Welfare Effects of Price Discrimination When Demand Curves are Constant Elasticity

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Introduction

Traditionally, the desirability of monopolistic "third degree" price discrimination has been evaluated in terms of its effect on output [Battalio and Ekelund 1972; Edward 1950; Finn 1974; Robinson 1933]. Recently, however, Yamey [1974] attempted to show that while a positive output effect of a two-price scheme was necessary to increase social welfare, it was not a sufficient condition. Price discrimination also works to misallocate goods from high to low value users and this cost is not necessarily outweighed by a positive quantity effect.

Yamey, however, did not prove that optimal price discrimination by a monopolist would in fact yield the perverse result—that is, output increasing but welfare reducing price discrimination. Moreover, he ignored recent contributions that suggest that monopoly profits may not be simple transfers [Ippolito 1976; Posner 1975; Tullock 1967]. The purpose of this paper is to examine the effects of simple price discrimination in the case of constant elasticity demand curves. Simulation results are provided over a wide range of parameter values. In every case, the quantity effect of price discrimination is positive. Yet, the welfare effect—whether profits are costs or transfers—is negative in a significant number of cases. Given the interpretation of profits as social costs or transfers, the critical determinants of the net welfare effect are the share of the most elastic market in the single-price equilibrium and the ratio of demand elasticities. Notably, the ratio of prices in the two-price equilibrium is found to be an unreliable index of the net welfare effect.

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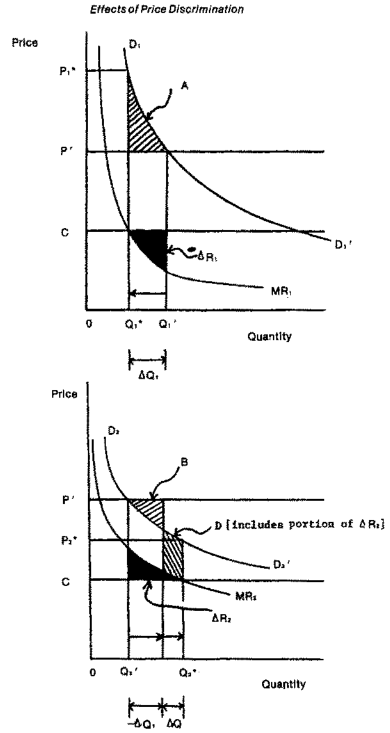
Effects of Price Discrimination

Assume that a monopolist can separate its market into two independent segments as shown by the schedules D_1D_1' and D_2D_2' in Figure I; market 2 exhibits a higher absolute elasticity than market 1. It is also assumed that the discrimination can be effected costlessly and that both markets are served at the optimal single price. The marginal cost of production is C and the optimal single price is P'.

FIGURE I

Effects of Price Discrimination



The monopolist will set price in either market so that marginal cost C equals marginal revenue MR_i; thus, prices are set equal to P_i^*, $i = 1,2$. Quantity in either market will therefore equal Q_i^* as shown in the figure. Price in the less elastic market increases from P' to P_1^* relative to the single price equilibrium, and therefore output falls from Q_1' to Q_1^*. In the more elastic market, price falls from P' to P_2^*, and hence output increases from Q_2' to Q_2^*. Profits also increase by the shaded areas $\Delta R_1 + \Delta R_2 = \Delta R$ in Figure I.

Assume first that profits are pure transfers from consumers to producers. Given this assumption, any change in social welfare from price discrimination is found by the direct summation of changes in producer and consumer surplus. The welfare effect is therefore initially separated into an output and misallocation effect; the rent erosion effect is subsequently considered.

Misallocation Effect

It is useful for expositional purposes to separate the total change in output in the second more elastic market into two parts. In particular, it must be true that $\Delta Q_2 = -\Delta Q_1 + \Delta Q$, where ΔQ is the change in total market output owing to price discrimination and ΔQ_1 $(=Q_1^* - Q_1')$ is the reduction in output in the first market in the two vis-a-vis the one-price scheme. Viewing output changes in this way, the reduction in output in the more inelastic market ΔQ_1 is seen as an increase in output in the more elastic market.

If the demand curve in either market is described by $P_i = P_i(Q_i)$, $i = 1,2$, the social loss owing to the transfer of output ΔQ_1 from high to low value consumers may be calculated as

$$A + B = \int_{Q_1^*}^{Q_1'} P_1(Q_1)\,dQ_1 - \int_{Q_2'}^{Q_2' - \Delta Q_1} P_2(Q_2)\,dQ_2 \qquad (1)$$

Since each marginal reduction in Q_1 is valued by some amount that is greater than P', while each marginal increase in Q_2 is valued by some amount that is less than P', the misallocation effect of price discrimination described in (1) is negative; that is, $A + B > 0$. The values of A and B are depicted graphically by the cross-hatched triangle-type areas in Figure I.

Quantity Effect

Having so described the misallocation effect from price discrimination, the output effect is easily calculated by considering the remaining change in output in the second market ΔQ. A priori, the sign of ΔQ is indeterminate [Robinson 1933], but a case has been depicted in Figure I where the quantity effect is positive. Since the marginal cost of expanding output in the second market is C, it follows that the net change in consumer and producer surplus owing to the change in output is described by

$$D = \int_{Q_2' - \Delta Q_1}^{Q_2^*} P_2(Q_2)\,dQ_2 - C(Q_2^* - Q_2' + \Delta Q_1) \qquad (2)$$

If the quantity effect is positive, i.e., if $Q_2^* > Q_2' + |\Delta Q_1|$, then the output effect on social welfare is also positive because the social valuation of the increase in output exceeds the marginal **social** cost; this effect is shown by the cross-hatched trapezoidal-type area labelled D in Figure I. Thus, price discrimination works to increase social welfare if (a) the output effect is positive, and (b) the positive output effect outweighs the unfavorable misallocation effect.[1]

[1] In the regulated milk market, a peculiar price discrimination scheme is enforced that generates a level of output that actually exceeds the socially optimal level; thus, in that model, the output effect works in the same direction as the misallocation effect, creating unambiguous social loss [Ippolito and Mason, 1977].

When profits are transfers, the welfare effect is written as

$$\Delta W_T = D - (A + B) \qquad (3)$$

Rent Erosion

The condition in (3) assumes that the excess rent earned by a monopolist is preserved. If the market for monopoly is competitive, however, then, *ex ante,* this assumption may be inappropriate. The process of competition for the promise of monopoly rent (as, say, in the case of patents) will tend to induce expenditures of resources in the hope of attaining these profits. It has been shown elsewhere that the degree of rent erosion will vary depending upon the particular nature of the competitive process [see Ippolito 1976; Tullock 1967].

In the case, for example, where a monopoly is established before it is realized that price discrimination can be profitably practiced, then the rent erosion effect will not apply to the creation of a discrimination scheme. As long as discrimination is seen *ex ante* as a possible corollary to the creation of monopoly, however, at least some portion of the incremental rent attributable to discrimination will be subject to erosion. For illustrative reasons, complete rent erosion provides a useful contrast to the pure transfer case. When profits are considered social costs, price discrimination improves welfare if the output effect is large enough to outweigh the misallocation effect and the rent erosion effect. In particular, the change in welfare owing to price discrimination is now denoted by

$$\Delta W_{SC} = D - (A + B) - \Delta R \qquad (4)$$

Simulation Results

We now consider the circumstances under which the expressions in (3) and (4) are likely to be positive. While these results depend upon the particular nature of the demand functions that characterize the relevant markets, some insight is gained if the welfare calculations ΔW_T and ΔW_{SC} are solved for the case where markets are characterized by constant elasticity. There are two motivations for this choice of functional form. First, a large number of empirical studies have shown that constant elasticity demand forms are reasonably consistent with observable price-quantity demand relations.

Second, the constant elasticity function is defined over the interior price-quantity space, giving it a significant analytical advantage over its empirical rival, the linear demand curve. In the latter case, it is easy to show that if both markets are served at the optimal single price, the quantity effect from price discrimination is zero. Hence, the welfare effect from discrimination cannot be positive.

If one market is not served at the optimal single price, however, the misallocation effect is zero and the quantity effect is positive; thereby yielding a positive welfare effect. Examination of the constant elasticity case provides an opportunity to verify the crucial role played by relative market shares at the optimal single price. More importantly, it offers an opportunity to analyze how the net results depend upon critical parameters along a continuous spectrum.

Specifying the demand functions in the constant elasticity form, $Q_i = A_i P_i^{E_i}$, where A_i is a constant and E_i is a constant elasticity parameter, the calculations ΔW_T and ΔW_{SC} were solved for the following parameter values: A_2/A_1 assumed values of .1 to 1.0 in increments of .1, then assumed assorted values to a bound of 10. For each of these values, the elasticity in the less elastic market, E_1, was varied from -2 to -10 in increments of -1; $E_1 = -25$ was picked as a limiting value. Finally, for each of these values, E_2 was set equal to $.1E_1$ to $.9E_1$, subject to the constraint that $E_2 < -1.0$. A total of 1312 observations were generated.

When profits are considered social costs, the welfare effect is positive in only 12 percent of the cases in the simulation exercise. When monopoly profits are considered as transfers, the welfare effect is positive in 31 percent of the cases. Additionally, in 19 percent of the cases, ΔW_T was positive while the measure ΔW_{SC} was negative.

The quantity effect was positive in every case, and hence was an unreliable index of the true welfare effect of discrimination. That is, the welfare measure ΔW_{SC} and the quantity effect were oppositely signed in 88 percent of the cases; the measure ΔW_T and the quantity effect were oppositely signed in 69 percent of the cases. Thus, when functional forms are constant elasticity, the quantity effect does not provide a useful index of the welfare effect of price discrimination. Hence, the linear case is shown to provide a special case in this regard.

Like the linear case, however, the welfare effect is highly correlated with the share of the more elastic market at the optimal single price. To see this, examine the results listed in Table 1. When the market parameters are such that the share of the more elastic market, *s*, at the optimal single price is less than .01, the welfare effect, using either measure is positive in almost all cases. When *s* exceeds .05, however, the welfare effect, using the measure ΔW_{SC} is positive in only three percent of the cases. The measure ΔW_T remains positive in a significant, although diminishing, proportion of cases until *s* reaches .30.

An additional useful market parameter in predicting the sign of the welfare effect was the magnitude of the relative elasticities E_1/E_2. This ratio was necessarily greater than 3.0 before welfare, measured by ΔW_{SC}, increased owing to discrimination. In addition, the probability that a positive welfare effect would occur, using the measure ΔW_T, increased monotonically with this parameter. These results do not imply, however, that relative prices served an equally useful role. Relative prices under a two-price scheme are functions of the absolute elasticities and relative elasticities; hence, relative prices and relative elasticities are not necessarily highly correlated. The sign of the welfare effect, using either measure, however, was generally negative when the price spread between markets was less than 100 percent.

These preliminary simulation results serve to illustrate that, for demand curves of constant elasticity form, monopolistic price discrimination can be desirable. In particular, two-price schemes are welfare increasing when the market share of the more elastic market is small and when relative elasticities between markets are substantially different. The possibility of positive welfare effects are considerably enhanced when monopoly profits are transfers. But the existence of a positive output effect per se is not sufficient to show that price discrimination is welfare increasing.

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TABLE 1

SIMULATION RESULTS: INDEXES OF POSITIVE WELFARE EFFECTS

Category	X_i	X_i^*/X_i	
		$\Delta W_{SC} > 0$;	$\Delta W_T > 0$
$.00 < s \leqslant .01$	115	.97	1.00
$.01 < \quad \leqslant .05$	115	.41	.96
$.05 < \quad \leqslant .10$	135	.02	.57
$.10 < \quad \leqslant .20$	155	.00	.45
$.20 < \quad \leqslant .30$	116	.00	.28
$.30 < \quad \leqslant .40$	147	.00	.04
$1.00 < E_1/E_2 \leqslant 1.50$	540	.00	.03
$1.50 < \quad \leqslant 2.00$	342	.00	.30
$2.00 < \quad \leqslant 3.00$	162	.00	.54
$3.00 < \quad \leqslant 4.00$	144	.47	.74
$4.00 <$	106	.87	.88
$1.00 < P_2/P_1 \leqslant 2.00$	1179	.07	.27
$2.00 < \quad \leqslant 3.00$	70	.66	.74
$3.00 < \quad \leqslant 4.00$	18	.22	.44
$4.00 <$	45	.62	.71

X_i: number of observations in the i^{th} category.

X_i^*: number of observations in the i^{th} category where the direct welfare effect was positive.

ΔW_{SC}: denotes the change in social welfare when monopoly profits are social costs.

ΔW_T: denotes the change in social welfare when monopoly profits are transfers.