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THE WEIGHTED OBLIMIN ROTATION

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Cureton & Mulaik (1975) proposed the Weighted Varimax rotation so that Varimax (Kaiser, 1958) could reach simple solutions when the complexities of the variables in the solution are larger than one. In the present paper the weighting procedure proposed by Cureton & Mulaik (1975) is applied to Direct Oblimin (Clarkson & Jennrich, 1988), and the rotation method obtained is called Weighted Oblimin. It has been tested on artificial complex data and real data, and the results seem to indicate that, even though Direct Oblimin rotation fails when applied to complex data, Weighted Oblimin gives good results if a variable with complexity one can be found for each factor in the pattern. Although the weighting procedure proposed by Cureton & Mulaik is based on Landahl's (1938) expression for orthogonal factors, Weighted Oblimin seems to be adequate even with highly oblique factors. The new rotation method was compared to other rotation methods based on the same weighting procedure and, whenever a variable with complexity one could be found for each factor in the pattern, Weighted Oblimin gave the best results. When rotating a simple empirical loading matrix, Weighted Oblimin seemed to slightly increase the performance of Direct Oblimin.

Key words: component analysis, factor analysis, oblique rotation, weighted rotation, Direct Oblimin, Promax, Varimax

1. The Weighted Oblimin Rotation

In factor analysis, the retained factors are often orthogonally and obliquely rotated in an attempt to eliminate the indeterminacy of their final position. The rotation aims to eliminate this indeterminacy rotating the pattern so that it becomes as simple as possible, enabling the solution obtained to be interpreted. Thurstone (1947) proposed an objective definition of a simple solution by means of a set of simple structure criteria. These criteria can be summarized as follows: A solution is a simple solution if (a) the variables in the analysis have loadings different from zero in only one factor in the pattern, and (b) all the factors in the pattern have some variables with loadings different from zero (Thurstone). The variables in such a simple solution are known as variables of complexity one, as they only have one loading different from zero in the pattern.

Since the simple structure criteria were proposed, many orthogonal and oblique rotation methods have been proposed to rotate solutions to a simple structure. Some of these are Normal Varimax (Kaiser, 1958), Harris-Kaiser's methods (Harris-Kaiser, 1964), Promax (Hendrickson & White, 1964) and Direct Oblimin (Clarkson & Jennrich, 1988). However, all these procedures may not find an interpretable solution if the solution has complex variables. An example of a solution in which these rotation procedures fail is the well-known 26 Box Problem (Thurstone, 1947). In this example, a considerable number of the variables have complexities larger than one: When the solution is rotated to a truly simple pattern, the variables have loadings different from zero in more than one factor.

Cureton & Mulaik (1975) summarize the situations in which Normalized Varimax can fail to obtain an interpretable solution. These situations are the following:

1. If a considerable number of variables have significant loadings on more than one factor, and only a few variables have one significant loading on each factor.

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2. If the number of variables with near-zero loadings on one factor in the initial orthogonal factor solution is larger than the number of factors.

One, or both, of these situations can be found when variables in the solution to be rotated have complexities larger than one. To deal with solutions in which this kind of variable is found, Cureton & Mulaik (1975) proposed the Weighted Varimax rotation procedure.

Weighted Varimax rotates the pattern so that the final position of the axes is mainly determined by the simplest variables in the pattern, while the complex variables in the pattern do not have much influence on this final position. So Weighted Varimax needs:

1. to detect the complex variables in the pattern before a rotation is performed,
2. to weight any variable in the pattern to be rotated, so that the simplest variables have a high weight in the rotation, while the complex variables (depending on how complex they are) have less.

Both procedures are described in detail below. Once the complex variables have been detected, and the weights for each variable computed, the Varimax rotation (Kaiser, 1958) is computed over the weighted pattern. It should be said that Weighted Varimax can successfully rotate such complex solutions as the 26 Box Problem (Cureton & Mulaik, 1975). However, as Cureton & Mulaik warned, the procedure may fail if the factors in the solution have a high correlation among one another. An anonymous reviewer noted that the procedure may also fail if a variable with complexity one cannot be found for each factor in the pattern.

Cureton & Mulaik (1975) combined Weighted Varimax with Promax (Hendrickson & White, 1964). Complex oblique data can be rotated and good results obtained if the orthogonal Weighted Varimax solution is taken as a starting point for Promax. This rotation procedure gives very good results when rotating Thurstone's 26 Box Problem (1947).

Weighted Varimax has also been incorporated in Promin (Lorenzo-Seva, 1999) and results have been good with complex solutions. Other rotation methods that can deal with complex variables are Orthosim and Oblisim (Bentler, 1977), Promaj (Trendafilov, 1994), and, in particular, Simplimax (Kiers, 1994). Although Simplimax can deal with very difficult situations, it is also the most complex to use.

The aim of this paper is to show that the weighting procedure proposed by Cureton & Mulaik (1975) can be applied to Direct Oblimin (Clarkson & Jennrich, 1988) and provide good results. As Direct Oblimin (Clarkson & Jennrich, 1988) is one of the most popular oblique rotation procedures, it seems reasonable to improve its performance using this weighting procedure. Not only can Weighted Varimax deal with complex solutions but it can also slightly improve Varimax rotation when rotating simple solutions (Cureton & Mulaik, 1975). So Weighted Oblimin may also improve the performance of Direct Oblimin (Clarkson & Jennrich, 1988) when rotating this kind of simple solution.

The algorithm for the Weighted Oblimin rotation is described below. Then, a simulation study is made with artificial data to test the performance of Weighted Oblimin. Thurstone's 26 Box Problem (1947) is rotated and compared to the solution obtained by Cureton & Mulaik (1975). As this problem is a very special case, a new data sample based on the Box Problem is also generated and rotated. Finally, Weighted Oblimin is applied to a real sample data set.

2. The Weighted Oblimin Rotation

Weighted Oblimin applies exactly the same weighting procedure as Weighted Varimax (Cureton & Mulaik, 1975). To understand this weighting procedure fully, the original paper should be read carefully. This paper describes the procedure briefly, adapted to the Direct Oblimin (Clarkson & Jennrich, 1988) rotation. As has been explained before, this procedure first detects the complex variables in the pattern, and then decides their weight in the rotation.

2.1. Detecting Complex Variables in the Pattern

Let the $r \times m$ matrix $\mathbf{P}$ be an orthogonal pattern matrix of r variables and m factors which is to be rotated, and the $r \times r$ matrix $\mathbf{R}^*$ the reproduced correlation between variables matrix,

$$\mathbf{R}^* = \mathbf{PP}' \tag{1}$$

If $\mathbf{R}^*$ is analyzed by Principal Components and m factors are retained, then the $r \times m$ matrix $\mathbf{F}$ will be the orthogonal pattern obtained from the factor extraction of $\mathbf{R}^*$.

Let $\mathbf{D}(r \times r)$ be a diagonal matrix that is computed as,

$$\mathbf{D} = \text{diag}(\mathbf{FF}')^{-1/2} \tag{2}$$

Then, the $r \times m$ matrix $\mathbf{G}$ is computed as the product

$$\mathbf{G} = \mathbf{DF} \tag{3}$$

Now $\mathbf{G}$ is the row-normalized matrix of $\mathbf{F}$. Let the $r \times r$ diagonal matrix $\mathbf{H}$ be a reflection matrix, where h_{jj} is the j-th diagonal element of $\mathbf{H}$. Each h_{jj} element has a value of 1, except if the element g_{j1} of $\mathbf{G}$ is negative. In this case the value of h_{jj} will be -1. Finally, the $r \times m$ matrix $\mathbf{A}$ is computed as,

$$\mathbf{A} = \mathbf{HG}. \tag{4}$$

Now the first column in $\mathbf{A}$ is a vector in an r-dimensional space that is placed approximately in the middle of the whole cluster of variables in $\mathbf{A}$, and the loadings on this factor are the projections of the variables on this vector.

Since each column in $\mathbf{A}$ pertains to a unit length factor, and each projected variable has now been normalized, the cosine of the angle between the i-th projected variable and this first factor is a_{i1}, where a_{i1} is the first loading of the i-th variable in the matrix $\mathbf{A}$.

If the elements in the first column of $\mathbf{A}$ are large when compared to the other factors (so the first factor explains more variance than any of the other factors), the first factor has another important feature: all the factors of an orthogonal approximation to simple structure will lie more or less symmetrically around this first factor. If the symmetry is perfect, the cosine of the angle between each factor and this first factor is $(1/m)^{1/2}$ (Landahl, 1938); it is smaller if the oblique configuration of the axes is acute, and greater if the oblique configuration of the axes is obtuse.

Now it can be decided which variables are complex: variables with a_{i1} loadings close to $(1/m)^{1/2}$ are the simplest in the pattern, while variables with a_{i1} loadings close to 1 are the most complex.

2.2. Weighting Variables in the Pattern

Let the $r \times r$ matrix $\mathbf{W}$ be a diagonal matrix. The w_{ii} is a diagonal element of $\mathbf{W}$ and its value is the weight of the i-th variable during the rotation procedure. Cureton & Mulaik (1975) proposed that each weight should be computed using two expressions. If a_{i1} is larger or equal to $(1/m)^{1/2}$,

$$w_{ii} = \cos^2\left(\frac{\arccos((1/m)^{1/2}) - \arccos(a_{i1})}{\arccos((1/m)^{1/2})}\arccos(0)\right) + 0.001. \tag{5}$$

If a_{i1} is smaller than $(1/m)^{1/2}$,

$$w_{ii} = \cos^2\left(\frac{\arccos(a_{i1}) - \arccos((1/m)^{1/2})}{\arccos(0) - \arccos((1/m)^{1/2})}\arccos(0)\right) + 0.001. \tag{6}$$

Variables with a_{i1} values equal to $(1/m)^{1/2}$ will have corresponding w_{ii} values of 1.001, while variables with a_{i1} values equal to 1 or 0 will have corresponding w_{ii} values of 0.001. Variables in the middle will have intermediate weight values. The most complex variables in **A** are the ones with least influence on the final position of the rotated factors.

As the Weighting Procedure is based on Landahl's (1938) expression for orthogonal factors, the procedure may fail if the ideal factors are correlated. Cureton & Mulaik (1975) implicitly suggested that their weighting procedure can be used when interfactor correlations are nonzero, since they used Weighted Varimax as an initial orthogonal rotation to Promax. Actually, the weighting procedure seems to give acceptable results when applied to highly oblique factors. Note that the maximum value of $w_{ii} = 1.001$ will be obtained by variables with complexity one in a pattern corresponding to perfectly orthogonal factors. As factors become more closely correlated, the values of all w_{ii} will usually decrease. In a correlated case, values w_{ii} will be small but more or less proportional to values w_{ii} in an orthogonal case. This means that the simplest variables will normally have larger w_{ii} values than the complex variables. In an extremely correlated case, all the complex variables will have values $w_{ii} = 0.001$, whereas the simplest variables will have values that are small but still higher than 0.001. When rotating correlated factors, all the values of the product **WA** to be rotated will have small values, but this does not seem to be a problem when computing Direct Oblimin (Clarkson & Jennrich, 1988).

2.3. *Obliquely Rotating the Orthogonal Weighted Pattern*

Once the weighting matrix **W** has been computed, the Direct Oblimin solution for the product **WA** is computed to obtain the $r \times m$ matrix **V** as

$$\mathbf{V} = \text{oblimin}(\mathbf{WA}, \gamma = 0) \tag{7}$$

where the matrix **V** is the Direct Oblimin pattern that has been weighted and reflected, and γ is the parameter of the Direct Oblimin rotation. The $r \times m$ final oblique pattern Λ is computed as,

$$\Lambda = \mathbf{HD}^{-1}\mathbf{W}^{-1}\mathbf{V}. \tag{8}$$

The $m \times m$ oblique transformation matrix **T** from the orthogonal pattern **P** is computed as,

$$\mathbf{T} = (\mathbf{P}'\mathbf{P})^{-1}\mathbf{P}'\Lambda. \tag{9}$$

So,

$$\Lambda = \mathbf{PT}, \tag{10}$$

and the $m \times m$ interfactor correlation matrix Φ is computed as,

$$\Phi = \mathbf{T}^{-1}\mathbf{T}^{-1'}. \tag{11}$$

As Weighted Oblimin has been presented with the γ parameter equal to zero, the rotation method that has been used here is Direct Quartimin (the simplest member of the Oblimin family). As results seems to be good when $\gamma = 0$ (Jennrich, 1979), in this paper Weighted Oblimin will always be computed in this way.

3. Simulation Study

The main purpose of this study is to compare the performance of Weighted Oblimin with Direct Oblimin (Clarkson & Jennrich, 1988) and other rotation procedures that seem to perform well with complex solutions. The software used in this study was Matlab 4.0 (The Mathworks, 1984).

The rotation procedures that are compared are:

1. Weighted Varimax (Cureton & Mulaik, 1975) followed by Promax ($k = 4$) (Hendrickson & White, 1964): this rotation procedure was proposed by Cureton & Mulaik (1975). Here it will be called Weighted Promax. Varimax (Kaiser, 1958) rotation computed in Weighted Varimax was started from five random positions and care was taken to prevent any possible permutation effect (ten Berge, 1995).
2. Promin (Lorenzo-Seva, 1999): this rotation procedure is based on Weighted Varimax (Cureton & Mulaik, 1975) followed by an oblique semi-specified Procrustes rotation (Browne, 1972; Meredith, 1977). Varimax rotation was computed as with Weighted Promax.
3. Promaj (Trendafilov, 1994): this procedure consists of orthogonally rotating the pattern with Orthosim (Bentler, 1977) and then using an Oblique Procrustes rotation (see Trendafilov, 1994, for more details). Here Promaj is applied in the same way. This procedure does not apply the Weighting technique proposed by Cureton & Mulaik (1975), and this makes the comparison quite interesting. Orthosim was started from five random positions. The algorithm for the Oblique Procrustes rotation is the one proposed by Browne (1972) but it is fully specified.
4. Direct Oblimin ($\gamma = 0$) (Clarkson & Jennrich, 1988): this rotation procedure might fail frequently, as the study is mainly designed to rotate variables with complexities larger than one. However, it is interesting to show that Direct Oblimin might fail with some kind of patterns, whereas Weighted Oblimin still performs well. Direct Oblimin rotation was started from five random positions.
5. Weighted Oblimin: this rotation was computed in the way described above, starting Direct Oblimin rotation from five random positions.

3.1. Design of the Simulation Study

As already observed, there is reason to suspect that Weighted Oblimin may fail if the factors are highly correlated or if a variable with complexity one cannot be found for each factor in the pattern. This simulation study tests Weighted Oblimin and the other rotation procedures taking into account both of these difficult situations. The steps of the study were the following:

1. The orthogonal factor solutions were constructed on the basis of a known simple oblique pattern Λ and interfactor correlations matrix.
2. Each solution was rotated from its orthogonal pattern using all the methods compared.
3. The rotation procedures which can recover accurately the oblique pattern Λ were determined.
4. The accuracy with which a rotation procedure recovers the oblique pattern Λ was determined.

To construct the oblique pattern Λ, besides a number of zero loadings, the following loadings were considered:

1. Loadings a: taken in a uniform random manner between 0.80 and 1.00.
2. Loadings b: taken in a uniform random manner between 0.50 and 0.69.
3. Loadings c: taken in a uniform random manner between 0.30 and 0.49.
4. Loadings d taken in a uniform random manner between 0.10 and 0.29.
5. Loadings e: taken in a uniform random manner between 0.01 and 0.09.

Using these loadings, seven kind of different Oblique Patterns Λ were constructed. These patterns are shown in Tables 1 and 2. For example, loadings a in Pattern 1 were all chosen in a uniform random manner, which means that it is unlikely that any of the loadings a are exactly equal but they are probably similar. The same is applied to loadings b, c, d and e in all patterns.

Each kind of pattern attempts to test the rotation procedures in different situations:

TABLE 1.
Patterns P1, P2 and P3 in the Study with Artificial Data

	Pattern P1			Pattern P2			Pattern P3		
	F1	F2	F3	F1	F2	F3	F1	F2	F3
V1	a	0	0	a	0	0	b	0	0
V2	a	0	0	0	a	0	0	b	0
V3	a	0	0	0	0	a	0	0	b
V4	a	0	0	b	b	0	b	b	0
V5	a	0	0	e	b	c	e	b	c
V6	0	a	0	0	d	b	0	d	b
V7	0	a	0	0	b	b	0	b	b
V8	0	a	0	e	c	c	e	c	c
V9	0	a	0	c	e	b	c	e	b
V10	0	0	a	b	d	d	b	d	d
V11	0	0	a	c	c	c	c	c	c
V12	0	0	a	b	d	e	b	d	e

1. Pattern 1 (P1) is a very simple pattern. All the methods should give good results when rotating solutions of this type. However, it is interesting to see if Weighted Oblimin can be safely used with this kind of pattern. Note that the number of variables with significant loadings on each factor differs so that there are no symmetries in the data.
2. Pattern 2 (P2) has three variables with complexity one, whereas all the other variables are complex variables. These three variables have larger loadings than those of the complex variables.
3. Pattern 3 (P3) has three variables with complexity one. Each of these variables has its significant loading on a different factor. All the other variables are complex variables like those found in pattern P2. However, the three simple variables have loadings similar to those of the complex variables.
4. Pattern 4 (P4) has a structure similar to pattern P3, but with 20 variables and 5 factors. Like before, each variable with complexity one has the significant loading on a different factor.
5. Pattern 5 (P5) is exactly like pattern P4, but variables 4 and 5 are complex variables, not variables of complexity one, and neither can factors 4 and 5 be associated to a single variable. Here Direct Oblimin and all the methods based on the weighting technique are expected to fail. No predictions can be made about the behavior of Promaj, as it is not based on this procedure.
6. Pattern 6 (P6) is exactly like pattern P4, but variables 2, 3, 4 and 5 are complex variables, not variables of complexity one, and neither can factors 2, 3, 4 and 5 be associated to a single variable. As this is a more degraded kind of pattern, the performance of the rotation procedures should be worse than with patterns P4 and P5.
7. Pattern 7 (P7) is exactly like pattern P4, but none of the variables have complexity one. This is a really difficult pattern, and none of the rotation procedures is expected to recover the Oblique Pattern Λ.

Four kinds of interfactor correlations matrices Φ were used in the study:

1. Low correlated factors: all the off-diagonal elements were fixed to 0.20.
2. Medium correlated factors: all the off-diagonal elements were fixed to 0.50.
3. High correlated factors: all the off-diagonal elements were fixed to 0.70.
4. Random correlated factors: all the off-diagonal elements were chose in a uniform random manner between 0.20 and 0.70. Each matrix Φ was checked to be positive semidefinite.

TABLE 2.
Patterns P4, P5, P6 and P7 in the Study with Artificial Data

	Pattern P4					Pattern P5				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
V1	b	0	0	0	0	b	0	0	0	0
V2	0	b	0	0	0	0	b	0	0	0
V3	0	0	b	0	0	0	0	b	0	0
V4	0	0	0	b	0	0	d	0	d	0
V5	0	0	0	0	b	0	0	c	0	d
V6	b	b	0	0	0	b	b	0	0	0
V7	0	b	0	0	b	0	b	0	0	b
V8	c	0	c	0	c	c	0	c	0	c
V9	0	c	c	d	0	0	c	c	d	0
V10	d	d	d	d	0	d	d	d	d	0
V11	e	d	e	e	d	e	d	e	e	d
V12	c	0	0	b	0	c	0	0	b	0
V13	e	e	b	0	0	e	e	b	0	0
V14	e	d	e	d	e	e	d	e	d	e
V15	e	c	d	e	e	e	c	d	e	e
V16	b	0	e	e	0	b	0	e	e	0
V17	0	e	0	e	0	0	e	0	e	0
V18	e	e	b	0	0	e	e	b	0	0
V19	0	0	0	c	c	0	0	0	c	c
V20	d	0	d	0	d	d	0	d	0	d

	Pattern P6					Pattern P7				
	F1	F2	F3	F4	F5	F1	F2	F3	F4	F5
V1	b	0	0	0	0	d	0	0	e	d
V2	c	0	c	0	0	0	e	0	0	e
V3	0	0	d	d	d	0	e	d	e	d
V4	0	d	0	d	0	0	d	e	d	0
V5	0	0	c	0	d	0	0	c	0	d
V6	b	b	0	0	0	b	b	0	0	0
V7	0	b	0	0	b	0	b	0	0	b
V8	c	0	c	0	c	c	0	c	0	c
V9	0	c	c	d	0	0	c	c	d	0
V10	d	d	d	d	0	d	d	d	d	0
V11	e	d	e	e	d	e	d	e	e	d
V12	c	0	0	b	0	c	0	0	b	0
V13	e	e	b	0	0	e	e	b	0	0
v14	e	d	e	d	e	e	d	e	d	e
V15	e	c	d	e	e	e	c	d	e	e
V16	b	0	e	e	0	b	0	e	e	0
V17	0	e	0	e	0	0	e	0	e	0
V18	e	e	b	0	0	e	e	b	0	0
V19	0	0	0	c	c	0	0	0	c	c
V20	d	0	d	0	d	d	0	d	0	d

Seven kinds of patterns Λ and four kinds of interfactor correlation matrices Φ were studied, so there were 28 conditions in the study. Each condition was analyzed by five rotation procedures, and the 28 conditions were replicated 50 times. This means that 1,400 solutions were constructed and 7,000 rotations computed.

3.2. Data Generation

Once these oblique patterns Λ and interfactor correlation matrices Φ had been built, an orthogonal pattern **P** was computed for each pair of oblique pattern and interfactor correlation matrix. This was based on the factor analysis model,

$$\mathbf{R} = \Lambda\Phi\Lambda' + \Psi = \mathbf{PP}' + \Psi = \mathbf{R}^* + \Psi, \tag{12}$$

where **R** is a correlation matrix, $\mathbf{R}^*$ is the reduced correlation matrix and Ψ is the diagonal matrix of unique variances, taken as $\mathbf{I} - \mathrm{Diag}(\mathbf{R} - \mathbf{PP}')$. The reduced correlation matrix $\mathbf{R}^*$ was constructed as $\mathbf{R}^* = \Lambda\Phi\Lambda'$ and its eigendecomposition computed as $\mathbf{R}^* = \mathbf{K}\Delta\mathbf{K}'$. So the orthogonal pattern, which is related to the principal factors, was computed as $\mathbf{P} = \mathbf{K}\Delta^{1/2}$.

Randomly choosing the loadings of the complex variables and the correlations between factors does not ensure that the diagonal elements of $\mathbf{R}^*$ will have values smaller or equal to one and the off-diagonal elements smaller than one, hence that Ψ is nonnegative. So, each pair of oblique patterns and interfactor correlation matrices was checked to verify that this was so. If a pair led to a negative unique variance, the pair was discarded and a new oblique pattern and true interfactor correlation matrix built. The orthogonal pattern **P** computed for each Λ and Φ pair was then obliquely rotated using all the rotation methods tested in the study.

3.3. Success Criteria

The main purpose of this first simulation study is to determine whether each rotation procedure is able to recover accurately the proposed patterns Λ from their orthogonal patterns **P**. It was arbitrarily determined that an oblique pattern Λ would be considered as accurately recovered by a rotation procedure from the orthogonal pattern **P**, when the congruence of the columns of Λ with the corresponding columns of the oblique pattern was equal to or larger than 0.90 for all the columns in the pattern. Before computing the congruence coefficient (Tucker, 1951), the columns of the oblique pattern obtained by the rotation procedure were permuted and reflected, if necessary, to maximize the congruence with the oblique pattern Λ. The number of times that a rotation procedure recovered accurately the patterns Λ was taken as a success index.

A second point of interest is to see how well each rotation procedure recovers the oblique pattern Λ from the orthogonal pattern **P**. This was evaluated by determining the factor congruence, that is to say, the mean of the congruence of the columns of Λ with corresponding columns of the rotated oblique pattern was computed. This congruence criterion (C) was computed as

$$C = m^{-1}\sum_{i=1}^{m}\phi(\lambda_i, \mathbf{Pt}_i), \tag{13}$$

where ϕ denotes Tucker's (1951) congruence coefficient, λ_i is the i-th column of Λ, and $\mathbf{t}_i$ is the i-th column of the oblique transformation matrix **T** obtained by the rotation procedure tested. If necessary, the columns of the oblique pattern obtained by the rotation procedure were permuted and reflected to maximize congruence with the oblique pattern Λ, so the values of C are in the range of [0, 1].

3.4. Results

Table 3 shows the number of patterns accurately recovered by each rotation procedure in the 28 conditions after 50 replications of this study. The rotation procedure that recovered most

TABLE 3.
Number of Solutions Recovered by Each Rotation Procedure

	Direct Oblimin	Promaj	Promin	Weighted Promax	Weighted Oblimin
$\Phi = 0.2$					
P1	50	49	50	50	50
P2	50	47	50	50	50
P3	1	27	50	46	50
P4	2	33	50	50	50
P5	2	12	25	25	16
P6	0	2	12	2	0
P7	0	0	7	2	0
Total	105	170	244	225	216
$\Phi = 0.5$					
P1	50	49	50	50	50
P2	50	45	50	50	50
P3	0	34	49	49	50
P4	6	16	50	50	50
P5	0	6	24	17	32
P6	0	5	7	4	0
P7	0	0	1	0	0
Total	106	155	231	220	232
$\Phi = 0.7$					
P1	50	47	50	50	50
P2	50	37	50	50	50
P3	0	36	50	48	50
P4	17	3	50	50	50
P5	1	0	21	6	35
P6	0	0	3	1	0
P7	0	0	3	0	0
Total	118	123	227	205	235
Φ = RANDOM					
P1	50	47	50	50	50
P2	50	45	50	50	50
P3	1	33	49	47	50
P4	7	16	50	49	50
P5	0	6	27	19	11
P6	0	6	11	3	1
P7	0	3	6	0	0
Total	108	156	243	218	212
Total	437	604	945	868	895

patterns was Promin (945 out of 1,400), followed by Weighted Oblimin (63.9%) and Weighted Promax (62.0%), whereas Promaj (43.1%) and Direct Oblimin (31.2%) were the procedures that recovered least.

As can be seen in Table 3, the patterns of type P1 and P2 were well recovered by Direct Oblimin, Promin, Weighted Promax and Weighted Oblimin. Despite being based on the Weighting procedure proposed by Cureton & Mulaik (1975), the last three gave good results even with highly correlated factors. However, Promaj based on the Orthosim rotation seems to have problems, particularly when rotating highly correlated factors.

Patterns of type P3 and P4 were well recovered by Promin, Weighted Promax and, particularly, Weighted Oblimin, which recovered most patterns. The performance of Promaj was rather poor with this kind of pattern, and that of Direct Oblimin was even poorer.

Promin recovered 48.5% of the patterns of type P5, and was the rotation procedure that achieved best results with this kind of pattern. Weighted Oblimin recovered 47.0% of the patterns, whereas Weighted Promax recovered 33.5%. Once again, Promaj and Direct Oblimin performed poorly with this kind of pattern.

When rotating patterns of type P6 and P7, all of the rotation procedures often gave unacceptable results. Promin, Weighted Promax and Promaj did recover some of the patterns. Weighted Oblimin recovered only one pattern of type P6, and Direct Oblimin was not able to recover any.

Whereas the performance of Promin, Weighted Promax and Promaj worsened as the indices of the inter-factor correlation matrix increased, the performance of Weighted Oblimin and Direct Oblimin seemed to improve slightly. So the correlation between factors did not seem to be a problem when applying Weighted Oblimin. Actually, the procedure that achieved best results when $\Phi = 0.7$ was Weighted Oblimin; it recovered 235 (out of 350) of the patterns.

Table 4 shows the average values of success criterion C after 50 replications, for each type of pattern and for each of the rotation procedures tested. It also shows how many times each of the methods performed best (denoted as "freq.best") in terms of the success criterion C. All the methods performed well when rotating P1 patterns. Weighted Oblimin is the rotation procedure that gave the best results when rotating P2, P3 and P4 patterns. Finally, Promin was the rotation procedure that gave best results when rotating P5, P6 and P7 patterns. It can also be seen that Weighted Oblimin was the procedure that performed best in 711 (out of 1,400) cases, Promin performed best in 35.4% of the cases, and the other procedures did so in 10.4%, or less, of the cases.

3.5. Conclusion

When a variable with complexity one can be found for each factor in the pattern (patterns of type P1, P2, P3 and P4 in the study), Weighted Oblimin gave good results even with highly correlated solutions. With this type of pattern, Weighted Oblimin seemed to perform better than any of the other rotation procedures tested. Even though Promin and Weighted Promax are based on Cureton & Mulaik's (1975) weighting procedure, they do not seem to give such good results as Weighted Oblimin. It can be concluded that, with this kind of pattern, Direct Oblimin rotation helps the weighting procedure to be more effective.

The results of Weighted Oblimin were bad when a variable with complexity one could not be found for each factor in the pattern. With this kind of pattern only Promin, Weighted Promax

TABLE 4.
Mean of Congruence Index for Each Rotation Procedure Across Replications and Φ Conditions

	Direct Oblimin	Promaj	Promin	Weighted Promax	Weighted Oblimin
P1	1.00	0.99	1.00	1.00	1.00
P2	0.99	0.97	0.99	0.98	1.00
P3	0.90	0.94	0.98	0.96	1.00
P4	0.93	0.94	0.99	0.98	1.00
P5	0.91	0.91	0.96	0.94	0.95
P6	0.85	0.90	0.91	0.89	0.90
P7	0.84	0.89	0.90	0.87	0.88
Freq.best	29 (2.07%)	146 (10.42%)	495 (35.36%)	19 (1.36%)	711 (50.79%)

and Promaj gave some good results but only on a few occasions. Finally, Promaj gave rather poor results when rotating complex patterns, and Direct Oblimin gave good results only when rotating the simplest kind of pattern.

4. Analysis of Thurstone's 26 Box Problem

To study the performance of Weighted Oblimin when rotating complex variables, the pattern matrix for Thurstone's 26 Box Problem (1947) was rotated. The unrotated pattern analyzed is the one given by Cureton & Mulaik (1975). To construct the 26 variables, a collection of thirty random boxes were measured by Thurstone. The three dimensions, x, y and z, were recorded for each box. Then a list of 23 arbitrary variables was designed using nonlinear combinations of the three dimensions measured. The formulas used by Thurstone are shown in Table 5. The correlation matrix between the 26 variables, including x, y and z, was computed and factored to retain three factors. The orthogonal pattern obtained was used as a test to assess whether a rotation procedure can recover the truly simple pattern that is expected.

The orthogonal pattern was obliquely rotated by Weighted Oblimin with $\gamma = 0$ and using five random starts. The solution obtained is shown in Table 5 with absolute elements larger than 0.30 set in bold face. The correlation between components 1 and 2 is 0.33, between components 1 and 3 it is 0.35 and between 2 and 3 it is 0.36. For comparison, Table 5 also displays the pattern obtained by Cureton & Mulaik (1975) from Weighted Promax rotation with $k = 4$. For

TABLE 5.
The Orthogonal Unrotated Pattern and the Oblique Patterns Rotated by Each Method in the 26 Box Problem

	Weighted Oblimin			Weighted Promax		
Variables	F1	F2	F3	F1	F2	F3
x	**0.99**	−0.01	−0.02	**0.95**	−0.03	−0.04
y	−0.07	**0.98**	0.02	0.04	**0.89**	0.05
z	−0.07	0.03	**1.00**	0.06	0.04	**0.90**
xy	**0.56**	**0.68**	−0.04	**0.60**	**0.60**	−0.03
xz	**0.55**	−0.00	**0.67**	**0.61**	−0.02	**0.58**
yz	−0.15	**0.62**	**0.65**	0.00	**0.57**	**0.61**
x^2y	**0.78**	**0.41**	−0.00	**0.79**	**0.35**	−0.02
xy^2	**0.31**	**0.85**	0.01	**0.36**	**0.76**	0.03
x^2z	**0.76**	−0.03	**0.43**	**0.78**	−0.04	**0.36**
xz^2	**0.39**	0.00	**0.88**	**0.48**	0.00	**0.78**
y^2z	−0.16	**0.78**	**0.45**	−0.01	**0.72**	**0.43**
yz^2	−0.14	**0.43**	**0.80**	0.01	**0.40**	**0.74**
x/y	**0.86**	**−0.86**	0.03	**0.74**	**−0.80**	−0.02
y/x	**−0.86**	**0.86**	−0.03	**−0.74**	**0.80**	0.02
x/z	**0.87**	0.04	**−0.85**	**0.73**	0.01	**−0.79**
z/x	**−0.87**	−0.04	**0.85**	**−0.73**	−0.01	**0.79**
y/z	−0.08	**0.91**	**−0.85**	−0.08	**0.82**	**−0.73**
z/y	0.08	**−0.91**	**0.85**	0.08	**−0.82**	**0.73**
$2x + 2y$	**0.46**	**0.75**	−0.03	0.51	**0.67**	−0.02
$2x + 2z$	**0.51**	−0.04	**0.71**	**0.58**	−0.04	0.62
$2y + 2z$	−0.13	**0.62**	**0.64**	0.02	**0.57**	**0.59**
$(x^2 + y^2)^{1/2}$	**0.45**	**0.74**	−0.03	**0.50**	**0.66**	−0.02
$(x^2 + z^2)^{1/2}$	**0.48**	−0.02	**0.70**	**0.55**	−0.03	**0.62**
$(y^2 + z^2)^{1/2}$	−0.11	**0.62**	**0.61**	0.04	**0.57**	**0.56**
xyz	**0.35**	**0.49**	**0.46**	**0.45**	**0.44**	**0.43**
$(x^2 + y^2 + z^2)^{1/2}$	**0.32**	**0.54**	**0.48**	**0.35**	**0.49**	**0.45**

this solution, the correlation between the components are 0.25, 0.25 and 0.27. Both solutions seem to agree with the original constituting rules used to construct the data, and there are no important differences between them. However, the solution obtained by Kiers (p. 574, 1994) from Simplimax with $p = 27$ agrees slightly better than the one obtained by Weighted Oblimin or Weighted Promax.

5. 35 Hyper-box Problem in a Four-Dimensional Hyperspace

The 26 Box Problem is a very special problem, because of its symmetries: the nonlinear combination functions of the variables are constructed to take into account all the combinations (for example, variables xy, xz and yz). In addition, because the example data are three dimensional, complex variables can only run from complexities two to complexities three.

To study an asymmetric situation as well, a new data sample based on the Box Problem was generated and rotated. This example is a four-dimensional hyper-box problem. A collection of 100 four-dimensional hyper-boxes in a four-dimensional space were virtually computed using Matlab 4.0 software (1984). For the virtual computing of the boxes, the distribution of the four dimensions (x, y, z and v) was normal and had an arbitrary mean of 30 and an arbitrary standard deviation of 10. The correlation between the four-dimensional matrix was arbitrarily taken. This correlation matrix is shown in Table 6.

Once the 100 hyperboxes had been virtually generated, 68 nonlinear combinations of the 4 variables (x, y, z and v) were computed following the kind of nonlinear combinations used by Thurstone (1947) in his 26 box problem. Of the 72 resulting variables, a sample of 35 was chosen: the four simplest variables (x, y, z and v), one variable with maximum complexity ($(x^2 + y^2 + z^2 + v^2)^{1/2}$), and 30 random variables of the other 67 variables built as nonlinear combinations, with complexities two, three or four. The final sample of variables in the example is shown in Table 7. The correlation matrix between the 35 variables was computed and analyzed to retain four factors. The factor extraction method applied was Principal Components.

The orthogonal pattern obtained was rotated using Weighting Oblimin ($\gamma = 0$ and five random starts), Weighted Promax ($k = 4$) and Simplimax. The Varimax (Kaiser, 1958) rotation computed in Weighted Promax was started from five random positions and care was taken to prevent any possible permutation effect (ten Berge, 1995). When computing Simplimax, the values $p = 50, \ldots, 60$ were chosen, and one rational start and ten random additional starts were used. The optimal function values were 0.000133, 0.000152, 0.000174, 0.000192, 0.000216, 0.090567, 0.164463, 0.235717, 0.307372, 0.383568 and 0.451825. As a large jump in the function values occurs after the 5th analysis, the solution corresponding to $p = 54$ was taken as the best solution.

Table 7 shows the orthogonal pattern after the factor extraction and the oblique pattern obtained by Simplimax.

Table 8 shows the rotated pattern obtained by Weighted Oblimin and the rotated pattern obtained by Weighted Promax.

Simplimax, Weighted Oblimin and Weighted Promax seem to provide a pattern that agrees with the nonlinear combinations used to build the variables in the solution. In fact, the patterns obtained by Simplimax and Weighted Oblimin agree perfectly with the original constituting rules

TABLE 6.
Arbitrary Inter-Dimension Correlation Matrix for the Generation of the 4 Dimensions in the 35 Hyper-Box Problem

	x	y	z	v
x	1.00	0.24	0.26	0.25
y	0.24	1.00	0.35	0.30
z	0.26	0.35	1.00	0.31
v	0.25	0.30	0.31	1.00

TABLE 7.
The Orthogonal Pattern and Simplimax Pattern in the 35 Hyper-Box Problem

	Orthogonal Pattern				Simplimax			
Variable	F1	F2	F3	F4	F1	F2	F3	F4
x	0.64	−0.58	0.11	−0.49	**1.00**	−0.01	0.00	0.00
y	0.71	0.33	−0.62	−0.07	0.00	**1.00**	0.00	0.00
z	0.72	−0.19	0.08	0.66	0.00	0.00	**1.00**	0.00
v	0.65	0.52	0.54	−0.16	0.00	0.00	0.00	**1.00**
xy	0.86	−0.16	−0.33	−0.36	**0.64**	**0.63**	0.00	0.00
xv	0.81	−0.04	0.41	−0.41	**0.64**	0.00	−0.01	**0.63**
yv	0.84	0.53	−0.04	−0.14	0.00	**0.62**	0.00	**0.62**
xz^2	0.85	−0.39	0.11	0.34	**0.41**	0.01	**0.81**	−0.01
x^2z	0.82	−0.55	0.12	−0.12	**0.82**	0.01	**0.40**	−0.01
yz^2	0.85	−0.02	−0.18	0.50	0.00	**0.40**	**0.79**	0.00
xyv	0.93	0.13	0.02	−0.33	**0.46**	**0.46**	0.00	**0.46**
yzv	0.93	0.30	0.00	0.20	−0.00	**0.45**	**0.45**	**0.45**
xy^2z	0.93	−0.04	−0.35	0.02	**0.34**	**0.67**	**0.34**	0.00
xy^2v	0.92	0.21	−0.20	−0.26	**0.34**	**0.67**	0.00	**0.34**
x^2zv	0.90	−0.28	0.29	−0.15	**0.68**	0.01	**0.33**	**0.34**
xyz^2	0.93	−0.21	−0.12	0.26	**0.33**	**0.34**	**0.67**	0.00
x^2yz	0.92	−0.35	−0.11	−0.12	**0.68**	**0.34**	**0.33**	0.00
yz^2v	0.92	0.16	0.03	0.36	−0.01	**0.33**	**0.65**	**0.33**
y^2zv	0.91	0.33	−0.20	0.12	0.00	**0.65**	**0.33**	**0.33**
yzv^2	0.89	0.40	0.18	0.09	0.01	**0.33**	**0.33**	**0.68**
y/x	0.04	0.74	−0.58	0.34	**−0.81**	**0.82**	−0.05	−0.01
v/x	0.00	0.89	0.35	0.28	**−0.81**	0.00	0.00	**0.82**
z/y	0.02	−0.46	0.61	0.64	0.00	**−0.88**	**0.87**	0.00
v/y	−0.05	0.15	0.98	−0.07	0.00	**−0.85**	0.00	**0.84**
z/v	0.07	−0.60	−0.39	0.70	0.00	0.00	**0.86**	**−0.85**
$2z+2v$	0.84	0.20	0.38	0.31	0.00	0.00	**0.62**	**0.62**
$2x+2y+2z$	0.96	−0.20	−0.20	0.05	**0.47**	**0.46**	**0.46**	0.00
$2x+2z+2v$	0.93	−0.11	0.34	0.01	**0.47**	−0.01	**0.46**	**0.46**
$2x+2y+2v$	0.93	0.13	0.02	−0.34	**0.47**	**0.46**	0.00	**0.47**
$2y+2z+2v$	0.93	0.30	0.01	0.20	−0.01	**0.33**	**0.33**	**0.66**
$(x^2+y^2)^{1/2}$	0.86	−0.16	−0.32	−0.36	**0.64**	**0.63**	0.00	0.00
$(x^2+y^2+z^2)^{1/2}$	0.96	−0.21	−0.19	0.05	**0.46**	**0.46**	**0.46**	0.00
$(x^2+z^2+v^2)^{1/2}$	0.93	−0.12	0.34	0.01	**0.47**	−0.01	**0.47**	**0.46**
$xyzv$	1.00	0.03	0.04	−0.01	**0.36**	**0.37**	**0.36**	**0.37**
$(x^2+y^2+z^2+v^2)^{1/2}$	1.00	0.03	0.05	−0.02	**0.37**	**0.36**	**0.37**	**0.37**

used to construct the data. Here, Weighted Promax seems to have some slight problems, particularly when recovering some loadings that should be zero in the rotated pattern and the high loadings of the four simplest variables.

The interfactor correlation matrices obtained by Weighted Oblimin, Weighted Promax and Simplimax are shown in Table 9.

This hyper-box study was replicated 50 times. Each time the variables were chosen as explained above. Hence, it was guaranteed that four variables had complexity one and one variable have complexities four, while 30 variables were taken in a uniform random manner from the other 67 variables built as nonlinear combinations with complexities two, three or four. The correlation matrix between the 35 variables was computed and analyzed to retain four factors. The

TABLE 8.
The Patterns Rotated by Weighted Oblimin and by Weighted Promax in the 35 Hyper-Box Problem

	Weighted Oblimin				Weighted Promax			
Variable	F1	F2	F3	F4	F1	F2	F3	F4
x	**1.01**	0.00	−0.02	−0.01	**1.13**	−0.08	−0.16	−0.06
y	−0.02	**1.00**	0.02	−0.01	−0.08	**1.06**	−0.03	−0.03
z	−0.02	−0.01	**1.01**	0.00	−0.09	−0.10	**1.10**	−0.02
v	−0.01	0.00	0.00	**1.00**	−0.04	−0.08	−0.11	**1.10**
xy	**0.63**	**0.64**	0.00	−0.01	**0.67**	**0.63**	−0.13	−0.07
xv	**0.63**	−0.01	−0.01	**0.63**	**0.68**	−0.11	−0.17	**0.66**
yv	−0.02	**0.61**	0.01	**0.62**	−0.08	**0.60**	−0.09	**0.66**
xz^2	**0.40**	−0.01	**0.81**	−0.01	**0.39**	−0.12	**0.83**	−0.04
x^2z	**0.81**	−0.01	**0.40**	−0.01	**0.88**	−0.11	**0.32**	−0.06
yz^2	−0.02	**0.39**	**0.81**	0.00	−0.10	**0.34**	**0.86**	−0.03
xyv	**0.45**	**0.46**	0.00	**0.47**	**0.46**	**0.42**	−0.14	**0.47**
yzv	−0.03	**0.44**	**0.46**	**0.45**	−0.10	**0.39**	**0.43**	**0.48**
xy^2z	**0.32**	**0.67**	**0.35**	−0.01	0.29	**0.65**	**0.30**	−0.05
xy^2v	**0.32**	**0.68**	0.01	**0.34**	**0.31**	**0.67**	−0.11	**0.33**
x^2zv	**0.67**	−0.01	**0.34**	**0.34**	**0.72**	−0.12	0.23	**0.33**
xyz^2	**0.32**	**0.33**	**0.68**	−0.01	0.29	0.26	**0.68**	−0.05
x^2yz	**0.67**	**0.33**	**0.34**	−0.02	**0.70**	0.27	0.26	−0.07
yz^2v	−0.03	**0.32**	**0.67**	**0.33**	−0.10	0.25	**0.67**	**0.34**
y^2zv	−0.03	**0.65**	**0.35**	**0.33**	−0.11	**0.63**	0.31	**0.34**
yzv^2	−0.03	**0.32**	**0.34**	**0.66**	−0.09	0.26	0.28	**0.71**
y/x	**−0.84**	**0.80**	0.04	0.00	**−0.99**	**0.91**	0.11	0.02
v/x	**−0.84**	0.00	0.02	**0.82**	**−0.96**	0.00	0.05	**0.94**
z/y	0.00	**−0.88**	**0.87**	0.00	0.00	**−1.02**	**0.99**	0.01
v/y	0.01	**−0.85**	−0.02	**0.86**	0.05	**−0.97**	−0.07	**0.96**
z/v	−0.01	−0.01	**0.86**	**−0.85**	−0.04	−0.02	**1.03**	**−0.95**
$2z + 2v$	−0.02	−0.01	**0.62**	**0.62**	−0.08	−0.11	**0.61**	**0.67**
$2x + 2y + 2z$	**0.45**	**0.45**	**0.47**	−0.01	**0.44**	**0.40**	**0.42**	−0.05
$2x + 2z + 2v$	**0.45**	0.00	**0.46**	**0.46**	**0.46**	−0.12	**0.38**	**0.47**
$2x + 2y + 2v$	**0.46**	**0.46**	0.00	**0.47**	**0.47**	**0.42**	−0.15	**0.47**
$2y + 2z + 2v$	−0.02	**0.44**	**0.47**	**0.45**	−0.10	**0.39**	**0.43**	**0.47**
$(x^2 + y^2)^{1/2}$	**0.64**	**0.63**	0.00	−0.01	**0.67**	**0.62**	−0.12	−0.06
$(x^2 + y^2 + z^2)^{1/2}$	**0.45**	**0.45**	**0.47**	−0.01	**0.45**	**0.40**	**0.42**	−0.05
$(x^2 + z^2 + v^2)^{1/2}$	**0.45**	0.00	**0.46**	**0.46**	**0.46**	−0.12	**0.38**	**0.47**
$xyzv$	**0.35**	**0.36**	**0.38**	**0.37**	**0.33**	0.29	**0.30**	**0.36**
$(x^2 + y^2 + z^2 + v^2)^{1/2}$	**0.36**	**0.36**	**0.37**	**0.36**	**0.34**	0.29	0.29	0.36

factor extraction method applied was Principal Components. Thus, 50 orthogonal patterns were obtained.

For each orthogonal pattern, an oblique true simple pattern was computed. As the positions for the expected zero loadings were known in each orthogonal pattern, 50 semispecified targets were built (where the specified values were the loadings expected to be zero values). Each orthogonal pattern was rotated by oblique semi-specified Procrustes rotation (Browne, 1972; Meredith, 1977) using its own semi-specified target matrix. The oblique rotated patterns obtained were considered as true simple patterns.

The 50 orthogonal patterns were also rotated using Weighted Oblimin, Promaj, Weighted Promax and Promin. All the rotation procedures were computed as described above. Finally, the success criterion C and the root mean square of the residuals (RMSR) between the patterns

TABLE 9.
Interfactor Correlation Matrices Obtained by Weighted Oblimin, Weighted Promax and Simplimax in the 35 Hyper-Box Problem

	Weighted Oblimin				Weighted Promax			
	F1	F2	F3	F4	F1	F2	F3	F4
F1	1.00	0.27	0.32	0.33	1.00	0.38	0.47	0.45
F2	0.27	1.00	0.29	0.28	0.38	1.00	0.38	0.41
F3	0.32	0.29	1.00	0.34	0.47	0.38	1.00	0.42
F4	0.33	0.28	0.34	1.00	0.45	0.41	0.42	1.00

	Simplimax			
	F1	F2	F3	F4
F1	1.00	0.35	0.24	0.30
F2	0.35	1.00	0.26	0.31
F3	0.24	0.26	1.00	0.24
F4	0.30	0.31	0.24	1.00

obtained by each rotation procedure and the corresponding true simple patterns were computed. Table 10 summarizes the results.

It can be observed that the oblique patterns which are most similar to the true simple pattern are the ones obtained by Weighted Oblimin, and that Promin and Weighted Promax give good results as well. Promaj seems to perform poorly with these data samples.

6. Rotation of Empirical Data

To study the performance of Weighted Oblimin with empirical data, the oblique solution given by Stankov, Roberts & Spilsbury (p. 278, 1994) was analyzed. These authors analyzed a correlation matrix of 22 variables by factor analysis. Three factors were retained using maximum likelihood factor extraction: crystallized intelligence (Gc), fluid intelligence (Gf) and short-term acquisition and retrieval function (SAR). An orthogonal pattern was rotated by Direct Oblimin (Stankov et al., 1994), to obtain the oblique pattern and the interfactor correlation matrix shown in Table 11. These matrices were used to compute the orthogonal pattern from which the Weighted

TABLE 10.
C and RMSR Indices Between the True and the Rotated Pattern (Replications = 50)

	Promaj	Promin	Weighted Promax	Weighted Oblimin
C				
Mean	0.8354	0.9885	0.9904	0.9993
Std	0.0665	0.0035	0.0031	0.0004
Best	0.9281	0.9948	0.9966	0.9997
Worse	0.6602	0.9792	0.9805	0.9977
RMSR				
Mean	0.2719	0.0804	0.0645	0.0142
Std	0.0718	0.0148	0.0111	0.0033
Best	0.1483	0.0525	0.0398	0.0090
Worst	0.4498	0.1110	0.0962	0.0231

TABLE 11.
Factor Pattern and Interfactor Correlation Matrices Obtained by Direct Oblimin and Weighted Oblimin

	Direct Oblimin			Weighted Oblimin		
Tests	Gc	Cf	SAR	Gc	Gf	SAR
1.–Information	**0.85**	−0.18	−0.04	**0.82**	−0.06	0.02
2.–Digit Span	0.07	0.06	**0.66**	0.04	0.24	**0.61**
3.–Vocabulary	**0.96**	−0.16	−0.04	**0.93**	−0.02	0.02
4.–Arithmetic	**0.37**	0.07	0.25	**0.35**	0.19	0.24
5.–Comprehension	**0.90**	−0.04	−0.03	**0.87**	0.10	0.01
6.–Similarities	**0.66**	0.05	−0.02	**0.64**	0.15	0.00
7.–Picture Completion	**0.33**	**0.38**	0.28	**0.32**	**0.51**	0.22
8.–Picture Arrangement	0.09	**0.53**	0.18	0.09	**0.60**	0.10
9.–Block Design	−0.02	**0.74**	0.07	−0.01	**0.77**	−0.04
10.–Object Assembly	0.10	**0.78**	0.07	0.11	**0.83**	−0.04
11.–Digit Symbol	−0.21	**0.66**	0.11	−0.19	**0.67**	0.00
12.–General Information	**0.60**	0.15	−0.02	**0.59**	0.24	−0.02
13.–Letter Series	−0.08	**0.67**	0.27	−0.07	**0.74**	0.15
14.–Concealed Words	−0.03	**0.30**	−0.03	−0.02	**0.30**	−0.07
15.–Digit Span Forward	0.05	0.10	**0.43**	0.03	0.22	**0.39**
16.–Digit Span Backward	0.02	0.08	0.23	0.01	0.14	0.20
17.–Esoteric Analogies	**0.57**	0.18	0.20	**0.55**	0.33	0.18
18.–Cattell's Matrices	0.20	**0.50**	0.00	0.21	**0.54**	−0.06
19.–Word Associations	0.14	**0.42**	−0.20	0.15	**0.40**	−0.24
20.–Letter Counting	0.01	0.25	**0.30**	0.00	**0.33**	0.24
21.–Synonym Vocabulary	**0.58**	0.00	0.22	**0.55**	0.15	0.22
22.–Crowder's Memory Span	0.07	−0.29	**1.00**	0.02	−0.03	**0.97**
	Interfactor Correlations					
Gc	1.00			1.00		
Gf	0.19	1.00		0.04	1.00	
SAR	0.30	0.31	1.00	0.25	0.19	1.00

Oblimin ($\gamma = 0$ and five random starts) was computed. The oblique pattern and the interfactor correlation matrices obtained by Weighted Oblimin are also shown in Table 11.

When comparing both oblique patterns, it can be seen that the main difference is that the loadings of the second factor are usually larger when rotated by Weighting Oblimin, whereas the loadings of the first and the third factors are usually smaller. The simplicity index (Bentler, 1977) for the oblique pattern obtained by Direct Oblimin was 0.9773, whereas the simplicity index for the oblique pattern obtained by Weighted Oblimin was 0.9891. Thus, the simplest oblique pattern was obtained by Weighted Oblimin. So, the weighting procedure (Cureton & Mulaik, 1975) might improve the performance of Direct Oblimin when rotating simple solutions.

Since the aim of this study is to discuss the differences between both rotation procedures in the analysis of real data sets, a substantive interpretation is not given here. For a more detailed explanation see Stankov et al. (1994).

7. Discussion

Cureton & Mulaik (1975) proposed a weighting procedure to allow Varimax (Kaiser, 1958) to reach simple solutions when variables in the solution have complexities larger than one. If the solution is expected to be factor correlated, they suggested prerotating the solution orthogonally by Weighted Varimax and taking this orthogonal solution as a starting point for Promax (Hendrickson & White, 1964). This rotation procedure has been called Weighted Promax here. The present paper shows that the weighting procedure proposed by Cureton & Mulaik (1975) can be applied to Direct Oblimin (Clarkson & Jennrich, 1988), and can provide good results. This procedure is denoted as Weighted Oblimin.

Weighted Oblimin has been tested in different complex data sets. Whereas Direct Oblimin (Clarkson & Jennrich, 1988) failed to find a good approximation to a simple solution in these data, Weighted Oblimin frequently obtained the expected simple solution. Weighted Oblimin also performed as well as, or better than, the Weighted Promax proposed by Cureton & Mulaik (1975) and other rotation procedures such as Promaj (Trendafilov, 1994) and Promin (Lorenzo-Seva, 1999). It should be said that Weighted Oblimin occasionally fails as well, but seems to give particularly good results when a variable with complexity one can be found for each factor in the pattern, even with complex, highly-correlated solutions. With this type of pattern, Weighted Oblimin seemed to perform better than any of the other rotation procedures tested. For example, Promin and Weighted Promax, that are based on Cureton & Mulaik's weighting procedure, do not provide such good results as Weighted Oblimin. It can be concluded that, with this kind of pattern, Direct Oblimin (Clarkson & Jennrich, 1988) rotation helps the weighting procedure to be more effective. The circumstances in which Weighted Oblimin seems to provide good results actually can be considered the most common ones when analyzing real data. So Weighting Oblimin can be considered appropriate for practical purposes.

Weighted Oblimin fails when a variable with complexity one cannot be found for each factor in the pattern. However, all of the rotation procedures tested with this kind of pattern performed poorly, with Promin (Lorenzo-Seva, 1999) giving best results. Only Simplimax (Kiers, 1994) is a rotation method that is expected to deal successfully with this kind of configuration, though it is more complex to use than Weighted Oblimin or Promin.

Cureton & Mulaik (1975) warned that the main drawback of the weighting procedure is that it may fail if the solution is highly oblique. However, in the studies carried out in this paper Weighted Oblimin did not seem to fail when rotating such highly correlated factors. These authors also noted that the weighted procedure could modestly improve Varimax (Kaiser, 1958) when rotating complex data and also simple data. It could also be expected when applied to Direct Oblimin (Clarkson & Jennrich, 1988). A rather simple real data set was analyzed with both Weighted Oblimin and Direct Oblimin, and the simplest oblique pattern was obtained by Weighted Oblimin. Therefore, the weighting procedure (Cureton & Mulaik, 1975) might improve the performance of Direct Oblimin when rotating simple solutions.

Jennrich (1979) advised to compute Direct Oblimin with the parameter γ set to zero. Then the rotation criterion computed is Direct Quartimin, the simplest member of the Oblimin family. However, the parameter γ could have other values, (see for example, Crawford, 1975; Jennrich, 1979; or Clarckson and Jennrich, 1988), and the rotation criterion computed would change. As Jennrich (1979) pointed out that the best performance of Direct Oblimin is obtained when $\gamma = 0$, his advice has been followed when computing Weighted Oblimin, so the rotation criterion applied is Weighted Quartimin in fact. However, the weighted procedure proposed by Cureton & Mulaik (1975) could be computed followed by Direct Oblimin with other γ values. Browne & Du Toit (p. 294, 1992) found that Direct Oblimin Varimax (Crawford, 1975) gave a slightly better fit than Direct Quartimin when applied to an example data set, but as the authors noted the differences were negligible in fact.

A new data set analogous to the 26 Box Problem (Thurstone, 1947) has been presented: the 35 Hyper-box Problem. The associated orthogonal pattern can be used to test other rotation procedures. It could be added that to have a standard library of patterns with complex structures, whose source features are well understood, could be useful for testing the performance of new extraction and rotation procedures. As an anonymous reviewer pointed out, to build such a library of test patterns could be well received.

As Direct Oblimin (Clarkson & Jennrich, 1988) is already implemented in the main statistical software packages, Weighted Oblimin should also be easy to implement. However, a Windows 95 program (WOblimin) can be obtained from the author at the following e-mail address: uls@fcep.urv.es.

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