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A MULTITRAIT-MULTIMETHOD MODEL WITH MINIMAL ASSUMPTIONS

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A new model of confirmatory factor analysis (CFA) for multitrait-multimethod (MTMM) data sets is presented. It is shown that this model can be defined by only three assumptions in the framework of classical psychometric test theory (CTT). All other properties of the model, particularly the uncorrelatedness of the trait with the method factors are logical consequences of the definition of the model. In the model proposed there are as many trait factors as different traits considered, but the number of method factors is one fewer than the number of methods included in an MTMM study. The covariance structure implied by this model is derived, and it is shown that this model is identified even under conditions under which other CFA-MTMM models are not. The model is illustrated by two empirical applications. Furthermore, its advantages and limitations are discussed with respect to previously developed CFA models for MTMM data.

Key words: multitrait-multimethod models, construct validity, classical test theory, structural equation modeling.

1. Introduction

Since Campbell's and Fiske's (1959) seminal work on convergent and discriminant validation, the multitrait-multimethod (MTMM) design has become a standard approach for validating personality questionnaires (for an overview, see Shrout & Fiske, 1995). Several methods for analyzing multitrait-multimethod data have been developed over the last forty years of research, in particular models of confirmatory factor analysis, models of covariance component analysis, and the direct product model (e.g., Browne, 1984; Cudeck, 1988; Millsap, 1995; Schmitt & Stults, 1986; Wothke, 1996). Among all methods proposed, the confirmatory factor analysis (CFA) approach has become the most widely applied alternative to the Campbell-Fiske criteria (e.g., Marsh & Grayson, 1995; Saris & Andrews, 1991; Saris & van Meurs, 1991; Widaman, 1985). Despite their popularity and sophistication, some CFA models for MTMM data, however, are affected by several problems, particularly nonproper solutions, nonidentification under specific conditions, and interpretation problems (Bagozzi, 1993; Brannick & Spector, 1990; Kenny & Kashy, 1992; Marsh, 1989; Marsh, Byrne, & Craven, 1992). In this paper, a new CFA-MTMM model is proposed that overcomes some of the problems in CFA of MTMM data. The paper is organized as follows: In the next section, a brief overview of the CFA approach to MTMM data and its problems is given. Then, a new model is introduced that was developed in the tradition of Zimmerman's (1975, 1976) and Steyer's (1989) work on classical psychometric test theory (CTT). It is shown that the trait and method factors can be defined as functions of the true-score variables. The covariance structure implied by this model is derived, and it is demonstrated that this model is identified even under conditions under which other CFA-MTMM models are not. Finally, this model is illustrated by applications to two data sets.

2. The CFA Approach to MTMM Data

Considering different trait and method structures, Marsh (1989) distinguished between 20 different CFA-MTMM models. To illustrate the problems of CFA-MTMM models relevant for understanding the advantages of the new model proposed in this paper, it is sufficient to consider

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only three CFA-MTMM models: the *correlated trait/correlated method* model, the *correlated trait/correlated uniqueness* model, and the *fixed method* model (Kenny & Kashy, 1992; Marsh, Byrne, & Craven, 1992; Marsh & Grayson, 1995).

In the correlated trait/correlated method model (CFA-CTCM model), each observed variable Y_{ik} designed to measure a trait i by a method k is a linear combination of a trait factor T_i, a method factor M_k, and an error variable E_{ik}. Assuming three traits ($i = 1, 2, 3$) and three methods ($k = 1, 2, 3$), for example, the model for mean-corrected data can be presented as follows:

$$\mathbf{y}_{(9\times 1)} = \begin{pmatrix} Y_{11} \\ Y_{21} \\ Y_{31} \\ --- \\ Y_{12} \\ Y_{22} \\ Y_{32} \\ --- \\ Y_{13} \\ Y_{23} \\ Y_{33} \end{pmatrix} = \left[\Lambda_{\mathrm{T}(9\times 3)} | \Lambda_{\mathrm{M}(9\times 3)}\right] \begin{pmatrix} \mathbf{t}_{(3\times 1)} \\ --- \\ \mathbf{m}_{(3\times 1)} \end{pmatrix} + \mathbf{e}_{(9\times 1)}$$

$$= \begin{bmatrix} \Lambda_{\mathrm{T}1} & | & \boldsymbol{\lambda}_{\mathrm{M}1} & 0 & 0 \\ \Lambda_{\mathrm{T}2} & | & 0 & \boldsymbol{\lambda}_{\mathrm{M}2} & 0 \\ \Lambda_{\mathrm{T}3} & | & 0 & 0 & \boldsymbol{\lambda}_{\mathrm{M}3} \end{bmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ --- \\ M_1 \\ M_2 \\ M_3 \end{pmatrix} + \mathbf{e}_{(9\times 1)}, \tag{1}$$

where $\mathbf{y}$ is a vector of observed variables Y_{ik}, Λ_{T} is a matrix of trait-related factor loadings $\lambda_{\mathrm{T}ik}$, Λ_{M} is a matrix of method-related factor loadings $\lambda_{\mathrm{M}ik}$, $\mathbf{t}$ is a vector of trait variables T_i, $\mathbf{m}$ is a vector of method factors M_k, $\mathbf{e}$ is a vector of error variables E_{ik}, $\Lambda_{\mathrm{T}i} = \mathrm{diag}(\lambda_{\mathrm{T}ik})$ is a (3×3) matrix of trait loadings, and $\boldsymbol{\lambda}_{\mathrm{M}k}$ is a 3×1 vector with elements $\lambda_{\mathrm{M}ik}$. Thus, in this model each observed variable is decomposed into a trait factor, a method factor, and an error variable. The trait- and method factor covariance matrix is assumed to be block-diagonal:

$$\mathrm{E}\begin{pmatrix} \mathbf{t} \\ -- \\ \mathbf{m} \end{pmatrix}\begin{pmatrix} \mathbf{t} \\ -- \\ \mathbf{m} \end{pmatrix}' = \Phi_{(6\times 6)} = \begin{bmatrix} \Phi_{\mathrm{T}} & 0 \\ 0 & \Phi_{\mathrm{M}} \end{bmatrix}, \tag{2}$$

where Φ_{T} denotes the (3×3) covariance matrix of the trait factors, and Φ_{M} denotes the (3×3) covariance matrix of the method factors. Furthermore, the error variables are presumed uncorrelated with each other, having a (9×9) diagonal covariance matrix Θ, and the error variables are uncorrelated with the trait and method factors. When Σ denotes the covariance matrix of the observed variables Y_{ik}, this model states that it is structured

$$\Sigma = \Lambda_{\mathrm{T}}\Phi_{\mathrm{T}}\Lambda_{\mathrm{T}}' + \Lambda_{\mathrm{M}}\Phi_{\mathrm{M}}\Lambda_{\mathrm{M}}' + \Theta.$$

The scaling indeterminacies are removed in the usual CFA manner by either constraining the covariance matrix Φ to be a correlation matrix or by constraining one factor loading for each factor to unity.

The CFA-CTCM model is widely applied to MTMM data, because it includes trait as well as method factors and assumes that both factors contribute independently to explaining the variation in an observed variable (Brannick & Spector, 1990). Therefore, the variance of an observed variable can be partioned into trait, method, and error components. The CFA-CTCM model, however, is affected by three shortcomings (Kenny & Kashy, 1992; Marsh, 1989). *(a) Nonproper* *solutions*. Iterative procedures often do not converge to a unique solution or result in estimates that are outside the permissible range of values, for example, negative variances of the method factors or the error variables (Bagozzi, 1993; Kenny & Kashy, 1992; Marsh, 1989; Marsh & Grayson, 1995). These improper solutions might often be due to underidentified models. Grayson and Marsh (1994) as well as Kenny and Kashy (1992) have shown that the CFA-CTCM model is not globally identified. Therefore, it is not applicable in general. For example, the CFA-CTCM with equal factor loadings for all observed variables that load on the same factor is not identified. *(b) Problems in the interpretation of trait and method factors*. In the CFA-CTCM model it is assumed that there are as many trait factors as traits considered and as many method factors as methods included in an MTMM study. The model, however, does not give an answer to the question what the factors really measure. In particular, it is not clarified what the difference between a trait and a method factor is. The only difference is that they are indicated by different indicators. It is unclear, however, whether there is a more fundamental difference in meaning between both types of factors. This causes some problems of interpreting the method and trait factors, particularly when all method factors are correlated. *(c) Uncorrelatedness of trait and method factors*. Although it is desirable to separate trait from method effects, the assumption of uncorrelated trait and method factors can be questioned, and less restrictive models with correlated trait and method factors are applied as well (e.g., Schmitt & Stults, 1986). It is still an open question under which conditions it is reasonable to assume that trait and method factors are correlated and under which conditions it is not.

To overcome the problems of the CFA-CTCM model, Marsh (1989) recommended to apply a model without method factors, but with correlated uniqueness variables. This model was originally proposed by Kenny (1979). The correlated trait/correlated uniqueness model (CFA-CTCU model; Kenny, 1979; Marsh & Grayson, 1995) for mean-corrected data is defined by the equation $Y_{ik} = \lambda_{Tik} T_i + E_{ik}$ and the assumptions (a) that all trait variables T_i are correlated, (b) all error variables E_{ik} with the same index i are correlated, and (c) all other latent variables are uncorrelated. This model converged to proper solutions in most applications. Marsh et al. (1992), however, discussed two limitations of this model: (a) Correlated method effects (across different methods) cannot be considered, and (b) this model is not parsimonious because of many error correlations. Furthermore, random error is confounded with method specificity (Bagozzi, 1993) and, consequently, the reliability coefficients of the observed variables might be underestimated.

To overcome the identification problems of the CFA-CTCM model and to get more appropriate estimates of the reliability coefficients than in the CFA-CTCU model, Kenny and Kashy (1992) proposed a fixed method model in which the number of method factors is restricted to one fewer than the number of methods included in the design. This model is defined by the assumption that the sum of method effects for each individual equals zero. The matrix of factor loadings has an effect coding structure, meaning that all manifest variables belonging to the same method have the same factor loadings on the method factors. Each method factor is indicated by two different methods with factor loadings of 1 and (-1), and the factor loadings of all other methods equal zero. According to Kenny and Kashy (1992), the major limitation of this model is that the method effects for each individual sum to zero. Consequently, each variable has to be measured in the same metric and the bias due to one method is exactly offset by the bias due to another method. Because of this restriction, Kenny and Kashy (1992) conclude that this model might be too restrictive for empirical applications. Furthermore, the trait and method factors are correlated in this model. Hence, the variance of an observed variable cannot be additively decomposed in components due to trait, method, and error effects, and these correlations might cause interpretative problems.

In this paper, a new CFA model will be introduced, that overcomes several shortcomings of previous models for MTMM data sets. Like the fixed method model, there are only $m-1$ method factors in this model, where m denotes the number of methods considered in a design. In contrast to the fixed method model proposed by Kenny and Kashy (1992), however, (a) a method factor is indicated by the variables belonging to only one method, (b) the loadings do not have to be equal

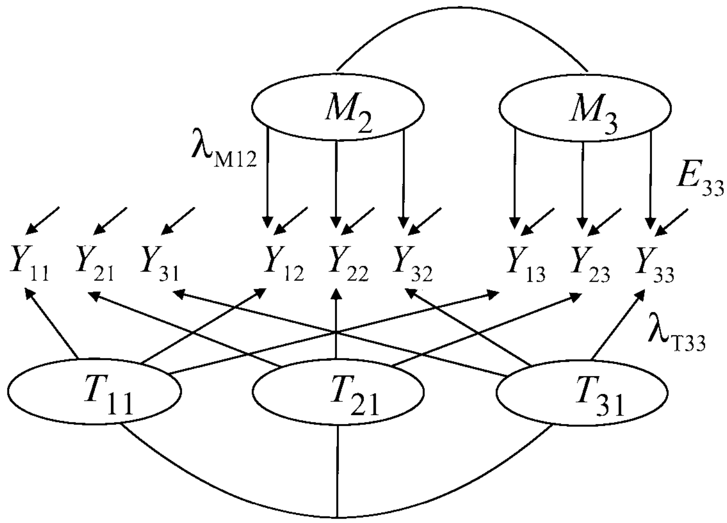

FIGURE 1.
A multitrait-multimethod model with twelve observed variables Y_{ik} measuring three traits ($i = 1, 2, 3$) with three methods ($k = 1, 2, 3$). The first method is taken as comparison standard. T_{ik}: true-score variables (trait factors); M_l: method factors; E_{ik}: error variables. The definitions of the latent variables are explained in the text.

for all variables of the same method, (c) the method and trait factors are uncorrelated, and (d) it is possible to estimate variance components due to trait, method, and error effects. In this model, one method k is chosen as a comparison standard for all other methods l. A method-specific latent variable is defined as that part of a true-score variable $T_{il} = Y_{il} - E_{il}$ measuring a trait i by a method l that cannot be predicted by the true-score variable T_{ik} measuring the same trait i with the method k that was chosen as comparison standard. Thus, the model structure equals a CFA-CTCM model with one method factor less than the number of methods included. For example, if the first method is chosen as comparison standard, the model is a special version of the CFA-CTCM model in (1) and (2) obtained by setting $\boldsymbol{\lambda}_{\text{M}1}$ to $\mathbf{0}_{(3\times1)}$, and setting the variance of the first method factor and the covariances between the first method factor and the two other ones to 0 in Φ_{M} (see Figure 1).

In contrast to the full CFA-CTCM model, however, the model proposed in this paper is identified, even under conditions under which the CFA-CTCM model is not. In contrast to the CFA-CTCU model, the model presented in this paper allows the estimation of variance components due to trait, method, and error variables. Furthermore, the model differs from all the CFA-MTMM models developed so far in the way it is defined. In contrast to previous CFA-MTMM models, the model is defined as a stochastic measurement model on the basis of classical psychometric test theory (CTT; Steyer, 1989), and all trait and method factors are defined as functions of the true-score variables.[1] Therefore, the trait and method factors are well-defined and have a clear meaning. As will be shown, there is a fundamental difference in meaning between the trait and the method factors. Furthermore, the model can be defined by only three assumptions. All other properties of the model, for example, the uncorrelatedness of trait variables with method factors and the uncorrelatedness of the error variables with all other latent variables, are logical

[1]In the CFA-CTCM model (and in other CFA-MTMM models) it might be implicitly assumed that an observed variable is decomposed into a true-score variable T_{ik} and an error variable E_{ik}, and each true-score variable is a linear function of trait factor T_i and a method factor M_k:

$$Y_{ik} = T_{ik} + E_{ik}, \quad \text{where}$$

$$T_{ik} = \lambda_{\text{T}ik}T_i + \lambda_{\text{M}ik}M_k.$$

However, the trait and method factors are not explicitly defined as functions of the true-score variables.

consequences of the definition of the model. Therefore, this model provides a rationale for the uncorrelatedness of trait and method factors in MTMM models.

3. CTT as a Formal Framework for Defining MTMM Models

In classical test theory, it is assumed that an observed variable Y can be decomposed into a latent true-score variable T and an error variable E (Lord & Novick, 1968): $Y = T + E$. Using the concepts of conditional expectation and conditional independence, it can be demonstrated that the main results of CTT can be derived with minimal assumptions. Using this approach, it was shown that some of the axioms in CTT are not assumptions, but immediate consequences of defining the true-score and error variables (Lord & Novick; Steyer, 1989; Zimmerman; 1975). For example, the uncorrelatedness of the true-score and the error variables is such a consequence that cannot be wrong in empirical applications.

In order to define an MTMM model on the basis of CTT, the starting point is the decomposition of each observed variable Y_{ik} into a true-score variable T_{ik} and an error variable E_{ik}:

$$Y_{ik} = T_{ik} + E_{ik}.$$

The general properties of the error variables are described by Lord and Novick (1968) as well as Steyer (1988). The most important consequences of these poperties are: (a) The expected value of an error variable as well as the conditional expected value of an error variable given a true-score variable T_{ik} are 0. (b) The error and true-score variables are uncorrelated. Consequently, the variance of an observed variable can be decomposed into the variance of the true-score variable and the error variable: $\text{var}(Y_{ik}) = \text{var}(T_{ik}) + \text{var}(E_{ik})$. Based on this decomposition, the reliability coefficient is defined as follows: $\text{rel}(Y_{ik}) := \text{var}(T_{ik})/\text{var}(Y_{ik})$, if $0 < \text{var}(Y_{ik}) < \infty$.

To define the models of essentially τ-equivalent variables and τ-congeneric variables, it is assumed that the true-score variables of all observed variables are perfectly correlated, because they are either translations (essentially τ-equivalent variables) or linear functions of each other (τ-congeneric variables). Both models imply the existence of one common factor (Steyer, 1989). In MTMM models, the assumption of unidimensionality is weakened, because the indicators measuring the same latent trait are not perfectly homogeneous and contain a method-specific component. To define an MTMM model, the true-score variables have to be decomposed. In this paper, an MTMM model is based on the decomposition:

$$T_{il} = \text{E}(T_{il}|T_{ik}) + M_{il}. \tag{3}$$

Thus, a true-score variable T_{il} is decomposed into the latent regression $\text{E}(T_{il}|T_{ik})$ and a residual M_{il}. The residual M_{il} indicates that part of a true-score variable T_{il} that is not due to another true-score variable T_{ik} supposed to measure the same trait i. Hence, this residual indicates the method-specific effect of a method l with respect to a method k that is chosen as a comparison method. The major advantage of defining a method-specific variable in this way is that the true-score variable T_{ik} and the method variable M_{il} are uncorrelated, because M_{il} is a residual with respect to the regression $\text{E}(T_{il}|T_{ik})$. Consequently, the variance of a true-score variable T_{il} can be additively decomposed in the following way:

$$\text{var}(T_{il}) = \text{var}[\text{E}(T_{il}|T_{ik})] + \text{var}(M_{il}). \tag{4}$$

The coefficient of determination $\text{con}(T_{il})k = \text{var}(\text{E}(T_{il}|T_{ik}))/\text{var}(T_{il})$, $0 < \text{var}(T_{il}) < \infty$, is the part of the variance of a true-score variable T_{il} that is explained by another true-score variable T_{ik} with the same index i. As this coefficient characterizes the consistency in the trait measurements across different methods, it is called *consistency coefficient of a true-score variable T_{il} with* *respect to the true-score variable* T_{ik}. This coefficient quantifies the *convergent validity*. The coefficient of indetermination $\mathrm{ms}(T_{il})_k = \mathrm{var}(M_{il})/\mathrm{var}(T_{il})$, $0 < \mathrm{var}(T_{il}) < \infty$, on the other hand, is the part of the variance of a true-score variable that is not explained by another true-score variable T_{ik}. As both variables should measure the same trait i, this variance component indicates that part of the variance of the true-score variable T_{il} that is due to the specific method l and is called the *method specificity coefficient of a true-score variable* T_{il} *with respect to the true-score variable* T_{ik}.

Because of (4) the reliability coefficient of an observed variable can be decomposed into two components as well:

$$\mathrm{rel}(Y_{il}) = \frac{\mathrm{var}(T_{il})}{\mathrm{var}(Y_{il})} = \frac{\mathrm{var}[\mathrm{E}(T_{il}|T_{ik})]}{\mathrm{var}(Y_{il})} + \frac{\mathrm{var}(M_{il})}{\mathrm{var}(Y_{il})},$$

if $0 < \mathrm{var}(Y_{ik}) < \infty$. The first variance component $\mathrm{con}(Y_{il})_k := \mathrm{var}[\mathrm{E}(T_{il}|T_{ik})]/\mathrm{var}(Y_{il})$ indicates the part of the variance of an observed variable that is due to interindividual differences on a true-score variable T_{ik} and is called *consistency coefficient of an observed variable* Y_{il} *with respect to the true-score variable* T_{ik}. The second component $\mathrm{ms}(Y_{il})_k := \mathrm{var}(M_{il})/\mathrm{var}(Y_{il})$ measures the proportion of variance of an observed variable that is due to true method-specific differences and is called *method specificity coefficient of an observed variable* Y_{il} *with respect to the true-score variable* T_{ik}.

All equations described so far do not depend on any restrictive assumption, and they cannot be wrong in any empirical application. However, without further assumptions the variances of the true-score variables, error variables, and method-specific residuals as well as the reliability, consistency, and method specificity coefficients are not identified. Hence, further assumptions have to be made that define specific MTMM models. Because the MTMM model defined in the next section is a CFA-CTCM model with one method factor less than methods considered, it is called *CFA-CTC(M-1)*$_k$ *model*, where k denotes the method chosen as comparison standard.

4. A CFA-CTC(M-1)$_k$ Model for MTMM Data

In order to define a MTMM model within the formal framework described in the last section, it is necessary to select one method as comparison standard. This means that the true-score variables of this method are taken as regressors and the true-score variables of the remaining methods as regressands in (3). The CFA-CTC(M-1)$_k$ model is defined by three assumptions.

Definition: CFA-CTC(M-1)$_k$ *model.* The variables $Y_{11}, \ldots, Y_{ik}, \ldots, Y_{tm}, i \in I = \{1, \ldots, t\}, k \in K = \{1, \ldots, m\}$, are variables of a CFA-CTC(M-1)$_k$ model, if and only if $T_{11}, \ldots, T_{ik}, \ldots, T_{tm}$ are the true-score variables, and $E_{11}, \ldots, E_{ik}, \ldots, E_{tm}$ are the error variables defined in Appendix A, $k \in K$ is the method chosen as comparison standard, and:

1. Given $k \in K$, there are one μ_{il} and one $\lambda_{\mathrm{T}il} \in \mathbb{R}$, $\lambda_{\mathrm{T}il} > 0$, for each pair (i, l), $i \in I, l \in K$, such that

$$\mathrm{E}(T_{il}|T_{ik}) = \mu_{il} + \lambda_{\mathrm{T}il} T_{ik}, \tag{5}$$

   where $\mathrm{E}(T_{il}|T_{ik})$ denotes the T_{ik}-conditional expectation of T_{il}.
2. For each pair (i, l), $i \in I, l \in K, l \neq k$, there is a $\lambda_{\mathrm{M}il} \in \mathbb{R}$, $\lambda_{\mathrm{M}il} > 0$, and for each $l \in K$, $l \neq k$, there is a variable M_l such that

$$M_{il} = \lambda_{\mathrm{M}il} M_l, \tag{6}$$

   where $M_{il} = T_{il} - \mathrm{E}(T_{il}|T_{ik})$.
3. $$\mathrm{cov}(E_{il}, E_{jl'}) = 0, (i, l) \neq (j, l'), \quad i, j \in I; l, l' \in K. \tag{7}$$

4. The vector $\boldsymbol{\mu}'$ and the matrices Λ_{T}, Λ_{M}, Φ_{T}, Φ_{M}, and Θ are defined as follows.

$$\boldsymbol{\mu}' = (\mu_{11}, \ldots, \mu_{i1}, \ldots, \mu_{t1}, \ldots, \mu_{1l}, \ldots, \mu_{il}, \ldots, \mu_{tl}, \ldots, \mu_{1m}, \ldots, \mu_{im}, \ldots, \mu_{tm})$$

with $\mu_{il} = 0, \quad \text{if } l = k.$

$$\Lambda_T = \begin{bmatrix} \Lambda_{\mathrm{T}1} \\ \vdots \\ \Lambda_{\mathrm{T}l} \\ \vdots \\ \Lambda_{\mathrm{T}m} \end{bmatrix} \quad \text{and} \quad \Lambda_{\mathrm{M}} = \begin{bmatrix} \boldsymbol{\lambda}_{\mathrm{M}1} & 0 & 0 & \ldots & 0 \\ 0 & \boldsymbol{\lambda}_{\mathrm{M}2} & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & 0 & \boldsymbol{\lambda}_{\mathrm{M}m} \end{bmatrix},$$

where $\Lambda_{\mathrm{T}l} = \mathrm{diag}(\lambda_{\mathrm{T}il})$ is a $(m \times m)$ matrix of trait loadings with $\Lambda_{\mathrm{T}k} = \mathrm{I}$ (unity) for $l = k$, and $\boldsymbol{\lambda}_{\mathrm{M}l}$ is a $(t \times 1)$ vector with elements $\lambda_{\mathrm{M}il}$ for $l \neq k$, and $\boldsymbol{\lambda}_{\mathrm{M}k} = \mathbf{0}$ for $l = k$. Φ_{T} is a fully free $(t \times t)$ matrix with elements $\mathrm{cov}(T_i, T_j), i, j \in I$. Φ_{M} is a $(m \times m)$ matrix with elements $\phi_{ll'} = \mathrm{cov}(M_l, M_{l'})$ for $l \neq k$ and $l' \neq k$, and $\phi_{ll'} = 0$ for $l = k$ or $l' = k$. Θ is a $(tm \times tm)$ diagonal matrix of error variances with diagonal elements $\mathrm{var}(E_{11}), \ldots, \mathrm{var}(E_{t1}), \ldots, \mathrm{var}(E_{1m}), \ldots, \mathrm{var}(E_{tm})$.

To define an MTMM model in the framework of CTT, it is necessary to choose one method k as a comparison standard. According to Assumption 1, the regressions of all true-score variables with the same index i on the true-score variable T_{ik} are assumed to be linear functions of T_{ik}. Hence, the true-score variable T_{ik} of the method k that is chosen as comparison standard is the *common latent trait variable* of all true-score variables with the same index i. Assumption 2 means that all residuals belonging to the same method l measure one common method factor M_l. This assumption is equivalent to the assumption that all residuals M_{il} belonging to the same method l are linear functions of each other (see Appendix B): $M_{il} = \lambda_{\mathrm{M}ijl} M_{jl}$. This means that all residuals M_{il} belonging to the same method l are perfectly correlated indicating a homogeneous method effect across traits. There is no additive constant (intercept) in (6) because the variables M_{il} are residuals and, therefore, $\mathrm{E}(M_{il}) = 0$. The common method factors M_l, however, are not uniquely defined by (6), because several combinations of $\lambda_{\mathrm{M}il}$ and M_l fulfill (6). The method factors M_l and the corresponding loading parameters $\lambda_{\mathrm{M}il}$ are uniquely defined only up to similarity transformations, that is, multiplying M_l with a real constant α and dividing $\lambda_{\mathrm{M}il}$ by the same constant α do not change (6). Consequently, some standardizations must be made to select specific representations. One possible standardization is to set either one item parameter $\lambda_{\mathrm{M}il}$ or the variances of the method factors M_l to any real value larger than 0 for each $l \in K$. Assumptions 1 and 2 show that there is a fundamental difference in what a trait and what a method factor is. Whereas the trait factors are the true-score variables of the comparison method, method factors are linear functions of true-score residuals. Assumption 3 means that all error variables are uncorrelated.

Because of (5) and (6), the following equation holds for all observed variables:

$$Y_{il} = \begin{cases} \mu_{il} + \lambda_{\mathrm{T}il} T_{ik} + \lambda_{\mathrm{M}il} M_l + E_{il}, & \text{for } l \neq k \\ T_{ik} + E_{ik}, & \text{for } l = k. \end{cases} \tag{8}$$

The CFA-CTC(M-1)$_k$ model implies that (a) the trait factors are uncorrelated with the method factors and the error variables, and that (b) the method factors are uncorrelated with the error variables (see Appendix C). Furthermore, the error variables are uncorrelated with each other because of (7). Consequently, the CFA-CTC(M-1)$_k$ model implies the following covariance structure:

$$\Sigma = \Lambda_{\mathrm{T}} \Phi_{\mathrm{T}} \Lambda'_{\mathrm{T}} + \Lambda_{\mathrm{M}} \Phi_{\mathrm{M}} \Lambda'_{\mathrm{M}} + \Theta,$$

where the matrices have the structure described in the definition of the CFA-CTC(M-1)$_k$ model. Using computer programs for structural equation modeling, for example, AMOS (Arbuckle, 1995), EQS (Bentler, 1992), LISREL (Jöreskog & Sörbom, 1993), or Mplus (Muthén & Muthén, 1998), it can be tested whether the covariance structure implied by the CFA-CTC(M-1)$_k$ model holds for an empirical application.

4.1. Identification

One major problem of the CFA-CTCM model is that it is not globally identfied. Grayson and Marsh (1994) discussed conditions for nonidentification of the CTCM model. They showed that the CFA-CTCM model is not identified if the MTMM loading matrix is rank deficient. Furthermore, they specified the conditions under which the MTMM loading matrix is rank deficient (Grayson & Marsh, 1994, Theorem 3). One condition leading to rank deficiency, for example, is the situation when all loading parameters are set equal to one. Other conditions are discussed by Grayson and Marsh (1994). Because there is one method factor less than methods included in an MTMM study, the CFA-CTC(M-1)$_k$ model is identified even in the case where all loading parameters are equal to 1.[2] The following theorem shows that the CFA-CTC(M-1)$_k$ model is globally identified when there are at least three traits and three methods. Furthermore, this theorem specifies the conditions under which the model is identified when there are less than three traits or less than three methods.

Theorem: Identification. If the variables $Y_{11}, \ldots, Y_{ik}, \ldots, Y_{tm}, i \in I, k \in K$, are variables of a CFA-CTC(M-1)$_k$ model, then the parameters of the vector μ and the matrices Λ_{T}, Λ_{M}, Φ_{T}, Φ_{M}, and E are identified, if either one factor loading $\lambda_{\mathrm{M}il}$ for each method factor M_l or the variances of the method factors are set to any real value larger than 0, and

(a) $t > 2$ and $m > 2$,
(b) $t > 2, m = 2$, and Φ_{T} is fully free with non-zero elements,
(c) $t = 2, m > 2$, and Φ_{T} and Φ_{M} are fully free with nonzero elements.

(Proof, see Appendix D).

According to this theorem, the CFA-CTC(M-1)$_k$ model is identified if there are at least three traits and three methods included in an MTMM study. If there are less than three traits or less than three methods, further conditions must be fulfilled to get an identified model. The minimum condition, however, is that either the number of traits or the number of methods is larger than 2. In the case of two traits and two methods the model is not identified without further restrictions on the parameters. For example, a model in which all loading parameters are set equal to one is identified in the case $t = 2$ and $m = 2$.

5. Applications

To illustrate the CFA-CTC(M-1)$_k$ model, it is applied to two data sets: (a) the Mount (1984) data set that was reanalyzed with CFA-MTMM models by Kenny and Kashy (1992), and (b) data from a study on sun protection behavior (Eid, 1997; Eid, Klusemann & Schwenkmezger, 1996). The applications refer to mean-corrected data sets.

5.1. Application I: The Mount (1984) Study

Mount (1984) compared self-ratings of managerial performance to those of supervisors and subordinates. Each rating was given on a nine point scale. The responses of the subordinates

[2] If the CFA-CTC(M-1)$_k$ model is defined by a reduced matrix Λ_{M} that does not contain $\boldsymbol{\lambda}_{\mathrm{M}k} = \mathbf{0}$, then the loading matrix of the CFA-CTC(M-1)$_k$ model is not rank deficient. In this article, $\boldsymbol{\lambda}_{\mathrm{M}k} = \mathbf{0}$ is included in Λ_{M} because it simplifies the description of the model. Both presentations are equivalent.

were aggregated for each manager. Kenny and Kashy (1992) reanalyzed a (3 × 3) MTMM matrix from the Mount (1984) study with CFA models ($N = 80$). The traits were *administrative ability*, *ability to give feedback to subordinates*, and *consideration when dealing with others*. The three methods were the self-ratings and the ratings of the supervisors and the subordinates. Kenny and Kashy (1992) reported several difficulties in fitting CFA-CTCM models with and without correlations between the method and the trait factors. Both, the application of the CFA-CTCU model as well as the fixed method model resulted in proper solutions and an appropriate fit of the model (CFA-CTCU model: $\chi^2 = 18.73$, $df = 15$, $p = .23$, CAIC = 180.02; fixed method model: $\chi^2 = 18.56$, $df = 15$, $p = .23$, CAIC = 180.19).

The (3 × 3) MTMM matrix was reanalyzed by the CFA-CTC(M-1)$_k$ model. In order to apply this model one of the three methods has to be chosen as comparison standard. Because there is one self-report method and two other-report methods it is reasonable to take the self-report as comparison standard and to contrast it with the two other-report methods. Therefore, the method factors measure deviations of the other-report variables from the values predicted by the self-report variables, and the correlations between the method factors indicate the degree of convergence of method-specific effects due to different other reports. The fit of the model with correlated method factors is $\chi^2 = 19.51$, $df = 17$, $p = .30$, CAIC = 170.26. In order to check the fit of the models if one of the other-report methods is chosen as comparison standard, these models are analyzed as well. The fit of the model with the supervisor ratings as comparison method is $\chi^2 = 23.68$, $df = 17$, $p = .13$, CAIC = 174.37, and the fit of the model with the subordinates ratings as comparison method is $\chi^2 = 24.73$, $df = 17$, $p = .10$, CAIC = 175.43. These results show that the fit of the CFA-CTC(M-1)$_k$ model depends on the choice of the method chosen as comparison standard. The model is not symmetrical and, therefore, the comparison method has to be adequately selected. Furthermore, the results confirm that the self-report method is a suitable comparison standard in the current application. Comparing the different MTMM models by their CAIC coefficients shows that the CFA-CTC(M-1)$_k$ models are more parsimonious than the other two CFA-MTMM models, the CFA-CTCU and the fixed method model. In contrast to the two other CFA-MTMM models, only the CFA-CTC(M-1)$_k$ model allows the estimation of variance components due to trait, method, and error effects. Taking the properties and the fit coefficients together, the CFA-CTC(M-1)$_k$ model with the self-report method taken as comparison standard seems to be the most appropriate model of all the CFA-MTMM models considered in this article for analyzing the managerial performance data set.

The estimated parameters of the CFA-CTC(M-1)$_1$ model (where $k = 1$ denotes the self-report method) and the estimated reliability, consistency, and method specificity coefficients are presented in Table 1. The estimated *reliability* coefficients range between .28 and .72. The method specificity coefficients are very large for the rating of the *ability to give feedback* indicating that both other reports differ largely from the self-report with respect to this trait. For the two other traits, the method specificity is lower, particularly for the supervisors. This shows that for these two traits the self-ratings are more similar to the supervisors' ratings than to the subordinate ratings. The correlation between the *administrative ability* and the *ability to give feedback* is relatively large, whereas the other correlations as well as the correlations between the method factors are comparably small.

5.2. *Application II: Sun Seeking Behavior*

The second application refers to a study on sun protection behavior (Eid, 1997; Eid et al., 1996). In this study $N = 531$ participants were asked to report various aspects of their sun-related behavior in the last summer, their health attitudes, and their health behavior in general. After reading a short text on sun-protection behavior, the participants had to answer several questions related to risk appraisal. Finally, they were asked to report how they intended to behave in the next summer. The CFA-CTC(M-1)$_k$ model was applied to three items (methods) measuring the

TABLE 1.
Parameter Estimtes for the Mount (1984) Data

Rating	Factor loadings						Variance components		
	T_{11}	T_{21}	T_{31}	M_2	M_3	Rel (Y_{ik})	Con (Y_{il})$_1$	MS(Y_{il})$_1$	Con (T_{il})$_1$
Self									
Administration	1.00					.60			
Feedback		1.00				.59			
Consideration			1.00			.42			
Supervisor									
Administration	.94			1.00		.59	.50	.08	.86
Feedback		.29		2.16		.65	.09	.56	.14
Consideration			.78	1.06		.56	.36	.18	.67
Subordinate									
Administration	.44				1.00	.28	.18	.10	.64
Feedback		.19			2.25	.52	.05	.47	.10
Consideration			.69		1.80	.72	.32	.40	.44

Factor Correlations

T_{11}-T_{21}: .43; T_{11}-T_{31}: .01; T_{11}-T_{31}: .26; M_2-M_3: .23

sun-related behavior in the last summer (Trait 1) and the intended behavior in the next summer (Trait 2). This application differs in two aspects from the first application: (a) The different items are considered as different methods, but all items were answered by self-report. Thus, method specificity in this application is due to item specificity, that is, differences in the formulation of the items that were constructed to measure the same latent construct. (b) The sample size is much larger than in the first application. Therefore, the test statistic has much more power to detect a misfit of the CFA-CTC(M-1)$_k$ model. The instruction for the assessment of the sun-related behavior in the last summer was: "This summer, I ... " and the items were: (i) " ... tried to get as dark a tan as possible" (suntan), (ii) " ... avoided the sun as far as possible," (sun avoidance) (iii) " ... I often lied out in the sun taking a sunbath" (sunbathing). The instruction for the items assessing the intended sun-related behavior in the next summer was: "Next summer, I ... " and the items were: (a) " ... will try to get as dark a tan as possible (suntan)," (b) " ... will avoid the sun as far as possible" (sun avoidance), (c) " ... I will often lie out in the sun taking a sunbath" (sunbathing). Each item was rated on a four-point rating scale with labels *is definitely true*, *is mainly true*, *is mainly wrong*, *is definitely wrong*.

In this application, it is less obvious which of the three methods should be selected as comparison standard. Therefore, the method that is most appropriate as comparison standard was detected by an empirical analysis. First, the total sample was randomly split into two subsamples of sizes $n_1 = 265$ (Subsample 1) and $n_2 = 266$ (Subsample 2). Then, the covariance matrices of the six observed variables were calculated for the two subsamples after listwise deletion of missing values (corrected n's: $n_1 = 256$, $n_2 = 259$). Finally, the fit of the three possible CFA-CTC(M-1)$_k$ models with different methods taken as comparison standard were analyzed by LISREL 8 (Jöreskog & Sörbom, 1993) for each subsample and the generalizability of the results across the two subsamples was analyzed. The results of these analyses are given in Table 2 (all method loadings are set equal to one in these applications). In both subsamples, the model with the suntan item as comparison standard shows the best fit, followed by the sun-avoidance item and the sunbathing item. After this cross-validation of the model fit coefficients it seems reasonable to take the suntan item as comparison standard. This model is also interesting for psychological reasons. The suntan item refers to more appearance-related *motivational* aspects (motivation to get tan) whereas the other two items assess sun-related *behavior* (sun-seeking vs. sun-avoiding behavior). Therefore, correlated method factors would indicate behavior-specific convergence. Hence, in this model motivational and behavioral aspects are contrasted.

For simplicity reasons, the model parameters are reported for the total sample. The covariance matrix of the total sample is depicted in Table 3 ($N = 515$). The CFA-CTC(M-1)$_1$ model (where $k = 1$ denotes the sun-tan item) fits the data well ($\chi^2 = 6.59$, $df = 5$, $p = .25$, CAIC = 122.50). The CAIC coefficient of the CFA-CTC(M-1)$_1$ model is smaller than the CAIC coefficients of the CFA-CTCU model ($\chi^2 = 9.06$, $df = 5$, $p = .11$, CAIC = 124.97) and the fixed method model ($\chi^2 = 0.66$, $df = 1$, $p = .42$, CAIC = 145.54). Furthermore, the

TABLE 2.
Fit Coefficients of Three CFA-CTC(M-1)$_k$ Models in Two Randomly Selected Subsamples

Comparison method	Subsample 1	Subsample 2
Sun tan	$\chi^2 = 7.11$, $p = .21$ CAIC = 111.83	$\chi^2 = 9.60$, $p = 0.9$ CAIC = 114.51
Sun avoidance	$\chi^2 = 19.17$, $p < 0.1$ CAIC = 123.89	$\chi^2 = 18.09$, $p < .01$ CAIC = 123.00
Sunbathing	$\chi^2 = 8.36$, $p = .14$ CAIC = 113.80	$\chi^2 = 11.70$, $p = .04$ CAIC = 116.61

Note. $df = 5$.

TABLE 3.
Covariance Matrix of the Sun Seeking Behavior Items

Behavior						
Suntan	1.08					
Sun avoidance	0.35	0.80				
Sunbathing	0.70	0.36	0.95			
Intentions						
Suntan	0.72	0.33	0.57	0.92		
Sun avoidance	0.37	0.51	0.37	0.40	0.94	
Sunbathing	0.62	0.38	0.68	0.64	0.42	0.90

Note. $N = 515$. The sun avoidance items are recoded.

application of the CFA-CTCM model resulted in improper solutions and identification warnings. Hence, the CFA-CTC(M-1)$_k$ model is a suitable model for the present application.

Table 4 contains the estimated parameters and coefficients of the CFA-CTC(M-1)$_1$ model. The reliabilities of the items are relatively large (between .57 and .83) and are clearly larger than the reliabilities estimated in the CFA-CTCU model (between .25 and .78, not reported in a table). The *sun-avoidance* items and the *sunbathing* items differ in their consistency and method specificity coefficients. The consistency is relatively large for the sunbathing items and relatively small for the sun-avoidance items. This result shows that the motivation to get a suntan is more closely related to the sunbathing behavior than to the sun-avoidance behavior. Both latent trait variables are highly correlated indicating that the intentions largely depend on previous behavior. The correlation between the method factors is moderately large and significant showing that both behavior related items have more in common than is explained by the correlations of the trait factors.

6. Discussion

Both applications demonstrate that the CFA-CTC(M-1)$_k$ model is a useful model for the analysis of MTMM data. In contrast to the CFA-CTCM model, it is globally identified. From this point of view, the problems of the CFA-CTCM model might be due to an overfactorization. This problem of overfactorization due to considering as many method factors as methods included in a MTMM design might not only be relevant for MTMM models but also for multimethod models of longitudinal data analysis (for a discussion of method factors in longitudinal studies, see Eid, 1996; Eid, Schneider & Schwenkmezger, 1999). Thus, the CFA-CTC(M-1)$_k$ model gives some reasons why the variance of one method factor is often not significantly different from 0 in an empirical application. Compared to the CFA-CTCU model, the CFA-CTC(M-1)$_k$ model (a) allows correlated method factors, (b) is parsimonious, and (c) provides more appropriate estimates of the reliability coefficients. In contrast to the CFA-CTCU model and the fixed method model, only the CFA-CTC(M-1)$_k$ model allows the estimation of variance components due to trait, method, and error effects. These variance components can be used to select items and scales with small method specificity. Furthermore, all latent variables of the CFA-CTC(M-1)$_k$ model can be defined on the basis of the true-score variables of classical psychometric test theory. This does not only clarify the meaning of the latent variables, but gives clear reasons for uncorrelatedness restrictions usually made in structural equation models of MTMM data.

Besides the advantages of the CFA-CTC(M-1)$_k$ model, there are three limitations that might restrict its applicability. The first limitation is that one method has to be selected as a comparison standard. The CFA-CTC(M-1)$_k$ model is not symmetrical and the choice of the comparison method affects the fit of the model as it was shown in the two empirical applications. The choice of an appropriate comparison standard might be a problem for the empirical application of the model. The selection of a comparison method is certainly best guided by theoretical assumptions

TABLE 4.
Parameter Estimates for the Skin Cancer Data

Variable	Factor loadings				Variance components			
	T_{11}	T_{21}	M_2	M_3	Rel (Y_{ik})	Con $(Y_{il})_1$	MS$(Y_{il})_1$	Con $(T_{il})_1$
Suntan								
Sun-seeking behavior	1.00				.80			
Sun-seeking intention		1.00			.80			
Sun avoidance								
Sun-seeking behavior	.42		1.00		.62	.19	.43	.31
Sun-seeking intention		.51	1.00		.57	.21	.36	.37
Sunbathing								
Sun-seeking	.80			1.00	.77	.58	.19	.67
Sun-seeking intention		.87		1.00	.83	.62	.21	.75
Factor Correlations								
T_{11}-T_{21}: .90; M_2-M_3: .33								

Note. All estimated loading parameters and all estimated variances and covariances are significantly different from 0 ($p < .01$). The sun avoidance items are recoded.

and research interests. In the first application of the CFA-CTC(M-1)$_k$ model, for example, the research interest was in contrasting self-report with other-report methods. In many applications, however, theoretical reasons for preferring one method to the others might not be obvious. In this case, a cross-validation strategy might be a suitable way to select an appropriate comparison method. This strategy was demonstrated in the second application. It is a worthwhile task for future research to explore the conditions that makes a method a good comparison standard.

The second limitation is that there is only one indicator for each trait-method combination. The very low reliabilities of some items in the first application show that the error variables might not only reflect measurement error but item-specific variance that is due to item-rater interactions. To get better estimates of the reliabilities, a model with multiple indicators for each trait-method combination might be more appropriate (Marsh & Hocevar, 1988). Thus, the development of an MTMM multiple indicator model in which all latent variables are defined as functions of the true-score variables seems to be a valuable task for future research. The second application, however, illustrates how the model can be used to estimate the degree of item specificity if all ratings are self-reports. In this case, different items were considered as different methods, but all items are developed as indicators of one latent construct. Hence, method effects are due to differences in the semantic formulation of the items. In this application the reliability coefficients are reasonably high and the model fits the data well. Thus, the CFA-CTC(M-1)$_k$ model is particularly useful for the analysis of item-specificity in multidimensional test models, in which the same items are administered under different conditions.

The third limitation might be due to the assumption of linear relations between the observed and the latent variables. In both applications, as in most CFA-MTMM studies, rating scales with ordered categories were analyzed. For this kind of response scale non-linear models of item response theory might be more appropriate. Therefore, the development of an item response MTMM model on the basis of the stochastic measurement theory considered in this paper is another important task for future psychometric research. In the present applications, however, it can be expected that the bias of analyzing rating scales with linear models is relatively small. All rating scales have more than three categories, the variables in the second application are only slightly skewed (between $-.87$ and $-.14$), and they are skewed in the same direction (the skewness of the managerial performance variables are not reported by Mount, 1984). Under these conditions, the normal theory maximum likelihood estimation method works well even for rating scales (West, Finch & Curran, 1995). In conclusion, the MTMM model developed in this paper supplements the class of CFA models for MTMM data, overcomes some limitations of previous MTMM models, demonstrates how trait and method factors can be defined as functions of the true-score variables, and gives important hints for the future development of further MTMM models.

Appendix A: Definition of the True-score and Error Variables

In this Appendix it is shown how the true-score and the error variables of the CFA-CTC(M-1)$_k$ model can be defined using the concepts of probability spaces and random variables. As the definitions do not differ from CTT, the basic principles of CTT will be shortly summarized following Steyer's (1989) and Zimmerman's (1975, 1976) definitions. The formal framework of CTT will shortly be described according to Steyer's formulation.

The kind of random experiment considered in CTT is defined by the following set Ω of possible outcomes: $\Omega = U \times A$. The set Ω of possible outcomes is the Cartesian product of two different types of sets: (1) U is the set of individuals from which a subject u is drawn, and A is a set of possible outcomes of the items or the possible values of a scale or another kind of measurement (e.g., hormone levels, etc.). The random variables $Y_{ik}:\Omega \rightarrow \overline{\mathbb{R}}$ map the possible outcomes into the set of real numbers $\overline{\mathbb{R}} = \mathbb{R} \cup \{\infty\} \cup \{-\infty\}$. The values of one random variable Y_{ik} are the scores on an item or scale measuring a trait i by a method k. The variances of the variables Y_{ik} are assumed to be positive and finite. For defining a latent variable model on the basis of CTT, the starting points are the conditional expectations $E(Y_{ik}|p_U)$. The values of the mapping $p_U:\Omega \rightarrow U$ are the individuals. A value $E(Y_{ik}|p_U = u)$ of the random variable $E(Y_{ik}|p_U)$ is the expected value of the intraindividual distribution of Y_{ik}. In CTT, this value is called true score and $E(Y_{ik}|p_U)$ is called *true-score variable*, with $T_{ik} := E(Y_{ik}|p_U)$, $i \in I$, $k \in K$. The error variables E_{ik} are defined as residuals: $E_{ik} := Y_{ik} - E(Y_{ik}|p_U)$.

Appendix B: Method Factors

In this Appendix it is shown that Assumption (2) of the Definition of the CFA-CTC(M-1)$_k$ model is equivalent to the following assumption:
(2′) For each tripel (i, j, l), $i, j \in I$, $l \in K$, $l \neq k$, there is a $\lambda_{\mathrm{M}ijl} \in \mathbb{R}$, $\lambda_{\mathrm{M}ijl} > 0$, such that

$$M_{il} = \lambda_{\mathrm{M}ijl} M_{jl}. \tag{9}$$

Proof. Because (6) holds for two variables M_{il} and M_{jl}, it follows that $M_{il} = \lambda_{\mathrm{M}il}(M_{jl}/\lambda_{\mathrm{M}jl})$. Hence, (9) results from (6) by defining $\lambda_{\mathrm{M}ijl} := \lambda_{\mathrm{M}il}/\lambda_{\mathrm{M}jl}$. Equation (6) results from (9) by defining, for example, $\lambda_{\mathrm{M}il} = \lambda_{\mathrm{M}i1l}$ and $M_l = M_{1l}$ for each $l \in K$ and $j = 1$. Inserting these new defined parameters in (9) results in (6). Therefore, (6) and (9) are equivalent. □

Appendix C: Uncorrelatedness of Trait Factors, Method Factors and Error Variables

The CFA-CTC(M-1)$_k$ model implies that

$$\mathrm{cov}(T_{ik}, M_l) = 0; \tag{10}$$

$$\mathrm{cov}(T_{ik}, E_{jl}) = 0; \tag{11}$$

$$\mathrm{cov}(M_l, E_{il'}) = 0. \tag{12}$$

Proof. (i) *Derivation of (10).* Because $M_l = (1/\lambda_{\mathrm{M}il})M_{il}$ (i.e., (6)), it follows that $\mathrm{cov}(T_{ik}, M_l) = (1/\lambda_{\mathrm{M}il})\mathrm{cov}(T_{ik}, M_{il})$. Because M_{il} is a residual with respect to T_{ik}, both variables are uncorrelated (Steyer, 1988, Equation (9)). Therefore, $\mathrm{cov}(T_{ik}, M_l) = 0$. (ii) *Derivation of (11).* See Steyer (1989, Equation (10)). (iii) *Derivation of (12).* From (5) and (6) it follows that $M_l = (1/\lambda_{\mathrm{M}il})[T_{il} - \mu_{il} - \lambda_{T_{il}} T_{ik}]$. Because the true-score variables T_{ik} and T_{il} are uncorrelated with the error variables $E_{il'}$, it follows that $\mathrm{cov}(M_l, E_{il'}) = 0$.

Appendix D: Proof of the Identification Theorem

For simplicity reasons, the identification of the CFA-CTC(M-1)$_k$ model is demonstrated for three traits and three methods. The proof of identification for more than three traits and methods is straightforward. Without loss of generality, the first method is chosen as comparison standard, and for each method l the loading parameter $\lambda_{\mathrm{M}1l}$ is set equal to one for standardization reasons. Then, the covariance matrix of the observed variables is partioned into $(m \times m)$ submatrices $\Sigma_{ll'}(l, l' \in K)$ of size $(t \times t)$ in the following way $(t = m = 3)$:

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} \\ \Sigma_{21} & \Sigma_{22} & \Sigma_{23} \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}$$

$$= \begin{bmatrix} \Phi_\mathrm{T} + \Theta_1 & \ldots & \ldots \\ \Lambda_{\mathrm{T}2}\Phi_\mathrm{T} & \Lambda_{\mathrm{T}2}\Phi_\mathrm{T}\Lambda_{\mathrm{T}2}' + \Lambda_{\mathrm{M}2}\Phi_\mathrm{M}\Lambda_{\mathrm{M}2}' + \Theta_2 & \ldots \\ \Lambda_{\mathrm{T}3}\Phi_\mathrm{T} & \Lambda_{\mathrm{T}3}\Phi_\mathrm{T}\Lambda_{\mathrm{T}2}' + \Lambda_{\mathrm{M}3}\Phi_\mathrm{M}\Lambda_{\mathrm{M}2}' & \Lambda_{\mathrm{T}3}\Phi_\mathrm{T}\Lambda_{\mathrm{T}3}' + \Lambda_{\mathrm{M}3}\Phi_\mathrm{M}\Lambda_{\mathrm{M}3}' + \Theta_3 \end{bmatrix},$$

where

$$\Lambda_{Tk} = \begin{bmatrix} \lambda_{T1l} & 0 & 0 \\ 0 & \lambda_{T2l} & 0 \\ 0 & 0 & \lambda_{T3l} \end{bmatrix}, \Theta_k = \begin{bmatrix} \mathrm{var}(E_{1k}) & 0 & 0 \\ 0 & \mathrm{var}(E_{2k}) & 0 \\ 0 & 0 & \mathrm{var}(E_{3k}) \end{bmatrix},$$

$$\Lambda_{M2} = [\mathbf{0}|\boldsymbol{\lambda}_{M2}|\mathbf{0}], \Lambda_{M3} = [\mathbf{0}|\mathbf{0}|\boldsymbol{\lambda}_{M3}], \quad \text{and} \quad \boldsymbol{\lambda}_{Mk} = \begin{bmatrix} 1 \\ \lambda_{M2k} \\ \lambda_{M3k} \end{bmatrix}.$$

In the definition of the CFA-CTC(M-1)$_k$ model, it is assumed that all trait and method loadings are larger than 1. However, there are no restrictions on the covariances of the trait factors and the covariances of the method factors. In this Appendix the identification of the CFA-CTC(M-1)$_k$ model is proved for four conditions: (1) Φ_T and Φ_M are fully free with nonzero elements, (2) Φ_T is fully free with nonzero elements and Φ_M is diagonal, (3) Φ_T is diagonal and Φ_M is fully free with nonzero elements, and (4) Φ_T and Φ_M are diagonal.

Condition 1: Φ_T and Φ_M are fully free with nonzero elements. The off-diagonal elements of Φ_T are identified from those of Σ_{11}. The elements of Λ_{Tl} are then identified from the off-diagonal elements of $\Sigma_{l1}(l > 1)$. The diagonal elements of Φ_T are then identified from $\Phi_T = [\Lambda_{Tl}]^{-1}\Sigma_{l1}(l > 1)$. The elements of Θ_1 are now identified from $\Theta_1 = \Sigma_{11} - \Phi_T$. The matrix $\Lambda_{M3}\Phi_M\Lambda'_{M2}$ is now identified from $\Sigma_{32} - \Lambda_{T3}\Phi_T\Lambda'_{T2}$. The matrix $\Lambda_{M3}\Phi_M\Lambda'_{M2}$ has structure $\mathrm{cov}(M_3, M_2)\boldsymbol{\lambda}_{M3}\boldsymbol{\lambda}'_{M2}$, and $\mathrm{cov}(M_3, M_2)$ is identified from the element in the first row and first column of $\Lambda_{M3}\Phi_M\Lambda'_{M2}$ (where $\lambda_{M13} = \lambda_{M12} = 1$). Then, $\boldsymbol{\lambda}_{M3}$ and $\boldsymbol{\lambda}_{M2}$ are identified from the first column and row, respectively. Given the $\boldsymbol{\lambda}_{Ml}$ are known, the diagonal elements of Φ_M (method variances) are identified from the off-diagonal elements of $\Sigma_{ll} - \Lambda_{Tl}\Phi_T\Lambda'_{Tl}(l > 1)$. Finally, the error submatrices Θ_l, $l > 1$, are identified from $\Theta_l = \Sigma_{ll} - \Lambda_{Tl}\Phi_T\Lambda'_{Tl} - \Lambda_{Ml}\Phi_M\Phi'_{Ml}$. These identification rules show that at least two different traits and three different methods are required.

Condition 2: Φ_T is fully free with nonzero elements and Φ_M is diagonal. The trait part of the model (Φ_T, Λ_T) and the matrix Θ_1 are identified according to the identification rules of Condition 1. The method part is identified from the matrices $\Sigma_{ll'}(l > 1)$:

$$\Sigma_{ll} = \begin{bmatrix} \mathrm{var}(Y_{1l}) & \cdots & \cdots \\ \mathrm{cov}(Y_{2l}, Y_{1l}) & \mathrm{var}(Y_{2l}) & \cdots \\ \mathrm{cov}(Y_{3l}, Y_{1l}) & \mathrm{cov}(Y_{3l}, Y_{2l}) & \mathrm{var}(Y_{3l}) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{var}(M_l) + \lambda^2_{T1l}\mathrm{var}(T_{11}) + \mathrm{var}(E_{1l}) & \cdots & \cdots \\ \lambda_{M2l}\mathrm{var}(M_l) + \lambda_{T2l}\lambda_{T1l}\mathrm{cov}(T_{21}, T_{11}) & \lambda^2_{M2l}\mathrm{var}(M_l) + \lambda^2_{T2l}\mathrm{var}(T_{21}) + \mathrm{var}(E_{2l}) & \cdots \\ \lambda_{M3l}\mathrm{var}(M_l) + \lambda_{T3l}\lambda_{T1l}\mathrm{cov}(T_{31}, T_{11}) & \lambda_{M31}\lambda_{M21}\mathrm{var}(M_l) + \lambda_{T3l}\lambda_{T2l}\mathrm{cov}(T_{31}, T_{21}) & \lambda^2_{M3l}\mathrm{var}(M_l) + \lambda^2_{T3l}\mathrm{var}(T_{31}) + \mathrm{var}(E_{3l}) \end{bmatrix}.$$

(See Figure 2 for a larger and more readable copy of this matrix). The loading parameter λ_{M2l} is then identified from $[\mathrm{cov}(Y_{3l}, Y_{2l}) - \lambda_{T3l}\lambda_{T2l}\mathrm{cov}(T_{31}, T_{21})]/[\mathrm{cov}(Y_{3l}, Y_{1l}) - \lambda_{T3l}\lambda_{T1l}\mathrm{cov}(T_{31}, T_{11})]$. The other loading parameter λ_{M3l} is identifed from $\mathrm{cov}(Y_{3l}, Y_{2l})$ and $\mathrm{cov}(Y_{2l}, Y_{1l})$ in an analogous way. Then, $\mathrm{var}(M_l)$ is identified from $[\mathrm{cov}(Y_{2l}, Y_{1l}) - \lambda_{T2l}\lambda_{T1l}\mathrm{cov}(T_{21}, T_{11})]/\lambda_{M2l}$. Finally, the error variances are identified from the diagonal. These identification rules show that at least three different traits and two different methods are required.

$$\Sigma_{ll} = \begin{bmatrix} \mathrm{var}(Y_{1l}) & \cdots & \cdots \\ \mathrm{cov}(Y_{2l}, Y_{1l}) & \mathrm{var}(Y_{2l}) & \cdots \\ \mathrm{cov}(Y_{3l}, Y_{1l}) & \mathrm{cov}(Y_{3l}, Y_{2l}) & \mathrm{var}(Y_{3l}) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{var}(M_l) + \lambda_{\mathrm{T1}l}^2 \mathrm{var}(T_{11}) + \mathrm{var}(E_{1l}) & \cdots & \cdots \\ \lambda_{\mathrm{M2}l}\mathrm{var}(M_l) + \lambda_{\mathrm{T2}l}\lambda_{\mathrm{T1}l}\mathrm{cov}(T_{21}, T_{11}) & \lambda_{\mathrm{M2}l}^2 \mathrm{var}(M_l) + \lambda_{\mathrm{T2}l}^2 \mathrm{var}(T_{21}) + \mathrm{var}(E_{2l}) & \cdots \\ \lambda_{\mathrm{M3}l}\mathrm{var}(M_l) + \lambda_{\mathrm{T3}l}\lambda_{\mathrm{T1}l}\mathrm{cov}(T_{31}, T_{11}) & \lambda_{\mathrm{M31}}\lambda_{\mathrm{M21}}\mathrm{var}(M_l) + \lambda_{\mathrm{T3}l}\lambda_{\mathrm{T2}l}\mathrm{cov}(T_{31}, T_{21}) & \lambda_{\mathrm{M3}l}^2 \mathrm{var}(M_l) + \lambda_{\mathrm{T3}l}^2 \mathrm{var}(T_{31}) + \mathrm{var}(E_{3,} \end{bmatrix}$$

FIGURE 2.
Matrix Σ_{ll} of Condition 2

$$\Sigma_{ll} = \begin{bmatrix} \mathrm{var}(Y_{1l}) & \cdots & \cdots \\ \mathrm{cov}(Y_{2l}, Y_{1l}) & \mathrm{var}(Y_{2l}) & \cdots \\ \mathrm{cov}(Y_{3l}, Y_{1l}) & \mathrm{cov}(Y_{3l}, Y_{2l}) & \mathrm{var}(Y_{3l}) \end{bmatrix}$$

$$= \begin{bmatrix} \mathrm{var}(M_l) + \lambda_{\mathrm{T}1l}^2 \mathrm{var}(T_{11}) + \mathrm{var}(E_{1l}) & \cdots & \cdots \\ \lambda_{\mathrm{M}2l} \mathrm{var}(M_l) & \lambda_{\mathrm{M}2l}^2 \mathrm{var}(M_l) + \lambda_{\mathrm{T}2l}^2 \mathrm{var}(T_{21}) + \mathrm{var}(E_{2l}) & \cdots \\ \lambda_{\mathrm{M}3l} \mathrm{var}(M_l) & \lambda_{\mathrm{M}3l} \lambda_{\mathrm{M}2l} \mathrm{var}(M_l) & \lambda_{\mathrm{M}3l}^2 \mathrm{var}(M_l) + \lambda_{\mathrm{T}3l}^2 \mathrm{var}(T_{31}) + \mathrm{var}(E_{3l}) \end{bmatrix}$$

FIGURE 3.
Matrix Σ_{ll} of Condition 3

$$
\Sigma_i = \begin{bmatrix} \text{var}(Y_{il}) & \cdots & \cdots \\ \text{cov}(Y_{i2}, Y_{i1}) & \text{var}(Y_{i2}) & \cdots \\ \text{cov}(Y_{i3}, Y_{i1}) & \text{cov}(Y_{i3}, Y_{i2}) & \text{var}(Y_{i3}) \end{bmatrix}
$$

$$
= \begin{bmatrix} \text{var}(T_{i1}) + \text{var}(E_{1l}) & \cdots & \cdots \\ \lambda_{\text{T}i2}\text{var}(T_{i1}) & \lambda_{\text{T}i2}^2\text{var}(T_{i1}) + \lambda_{\text{M}i2}^2\text{var}(M_2) + \text{var}(E_{i2}) & \cdots \\ \lambda_{\text{T}i3}\text{var}(T_{i1}) & \lambda_{\text{T}i3}\lambda_{\text{T}i2}\text{var}(T_{i1}) + \lambda_{\text{M}i3}\lambda_{\text{M}i2}\text{cov}(M_3, M_2) & \lambda_{\text{T}i3}^2\text{var}(T_{i3}) + \lambda_{\text{M}i3}^2\text{var}(M_3) + \text{var}(E_{i3}) \end{bmatrix}
$$

FIGURE 4.
Matrix Σ_i of Condition 3

Condition 3: Φ_T is diagonal and Φ_M fully free with nonzero elements. The variances of the method factors and the method loadings are identified from the matrices $\Sigma_{ll}(l > 1)$:

$$\Sigma_{ll} = \begin{bmatrix} \text{var}(Y_{1l}) & \ldots & \ldots \\ \text{cov}(Y_{2l}, Y_{1l}) & \text{var}(Y_{2l}) & \ldots \\ \text{cov}(Y_{3l}, Y_{1l}) & \text{cov}(Y_{3l}, Y_{2l}) & \text{var}(Y_{3l}) \end{bmatrix}$$

$$= \begin{bmatrix} \text{var}(M_l) + \lambda_{T1l}^2\text{var}(T_{11}) + \text{var}(E_{1l}) & \ldots & \ldots \\ \lambda_{M2l}\text{var}(M_l) & \lambda_{M2l}^2\text{var}(M_l) + \lambda_{T2l}^2\text{var}(T_{21}) + \text{var}(E_{2l}) & \ldots \\ \lambda_{M3l}\text{var}(M_l) & \lambda_{M3l}\lambda_{M2l}\text{var}(M_l) & \lambda_{M3l}^2\text{var}(M_l) + \lambda_{T3l}^2\text{var}(T_{31}) + \text{var}(E_{3l}) \end{bmatrix}.$$

(See Figure 3 for a larger and more readable copy of this matrix). The loading parameter λ_{M2l} is identified from $\text{cov}(Y_{3l}, Y_{2l})/\text{cov}(Y_{3l}, Y_{1l})$. The other loading parameter λ_{M3l} is identifed from $\text{cov}(Y_{3l}, Y_{2l})$ and $\text{cov}(Y_{2l}, Y_{1l})$ in an analogous way. Then, $\text{var}(M_l)$ is identified from $\text{cov}(Y_2, Y_{1l})/\lambda_{M2l}$. The method covariances are identified from the matrix Σ_{32}:$\text{cov}(M_3, M_2) = \text{cov}(Y_{13}, Y_{22})/\lambda_{M22}$.

The trait part of the model is then identified from

$$\Sigma_i = \begin{bmatrix} \text{var}(Y_{i1}) & \ldots & \ldots \\ \text{cov}(Y_{i2}, Y_{i1}) & \text{var}(Y_{i2}) & \ldots \\ \text{cov}(Y_{i3}, Y_{i1}) & \text{cov}(Y_{i3}, Y_{i2}) & \text{var}(Y_{i3}) \end{bmatrix}$$

$$= \begin{bmatrix} \text{var}(T_{i1}) + \text{var}(E_{i1}) & \ldots & \ldots \\ \lambda_{Ti2}\text{var}(T_{i1}) & \lambda_{Ti2}^2\text{var}(T_{i1}) + \lambda_{Mi2}^2\text{var}(M_2) + \text{var}(E_{i2}) & \ldots \\ \lambda_{Ti3}\text{var}(T_{i1}) & \lambda_{Ti3}\lambda_{Ti2}\text{var}(T_{i1}) + \lambda_{Mi3}\lambda_{Mi2}\text{cov}(M_3, M_2) & \lambda_{Ti3}^2\text{var}(T_{i3}) + \lambda_{Mi3}^2\text{var}(M_3) + \text{var}(E_{i3}) \end{bmatrix}.$$

(See Figure 4 for a larger and more readable copy of this matrix.) The parameter λ_{Ti3} is identified from $[\text{cov}(Y_{i3}, Y_{i2}) - \lambda_{Mi3}\lambda_{Mi2}\text{cov}(M_3, M_2)]/\text{cov}(Y_{i2}, Y_{i1})$. The parameter λ_{Ti2} is identified in an analogous way from $\text{cov}(Y_{i3}, Y_{i2})$ and $\text{cov}(Y_{i3}, Y_{i1})$. The trait variance $\text{var}(T_{i1})$ is then identified from $\text{cov}(Y_{i2}, Y_{i1})/\lambda_{Ti2}$. Finally, the error variances are identified from the diagonal of Σ_i. These identification rules show that at least three different traits and three different methods are required.

Condition 4: Φ_T and Φ_M are diagonal. The identification rules are the same as the identification rules of Condition 3 with the additional restriction that $\text{cov}(M_3, M_2) = 0$.

Identification of the constants μ_{il}. Under all four conditions, the constants μ_{il}, $l > 1$, are identified from the expectations of the observed variables and the loading parameters whose identification has been shown above: $\mu_{il} = \text{E}(Y_{il}) - \lambda_{Til}\text{E}(Y_{i1})$, because $\text{E}(Y_{il}) = \text{E}(\mu_{il} + \lambda_{Til}T_{i1} + \lambda_{Mil}M_l + E_{il}) = \mu_{il} + \lambda_{Til}\text{E}(T_{i1})$ and $\text{E}(Y_{i1}) = \text{E}(T_{i1} + E_{i1}) = \text{E}(T_{i1})$.

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