# A NEW COEFFICIENT: APPLICATION TO BISERIAL CORRELATION AND TO ESTIMATION OF SELECTIVE EFFICIENCY\*

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A coefficient of selective efficiency is proposed which can be usefully applied to selection problems involving the evaluation of the validity of (1) dichotomous predictors and (2) continuous predictors at a particular or at successive points of cut. Previously the author has shown that the product-moment correlation can be interpreted as a direct index of selective efficiency if the distribution forms of the criterion and the predictor are similar and the regression of the criterion on the predictor is linear. The coefficient proposed in the present article may be employed to evaluate selective efficiency of a continuous predictor at particular points of cut even when these assumptions are not tenable or are not applicable. It also is demonstrated that the proposed coefficient of selective efficiency may--with somewhat simpler and more generally applicable assumptions than those required in deriving the conventional formula--be employed as a substitute for the biserial correlation coefficient.

In a previous paper (1), the author demonstrated that the product-moment correlation coefficient could be interpreted as a direct index of selective efficiency. Selective efficiency was defined in that paper as the gain over random selection in mean criterion score achieved by selecting with the given predictor, divided by the gain that would have been achieved with perfect selection or selection on the criterion itself. This definition resulted from a logical examination of the objectives of selection and was regarded merely as one step in the solution to the problem of determining the proper interpretation of a product-moment validity coefficient. It was, however, briefly indicated in a footnote to this paper that this definition provided the basis for deriving a new biserial correlation coefficient.

Subsequently, it has seemed evident to the author that this definition also provided a direct basis for a coefficient with quite general application in estimating selective efficiency. It may, in other words, be applied in situations in which the assumptions required to demon-

<sup>\*</sup>The opinions expressed in this paper are those of the author and are not to be interpreted as representing official Department of the Army policy.

strate that the product-moment correlation coefficient is an index of selective efficiency do not apply.

In the present paper, this coefficient of selective efficiency-which will be designated  $S$ -will be derived as a substitute for the biserial correlation coefficient. The argument for direct application of S as an index of selective efficiency will also be presented, together with some indication of situations in which it will prove useful.

A distinction between what is desired in a biserial correlation coefficient and what is desired in a coefficient of selective efficiency should be made explicit before these two separate problems are approached. A biserial correlation coefficient is an estimate of a productmoment coefficient, an indication of the product-moment coefficient that would have been obtained if the dichotomous variable were continuous. The significance and interpretation of the coefficient vary according to the goodness with which it estimates the product-moment and as the significance and interpretation of the product-moment coefficient vary. The problems involved in deciding whether to use and how to interpret such a coefficient are largely mathematical in nature and have to do with assumptions involved in the derivation of the biserial and in the derivation of the product-moment coefficient.

On the other hand,  $S$  as a coefficient of selective efficiency has been developed with the specific objective of measuring the efficiency with which a predictor will accomplish the objectives of selection. The absence of restrictive assumptions in its derivation allows general application within the general area of validation studies where a coefficient of selective efficiency is usually required.

In general, it is probably true that statistical formulas are not developed with the primary objective of providing interpretations most meaaingful for a research worker having problems peculiar to a given area of research. The formula is more apt to be developed as an expression of certain mathematical relationships. In the derivation, assumptions-often highly limiting in nature--are introduced as necessary to the development of the given formula. Applications are sought at a later date. Very often it is found that the assumptions are so restrictive that the coefficient can legitemately be used in only a small proportion of certain types of applications. In other instances the coefficient may have legitimate application but may not provide the interpretation needed.

## *Derivation of the Conventional Biserial*

It should help in further discussion if, first of all, we consider a derivation of the conventional biserial formula.

The following notation will be employed in this derivation and throughout this paper.

- $X$  the dichotomous variable.
- $X'$  -- a hypothetical continuous variable corresponding to X.
- $Y$  the continuous variable in raw score form.
- $Z_{X'}$ ,  $Z_{Y}$  the standard score form (mean of zero, and *S.D.* of one) of  $X'$  and  $Y$ , respectively.
	- $P_x$  proportion of cases in the upper category of X.
		- $u-a$  subscript indicating that the symbol it modifies refers to those cases in the upper category of  $X$ .
		- $v-a$  subscript indicating that the symbol it modifies refers to those cases in an upper category of Y equal in number to those in the upper category of X.

The three assumptions involved in this derivation of the conventional biserial formula are:

- a. A continuous variable underlies the obtained dichotomous variable X.
- b. The regression of the continuous variable,  $Y$ , on  $X'$  is linear. (We might note here that as a consequence of this assumption of linearity predicted  $Y$  values for any given  $X'$  value will fall on the regression line and, in addition, the predicted Y value for any linear combination of  $X'$  values will likewise fall on the regression line.)
- c. The distribution of  $X'$  is normal.

Note: Nothing is directly assumed regarding the distribution of the continuous variable,  $Y$ , or regarding the regression of  $X'$  on  $Y$ . The regression of Y on  $X'$  may be expressed as

$$
Z_{\mathbf{Y}} = r_{\mathbf{X}'\mathbf{Y}} Z_{\mathbf{X}'}\,. \tag{1}
$$

The  $\overline{Z}_Y$  of (1) is both the predicted value of  $Z_Y$  for the  $Z_{X'}$  value of a given individual and the mean of the  $Z<sub>r</sub>$  values in an X' array. Equation (1) will hold in predicting an individual's  $Z<sub>y</sub>$  value from his  $Z_{x'}$  value and any sum of different values since the regression of Y on  $X'$  is linear. Hence, with this assumption of linearity, Equation (1) may be employed to predict Y values for any individual above some point of cut on  $X'$ , and the sum total for all individuals above the point of cut may be expressed as follows:

$$
_{u}(\sum Z_{Y})=r_{X^{\prime}Y\,u}(\sum Z_{X^{\prime}}). \tag{2}
$$

The bar over Z in  $_4(\Sigma \bar{Z}_Y)$  may be dropped, since in summing over a sub-population the errors of estimating individual  $Z<sub>x</sub>$  scores will "average out." Dividing both sides by  $N_{\mu}$ , the number above the point of cut, and reducing, we obtain

$$
_{u}M_{z_{v}}=r_{x^{\prime}Y}uM_{z_{x^{\prime}}}
$$
 (3)

or

$$
r_{X'Y} = \frac{1}{u} M_{Z_{Y}} / \frac{1}{u} M_{Z_{X'}} \tag{4}
$$

Thus, with the assumption indicated,  $r_{X'Y}$  may be estimated as the ratio of two means. If X is dichotomous,  $_{u}M_{z_{y,i}}$  is unknown, but may be estimated if it is assumed that the continuous variable  $(X')$  is normally distributed. The mean of the tail of the normal curve is given by the formula  $Y/P_x$ , where Y is the height of the ordinate at the point of cut and  $P_x$  is the proportion of cases in the tail of the curve. After making the indicated substitution in (4) and converting to raw scores

$$
r_{\text{bis}} = \frac{uM_Y - M_Y}{\sigma_Y} P_X/Y.
$$
 (5)

This is the conventional form of the biserial.

# *Limitations of the Conventional Coefficient; Derivation of Alternative Formulas*

An elaboration of the implications of assuming normality of  $X'$ when  $Y$  is not normally distributed will aid in understanding why coefficients over unity are obtained with (5), even though the explicitly stated assumptions involved in its derivation are satisfied.

It is evident that the assumption of normality in the distribution of  $Z_{X'}$  coupled with that of linear regression of  $Z_{Y}$  on  $Z_{X'}$  requires that the  $\overline{Z}_Y$  values predicted from this linear regression line be normally distributed. As a consequence any lack of normality in the distribution of  $Z<sub>y</sub>$  must be accounted for in the distribution of the errors of estimation  $(Z_Y - \overline{Z}_Y)$ . Since there are limits to the extent of the influence of the distribution of  $(Z_Y - \overline{Z}_Y)$  on the distribution of  $Z_Y$ , particularly when  $r_{X'Y}$  is high and the variance of  $(Z_Y - \overline{Z}_Y)$  is small, there are apparently situations in which the two assumptions of linear regression of  $Z_Y$  on  $Z_{X'}$  and normality of X' are mutually inconsistent. The appearance of biserial correlations above unity  $-$  which seem to occur with an anormal distribution of  $Z_{Y}$ -is undoubtedly reIated to inconsistency between these two assumptions. In the limiting case when the correlation is unity and when the regression of  $Z<sub>r</sub>$  on  $Z_{x'}$ , is linear, it is quite apparent that the distribution of  $Z_{x'}$  and  $Z_{y}$ must be of the same form. If the distribution of  $Z<sub>y</sub>$  is not normal it is equally apparent that at least in this limiting case the substitution of an estimate of  $M_{z_{\nu}}$ -such as *Y/P*-must bias  $r_{b_{i}}$  as an estimate of  $r$ . Distortion or bias will very probably occur in other than the limiting case. While the presence of such bias is stressed, no attempt will be made here to trace the exact mechanism or to show the exact nature of its effect.

If it is agreed that the assumptions involved in the derivation of the conventional coefficient are unreasonable when the distribution of Y is not normal, examination of possible alternative assumptions should be of interest.

Two possibilities are suggested, both of which amount, in effect, to equating the distribution form of  $X'$  and  $Y$ . First of all it could be argued that if normality of the  $X'$  distribution is assumed, together with linear regression of Y on  $X'$ , Y should also be assumed to be normally distributed. In effect, this is assuming bias in the units of the obtained Y distributions. The implications of such an assumption on computation are straightforward. The Y distribution may be normalized by use of appropriate tables. Additional computational labor required to normalize Y would not be excessive, especially if a considerable number of biserials were to be computed against each continuous variable. The actual computational process would involve determining the normalized values for the midpoints of the frequency intervals by the formula  $(Y_1 - Y_2)/P_x$ , where  $Y_1$  and  $Y_2$  are heights of the ordinates at 'the limits of the class interval as determined from the proportions exceeding these points. In the case of the intervals at the two extremes of the test, the above formula would reduce to *Y/Px.* In both instances the P value in the denominator is the proportion of cases in that class interval. Since, for a distribution of such normalized values,  $M_Y$  of (5) would become  $M_{Z_V}$  or zero and  $\sigma_Y$ would become  $\sigma_{z_v}$  and equal unity, (5) with Y in standard score form would become

$$
r_{bis} = {}_u M_{Z_v} \cdot P/Y. \tag{6}
$$

While  $M_{z_{\rm v}}$  is assumed to be zero and  $\sigma_{z_{\rm v}}$  to be unity, it would probably be advisable, as a check, to calculate the mean and  $\sigma$  of the normalized values. With a small number of categories  $\sigma_{Z_v}$  may fall below unity. In this event the value obtained from (6) should be di-

vided by the obtained  $\sigma_{z_v}$  value to avoid the overestimation of  $r_{bis}$ that would result from underestimating  $\sigma_{z_{\alpha}}$ .

Exact correspondence of the distribution form of  $X'$  and  $Y$  is a second and possibly the most plausible of the several possible assumptions regarding the nature of the distribution of  $X'$  when the distribution of Y is not normal. With such an assumption, symmetry of the regression lines and of the frequency surface is plausible when the continuous variable is anormally distributed. This assumption leads to the derivation of S as an  $r_{his}$  formula.

The specific assumptions involved in deriving the coefficient S are as follows:

- (a) The dichotomous variable may be regarded as continuous.
- (b) The regression of Y on X' is linear.
- (c) The distribution form of  $X'$  is the same as the distribution form of Y.

The derivation follows that for the conventional coefficient through  $(4)$ . Equation  $(4)$  will be repeated here for the convenience of the reader. In (4)

$$
r_{X'Y} = {}_u M_{Z_Y}/{}_u M_{Z_{X'}}.\t\t(4)
$$

 $_{u}M_{z_{v}}$  is the mean of the tail of the frequency distribution of Y. If, at this point, we assume that the  $X'$  has the same distribution form as Y, it follows that an equal tail of the  $X'$  distribution would have the same mean. Thus  $_{u}M_{x'} = _{v}M_{y}$ , and substituting in (4) we have

$$
r_{X'Y} = {}_u M_{Z_V}/{}_v M_{Z_V} \,.
$$

Substituting raw scores, reducing, and designating the resulting coefficient *S,* we obtain

$$
S = \frac{uM_Y - M_Y}{vM_Y - M_Y}.\tag{8}
$$

Computations for (8) may be made rather rapidly.  $_{u}M_{Y}$  may be computed from a frequency distribution of Y. If large numbers of correlations against a single criterion are being calculated, a table of such values may be prepared for each possible percentage above the cutting point on Y. It may, of course, be necessary to interpolate to obtain  $_{v}M_{Y}$  for the appropriate percentile value. Linear interpolation would seem sufficiently accurate for most purposes.

A computational exampIe is given below.

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#### COMPUTATIONS

1.  $M_v = 4$ 

2. 
$$
u^M = \left[\frac{2}{7} + \frac{2}{6} + \frac{3}{5} + \frac{3}{4} + \frac{2}{3}\right] \times 12
$$
\n $= 4.91$  or the mean *Y* value of those cases in the upper category of the dichotomous variable.

3.  $vM_y = [(3) \ 7 + (5) \ 6 + (4) \ 5.3^*]/12$ 

 $=6.02$  or the mean Y value of the 12 cases selected as highest on the continuous variable  $Y$ , 12 being the number in the upper category of the dichotomous variable.

4. 
$$
S = \frac{4.91 - 4.00}{6.02 - 4.00} = .450
$$

\*To obtain  $vMy$  we need the average of the 12 cases highest on Y. This obviously includes the 3 cases having Y values of 7, the 5 cases having Y values of 6, and 4 of the 10 cases having Y values of 5. If we assume Y to be continuous, and the 10 cases having Y values of 5 to vary uniformly between 4.500 snd 5.499, then the 4 cases highest within this interval can be presumed to range between 5.100, and 5.499, with a mid-point of 5.3.

## *S as a Direct Index of Predictive Efficiency; Applications to Curvilinear Relations and to Distributions of Predictors and Criteria Which Are Not Normal*

In the introduction, it was stressed that S has direct meaning as a coefficient of selective efficiency apart from any relation which it may bear to the product-moment coefficient of correlation. S is justified directly from the definition of selective efficiency. It will be desirable consequently to review the nature of the selection process to determine exactly its objectives and to insure that the definition chosen is the most logical and appropriate.

By selection we refer to the process of identifying by means of predictor variables that portion of a general population which will be found to have high criterion scores. A predictor variable is usually employed in place of the criterion itself as a selector for reasons of economy or time or simply because the criterion values are not obtainable at the time at which selection must be made.

The nature of an adequate criterion is determined by the objectives of the organization for which selection is to be made. Thus, in an employment situation the objective would usually be increased quantity and quality of production; in a school or university, increased academic achievement. Criteria for the employment situations should, then, measure the differential effect of individual employees on the over-all productivity of the organization. In the school situation, academic achievement should be measured. Assuming that perfect criteria could be devised, the objectives of selection would be maximized by selection on the criterion itself.

Whatever the means of selection, the average gain achieved by selection is the difference between the mean criterion score of those selected and the mean criterion score of the total group from which they were selected. The latter gives the average or expected on-thejob productivity if selection were made at random from the total population. The former is the average on-the-job productivity of members of the selected group. This means, in other words, that with a given predictor or predictor battery employed at a given selection ratio the above difference in mean values shows the estimated absolute gain in productivity, per selected individual, resulting from the selection process. This value has meaning in its own right. However, it is a function of the selection ratio as well as the validity of the selection instrument. To obtain an index of selective efficiency of a predictor, the increase in productivity obtained with the predictor should be divided by the increase over random selection that would have been obtained with perfect (criterion) selection of the same number of applicants. This would give the *percentage* of possible gain actually achieved. If we translate this verbal definition of selective efficiency into statistical symbols, designating Y as the criterion, we obtain the coefficient

$$
S=\frac{{_uM_x}-M_x}{_vM_x-M_x}.
$$

 $_{u}M_{y}$  would now be defined as the mean criterion score of those selected by the predictor and  $_{v}M_{y}$  as the mean criterion score of a group of the same number selected by the criterion.

It has already been shown how S may be directly computed, given test and criterion scores and the number or proportion of applicants to be hired. In addition it has been shown that the product-moment  $r$ is equal to  $S(1)$  if the regression of the criterion on the predictor is linear and the two distribution forms are the same. In stating that  $r$ equals S, we mean that this equality will hold no matter what point of cut on the predictor is chosen for computation of  $S$ . There should be no problem, then, as to the evaluation of selective efficiency when these two assumptions are satisfied; the product-moment  $r$  is directly applicable.

While the assumption of linear regression of the criterion on the predictor is readily understood, the assumption of equal distribution forms implies acceptance of certain principles which should be clarified. Such clarification is important because the applicability of either  $r$  or S as an index of selective efficiency hinges upon acceptance of these principles. First of all it is implied that the criterion distribution has meaning in its own right or that the criterion scale units represent equal increments of the variable measured. Where the criterion scale consists of such production units as number of objects produced or number of errors made, this assumption is apparently quite legitimate. Errors or objects produced are units having definite and standard significance relative to the objective of the selection program--improvement of the efficiency of operation of the organization for which selection is made. If ratings are employed as criteria, the experimenter will have to decide from knowledge of the particular scale whether or not sufficient bias in scale units exist as to make this assumption unjustifiable. Unfortunately, it will probably be impossible to arrive at a definite decision.

To digress for a moment, we might note that a coefficient dependent upon the assumption of equal scale units has definite advantages over coefficients such as the conventional biserial which tend toward biased estimates of validity without normality of the criterion distribution. (See discussion on page 172). From the viewpoint of the objective of selection, the need for normalizing anormal criterion distributions before an index of selective efficiency will properly apply is equivalent to the necessity of converting to non-meaningful units before selective efficiency can be determined. The experimenter is faced with the dilemna of being unable to determine selective efficiency or of applying a coefficient which will result in a distortion of the proper answer to his problem.

A second implication of the assumption of equal distribution forms is that the predictor scale units have no direct meaning for the purpose of evaluating selective efficiency. Thus, if the distribution form of the predictor is the same as that of the criterion--and the regression of the criterion on the predictor is linear—the productmoment coefficient may be appropriately employed as an index of selective efficiency. When  $r$  is not appropriate,  $S$  should be employed to determine selective efficiency at various cutting points throughout the range of predictor scores. From the formula for  $S$  it can be seen

that the distribution form of the predictor is, in the latter instance, ignored.

This point needs little elaboration. We might, however, note that the predictor distribution form necessary to use of  $r$  as an index of selective efficiency has no necessary relation to the distribution form which will provide maximum predictive efficiency. The latter problem, in the case of test scores which are sums of dichotomous items, is a problem in the proper distribution of item difficulties.

When the assumptions of linear regression and equal distribution forms are known not to be true, or suspected not to be true, S will provide directly the desired index of selective efficiency for particular points of cut or particular selection ratios.  $S$  may also be employed in certain additional situations where the product-moment correlation is obviously not applicable. Thus, with dichotomous predictors, S provides, without the assumptions involved in its derivation as a biserial correlation coefficient, a direct index of selective efficiency. This is directly evident only if the proportion in the upper category of the predictor corresponds to the proportion selected. However, S may be readily adapted to estimating selective efficiency when these two proportions do not correspond. This may be done by redefining the number of cases selected on the criterion, in computing  $_{v}M_{y}$ , as the number of applicants it is desired to select rather than the number in the upper category of the dichotomous variable.

If multiple cutting scores have been set for several predictors, S may be employed as an index of the selective efficiency obtained when a battery of tests is utilized in this manner.\*

Application of S with dichotomous predictors or in the case of multiple cutting scores is not in need of further elaboration. Its application to continuous predictors when  $r$  is not applicable because of non-linear regression or inequality of distribution forms will bear further discussion.

When the product-moment correlation coefficient is not applicable, its inapplicability means in effect that  $r$  would not equal  $S$  if the latter were computed for all points of cut on the predictor. It follows consequently that these assumptions may be tested by computing S for points of cut covering the range of the predictor. Such computations would not only test the applicability of  $r$  as an index of selective efficiency but would indicate the extent of error introduced by employing  $r$  as an index of selective efficiency and allow estimation of improvement in selective efficiency resulting from choice of

\*For such application  $_{n}M_{y}$  is the mean criterion score of those "accepted" after application of the multiple cutting score procedure, while  $M_v$  is the mean criterion score of a group of comparable size selected on the criterion itself.

particular cutting points. If several alternative predictors, or predictor composites, are available, that one most suitable for selection at a predetermined selection ratio could be chosen. With a predetermined selection ratio, S may also prove of value in combining predictor variables into a composite. The exact manner of its application to this problem is not clear. It is apparent from perfunctory review of the derivation of partial regression coefficients that it cannot be readily proved that S may be employed for computing validity coefficients to be used in multiple regression analysis. However, if several weighted combinations of predictors were tried, S could be employed to determine that yielding the highest predictive efficiency for the given predetermined selection ratio.

A plot of particular S values against the percentage above the point of cut involved, which might be termed a curve of selective efficiency, is suggested as an aid in determining whether or not descrepancies between  $S$  and  $r$  show systematic trends or merely chance deviations. Such a curve should be useful in deciding upon the selection ratio and in choosing from several possible predictors that one most suited to selection at a predetermined selection ratio.

A curve of selective efficiency, as determined by computing S at various points of cut, does not provide information corresponding either to that provided by eta or to that provided by fitting a curvilinear regression line. Eta provides a single estimate of correlation for the entire range of the scatter-plot. A curvilinear regression line provides both such an estimate of correlation and the predicted criterion score for any given test score. In actual practice, however, it is usually not desired to select a group having a particular test score but a group above some given test score. Additional computation would be necessary to obtain the mean score of such a group from a curvilinear regression line. The coefficient S provides directly the information desired in evaluating selective efficiency and should, in the author's opinion, be preferable to the alternative mentioned in evaluating selective efficiency whenever curvilinear regression of the criterion on the predictor is known or suspected to exist.

A point previously made might be stressed again in this connection. Curvilinear regression may be and sometimes is linearized by alterations in the predictor scales. An equation may be employed for this purpose or each predictor value may be assigned its actual  $\overline{Y}$ Value. Such converted predictor scales will maximize the productmoment  $r$ . The S values computed at various points of cut will, however, be unaffected, since  $S$  is computed entirely from criterion scores. This further implies that solution to the problem of obtaining maximum selective efficiency of a composite or weighted sum for a predetermined selection ratio cannot be fully solved by altering the scale values of the component predictors.

Curvilinear regression lines have not been widely used in practice. This is undoubtedly due in part to the labor required in their application. It has often been found, however, that even with apparently marked curvilinear regression little improvement is found over the predictive efficiency obtained with the best straight-line approximation to the curvilinear regression line. The author would agree and even emphasize the poor expectancy of improved efficiency from application of curvilinear regression when it is desired to predict over the entire range of the variables in question. When it is desired to employ a definite selection ratio (or a definite point of cut) or when it is possible to modify the selection ratio to take advantage of any variation in selective efficiency that may be discovered, more fruitful results may be expected. A correlation coefficient computed from eta or from a curvilinear regression line, since it does provide an "average" selective efficiency coefficient, conceals and ignores these differences in selective efficiency which may be identified by use of  $S$ and used to advantage.

The point made above may be effectively illustrated by a numerical example. In Figure 1 we have a scatter plot with the regression of Y on X indicated, with  $r$  and eta computed for the entire range and S determined for various points of cut. It will be noted that while the regression appears to be definitely curvilinear,  $r$  and eta do not differ markedly. However, the differences in the value of S for the several points of cut are of considerable magnitude.

To utilize with confidence the differences between the values of S at different points of cut, the number of cases would have to be much greater than in this example. The technique of cross-validation should probably be applied in such circumstances in order to obtain an unbiased estimate of validity if selection of the particular point of cut were dependent upon the obtained values of S. If the point of cut were predetermined, cross-validation would be unnecessary.

A special problem in which S has particular application occurs in estimating selective efficiency of tests constructed for the particular purpose of selecting at a predetermined point of cut. In obtaining an efficient test for that purpose items should be selected which have P values approximately equal to the per cent of the population to be eliminated (2). Such a selection will obviously lead to a distribution of test scores whose mean, standard deviation, and distribution form have no relation to the distribution of "true" ability in the function measured. Since the product-moment  $r$  will be influenced by the standard



**⊁** 



deviation and the distribution form, it should probably never be employed for evaluating selective efficiency of such a test. S will in such circumstances give the evaluation desired.

If a curve of selective efficiency were obtained by computing  $S$  at successive points of cut, it would not only be possible to measure the validity of S at the point or in the area for which the test is designed, but it should be possible to detect "wasted" efficiency in the sense of discrimination in areas where such a specialized test was not intended to function. It is realized that there would be, at the present time, little basis for deciding the optimal form of the curve of selective efficiency for such a test. Probably a test with items of low reliability would show a shallow curve of selective efficiency, while a test with items of high reliability would be more markedly curvilinear and would show a more definite optimal point. Any decision as to deletion of items made on the basis of such a curve of selective efficiency should probably be checked empirically by recomputing the curve of selective efficiency after item selection.

From the two assumptions required to show equality between  $r$ and  $S$  at all points of cut, it is apparent that variation of  $S$  for different points of cut may be due to differences in the distribution forms of the predictor and criterion as well as to curvilinear regression of  $Y$  on  $X$ . Of course, either or both of these two observed phenomena may be due in turn to other characteristics of the correlation surface.

If the criterion distribution is highly skewed or otherwise lacking in normality, the possibility of obtaining differential selective efficiency at different points of cut is of considerable interest. As in the case of the suggested application of  $S$  in curvilinear regression, the extent of the differences may be calculated for each possible selection instrument or battery and used to advantage in selecting tests or determining selection ratios. In practice, as was mentioned before, bias in the criterion scale units may mislead the investigator in this respect. Where production units are available as criteria, the investigator can often accept the distribution as having meaning for his purpose-regardless of the relation of production units to any hypothetical underlying ability. When ratings or achievement test scores are the criteria to be predicted, the obtained curve of selective efficiency will have to be interpreted in the light of known biases in the scale units involved.

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