduction

The complex-valued neural networks are the extended version of conventional real-valued neural networks [1, 2]. Input and output signals, weighting factors, and neuron nonlinear functions are determined using complex number so that the information geometry of the network is constructed in complex space. This feature is advantageously used especially for smooth learning and expression of dynamical attractors [3]. The complex-valued neural networks can treat the signal phase in microscopic neurons as well as macroscopic neural networks [4].

On the other hand, chaotic aspects in biological and artificial neural networks are very attractive when we deal with time-sequential behaviour of the networks [5, 6]. Such chaotic characteristics of the neural networks are sometimes related to spontaneous process in brains or probabilistic operations of biological networks. In many cases, the chaotic behaviour originates from the neuron dynamics having negative-resistance-type nonlinearity.

In this paper, a fractal variation of attractors in complex-valued neural networks is reported. When a negative-resistance nonlinearity is introduced in a complex-valued associative memory which has originally periodic attractors, the phase relation between neuron outputs becomes unstable. As a loose nonlinearity varies continuously to a steeper one, the attractor-variation diagram shows a kind of fractal pattern between the parameter ranges of deterministic and non-deterministic attractors. The fractal pattern converges at a critical point, at which the deterministic attractor also changes into the non-deterministic one. This result suggests that the complex-valued neural networks having negative-resistance nonlinearity present the dynamics complexity at the so-called edge of chaos [7].

2. Network contruction

Figure 1 shows the construction of the complex-valued associative memory. The memory has one connection layer, where both the weighting factors $W \equiv [w_{kj}]$ and the neuron nonlinearity are determined in complex space. A complex-valued input signal vector $x \equiv [x_j]$ is fed to the associative memory as a trigger. Obtained output signals are iteratively fed to and transformed through the network. After a certain number of iteration, an attractor is recalled depending on the input signals x and the weighting matrix W.

In the complex-valued neural networks, the signal amplitude is transformed nonlinearly at each neuron, whereas the phase information is left almost unchanged. When the neuron nonlinearity is a simple saturation function such as sigmoidal function, we

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obtain stationary, circular, or quasi-periodic attractors. Such smooth dynamical attractors arise mainly from the phase information of the signal vector and the weighting factors.

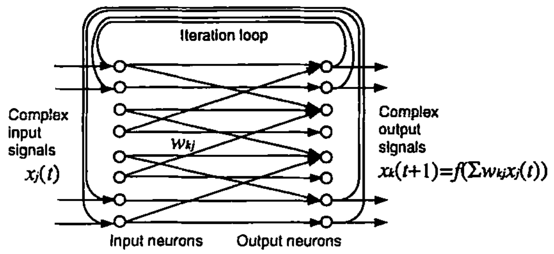

Fig. 1. Basic construction of complex-valued associative memory.

In this paper, on the other hand, we introduce a negative-resistance nonlinearity. The nonlinear function f, which is used at each neuron in this network, is defined as

$$f\left(\sum_j w_{kj}x_j\right)=r\left(\left|\sum_j w_{kj}x_j\right|\right)\exp\left[i\arg\left(\sum_j w_{kj}x_j\right)\right], \tag{1}$$

where the real-number function $r(u)$, which determines the output-signal amplitude, is a negative-resistance nonlinear function that is a modified version of the sigmoidal function with a parameter a:

$$r(u)=\begin{cases}1-u/a & (0\le u\le a)\\ \tanh(u/a-1) & (a\le u).\end{cases} \tag{2}$$

In the range that the amplitude of weighted and summed input $u \equiv |\sum_j w_{kj}x_j\ |$ is less than the parameter a, the function $r(u)$ is monotonically reducing (negative-resistance characteristic); otherwise it is a conventional sigmoidal function. When the parameter a becomes smaller, the gradient in both the negative-resistance and sigmoidal regions becomes steeper.

3. Experiment

In this experiment, the numbers of both the input and output neurons are 50, respectively, and all operations are synchronous in discrete time steps. Before the negative-resistance nonlinearity is introduced, the neurons have learned sinusoidal trajectories having different frequencies. The output signal of each output neuron $x_k(t)$ is sinusoidal oscillation expressed as

$$x_k(t) = A\exp[i\omega_k t], \tag{3}$$

where the angular frequencies are chosen $\omega_k \equiv k \times 2\pi/T$ depending on the neuron index k ($1\le k\le 50$) with a long-term time step T (= 64 unit time steps). In equation (3), A denotes an amplitude constant and chosen at 0.64 in this case.

Figure 2 shows a typical attractor variation obtained for the complex-valued associative memory with a negative-resistance nonlinearity expressed by (2) when the parameter a is varied. In figure 2, small dots present the real part of one of the output signals $\mathrm{Re}[x_1(t)]$ which is recorded when the output signal value of a neighbour neuron $x_2(t)$ crosses the real axis (i.e. $\arg[x_2(t)]=0$). This attractor-variation diagram shows the output behaviour in a temporal term of $50 \times T$ which follows a long period ($20 \times T$) after the recalling trigger, so that the recorded data present attractors without transient.

Since the original attractor is periodic as is expressed by (3), the phase relation between the two neurons ($x_1(t)$ and $x_2(t)$) is stable when the negative-resistance nonlinearity is loose. This loose parameter range corresponds to the larger values of parameter a in figure 2 ($1.19 < a$). In this range, two lines of dots express the doubled frequency of $x_2(t)$ compared with that of $x_1(t)$, i.e $\omega_2 = 2\omega_1$. On the other hand, as the parameter a decreases, the steepness of the nonlinearity expressed by (2) increases. In such a small-a range in figure 2 ($a < 0.82$), the output signal is extremely noisy.

In the middle range ($0.99 < a < 1.18$), it is found that the dotted region shows a kind of fractal pattern. Each attractor (which corresponds to a certain value of parameter a) is recorded as dots within restricted segments of lines, and the overall attractor variation constructs the fractal pattern. The fractal pattern has a convergence point at $a \approx 1.18$. (Two-dimensionally on the variation diagram, there are 3 convergence points for the same value of parameter a.) This convergence point is recognized also as a critical point at which deterministic attractors ($1.18 < a$) change into chaotic attractors ($a < 1.18$). Accordingly it is found that the complex-valued neural networks having negative-resistance nonlinearity present clearly the so-called edge of chaos. This result also suggests that this network has the dynamics complexity at the edge of chaos.

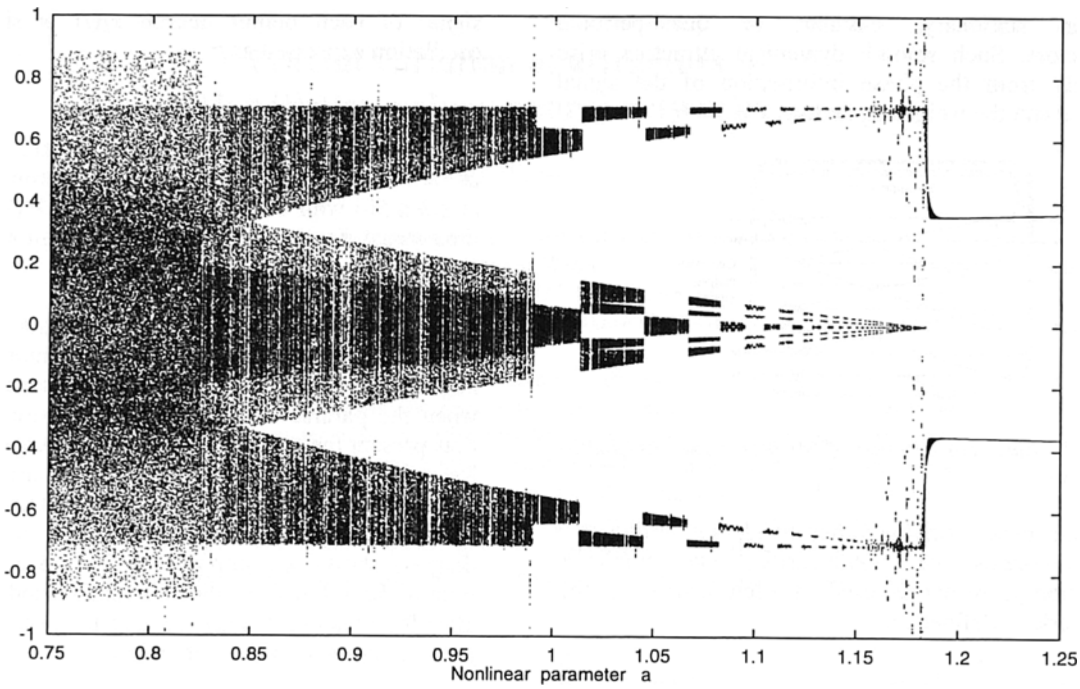

Fig. 2. Attractor-variation diagram dotted for a neuron output value $Re[x_1(t)]$ when the output of a neighbour neuron $x_2(t)$ crosses the real axis; i.e., $arg[x_2(t)] = 0$.

4. Conclusion

Fractal variation of dynamical attractors has been observed in complex-valued neural networks when a parameter of negative-resistance nonlinearity is continuously changed. The fractal pattern has a clear convergence point in the attractor-variation diagram. This parameter point is also the critical point at which deterministic attractors change into chaotic attractors. This result suggests that this network presents the dynamics complexity at the edge of chaos.

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Acknowledgments

The author is grateful very much to Prof. Rolf Eckmiller at the Institute for Neuroinformatics, University of Bonn, and Prof. Yoichi Okabe at the Research Center for Advanced Science and Technology (RCAST), University of Tokyo, for their continuous encouragement. This work is partly supported by the Japan Society for the Promotion of Science (JSPS).