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THE FOUNDATIONS OF PSYCHOMETRY: FOUR FACTOR SYSTEMS.

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Four methods of factorizing the fundamental matrices used in factor analysis are described and illustrated. The first is represented by the techniques already developed. The second is the obverse factor technique. The third and fourth methods are variants of the first and second. The implications of each method for different schools of psychology are pointed out. The methods are complementary, not competitive.

Contents:

Introduction.

I am to draw attention to four different ways in which matrices of the kind used in factor analysis can be standardized and analyzed for factors. In the past only one of these four has been clearly distinguished, namely, that for the factor technique of Spearman, Thurstone, Kelley and others in which tests etc. are variables, and persons are the population. The second, the obverse or inversion of the first, was described recently;* it employs persons as variables instead of tests or other single attributes. Two others, of minor immediate interest and subsidiary to the first pair, will be described in the sequel. The four together, I believe, supply the foundations for a considerable part of the future science of psychometry.

In the present paper , after defining the systems and supplying an example of each, I proceed to examine the modes of thought that each can subserve. The four help us to see in perspective what, I believe, has not been perceived clearly hitherto. Two of the four are statistical statements of a relativistic standpoint, and may be of help to gestalt theorists, whilst the other two are parallel foundations for

*Stephenson, W. *The Inverted Factor Technique.* Brit. J. Psychol. XXVI, 1936, p. 344. (Other papers are referred to in the sequel).

'elemental', process,* or ability psychology. Psychologists of the former school of thought have claimed that theirs supplies the only scientific procedure.† Psychologists of the latter school draw attention to the "irrelevancies of experience" in gestalten, and hold that it is precisely characteristic of scientific methodology that it does not engulf the "total situation".§ I shall not need to take sides in this controversy. Of the four systems two subserve the former viewpoint, and two the latter; but no one of the four systems, nor either of the psychological standpoints, is sufficient or adequate alone. The future psychometry cannot hope to be complete unless it uses all four systems, each for ends which it is best fitted to serve.

The Four Systems

We are to be concerned with matrices of the following kind: Each is to contain persons in the row (persons $A, B, C \cdots N$) and items such as personality traits, tests, measurements or attributes in general, in the column (attributes $a, b, c, \cdots n$) .

Persons

		A	B	C	D	..	..	N
Attributes etc.	a	${}_aX_A$	${}_aX_B$	${}_aX_C$	..	..	..	${}_aX_N$
	b	${}_bX_A$	${}_bX_B$	${}_bX_C$	..	..	..	${}_bX_N$
	c	${}_cX_A$	${}_cX_B$	${}_cX_C$	..	..	..	${}_cX_N$
	..	..	..	..	..	..	..	
	n	${}_nX_A$	${}_nX_B$	${}_nX_C$	..	..	..	${}_nX_N$

The variate of person A for item a, or of item a for person A, is represented by ${}_aX_A$, and the same connotation is held throughout. The four systems or ways of treating such matrices are as follows:—

Variates can be entered in the matrix for single attributes or tests etc. (such as a), for standardization with respect to the population of persons. This is what has been done, of course, in all previous factor analysis. Data is standardized along the row, so that for each attribute or quality a to n respectively, the following holds:—

*Line, W. *Process-Psychology — Individual and General.* Psychol. Rev. XL. p. 256.

†Lewin, K. *Dynamic Theory of Personality,* Chap I. (McGraw-Hill) 1935.

§Thurstone, L. L. *The Vectors of Mind.* p. 46. (Chicago Univ. Press)1935.

$$\sum {}_a x^2 = \sum {}_b x^2 = \cdots = \sum {}_n x^2 = 1 \ . \quad (1)$$

(where the summation is from x_A to x_N in each case). Rows are correlated, and analyzed by way of factor theorems such as those of Spearman, Thurstone, Hotelling, Kelley, and others.

Secondly, variates can be entered into the matrix for standardization in columns, for a population of attributes, so that for each person the following holds:—

$$\sum x_A^2 = \sum x_B^2 = \cdots = \sum x_N^2 = 1 \ . \quad (2)$$

(where scores are summed in each column, from ${}_a x$ to ${}_n x$). Correlations are calculated between columns, for persons or whole aspects of persons as variables, and factorized by way of inverted factor theorems, that is, obverses of the theorems used in system (1).

Thirdly, variates which have been standardized as at (1) can be entered into the matrix, and thereupon *re-standardized* by way of system (2). That is, using y to stand for variates that have already been standardized in rows, the same variates can be given the following standardization per column:—

$$\sum y_A^2 = \sum y_B^2 = \cdots = \sum y_N^2 = 1 \quad (3)$$

(where the summation is per column, from ${}_a y$ to ${}_n y$). Correlations are for columns, and these, as in system (2), are factorized by way of inverted factor theorems.

Fourthly, scores already standardized as at (2) can be entered into the matrix and used for re-standardization by way of system (1). Re-standardization is effected with respect to the population of persons, for traits now considered separately as variables, the crude variates for which were standard scores with respect to a column. Using z to stand for variates already standardized in columns, the following represents this system:—

$$\sum {}_a z^2 = \sum {}_b z^2 = \cdots = \sum {}_n z^2 = 1 \quad (4)$$

(where the summation is now per row, from z_A to z_N). Correlations are between rows, and are factorized by way of the usual Spearman or other factor theorems in their initial form, as in the case of system (1).

Systems (1) and (4) deal with separate* attributes as variables; (2) and (3) with persons. Factor measurements for an individual are regression estimates by way of systems (1) and (4); they are

*The word does not imply that the attributes are distinct, unique, isolated or unitary, in the sense of not having correlation with any others.

dependent upon the statistics of a population of persons. Those for systems (2) and (3) are correlation coefficients (loadings, or saturation coefficients). The original variates might be identical for all four systems of standardization, in which case the factor systems issuing from each will not be independent, although each has its own point of interest for psychometry. But the variates can also be quite independent for systems (1) and (2). Systems (3) and (4), on the other hand are not independent of (1) and (2) respectively.*

Systems (1) and (4) are what I called *r*-technique elsewhere, since they deal with the correlations (r) between tests or separate traits etc., and analysis is by way of direct factor theorems. Systems (2) and (3) have been called *Q*-technique, their concern being with correlations (Q) between *persons* or whole aspects of persons, analyzed by inverted factor theorems.

An Example.

The following example will illustrate the above systems, and supply at the same time a standard of reference by which to judge past and future investigations involving factor analysis.

Suppose that we make 100 measurements (a_1 to a_{100}) for each of a number of persons A to N. Let the measurements all relate to physical features of the body, height (a_1), arm-length (a_2), leg-length (a_3), trunk diameter (a_4), neck-circumference (a_5), finger length (a_6), and so forth. For the purposes of a controlled study, let the persons be of one age, and all comparable in point of nourishment and freedom from obesity etc. We can say a great deal about the factor systems for such measurements, purely on a priori grounds.†

First let the physical measurements be standardized in *rows* of the matrix of data (system (1)). That is, the measurements for height (a_1) are reduced to standard scores, for the population of N persons, and similarly for each row separately. Such rows can be correlated, and it is obvious that the correlations must often be positive, and that a factor or factors will emerge for some of the variables, because the person with the greatest height will also tend to have the longest arm-length etc. The interpretation to be given to such factors will be discussed later.

But the crude measurements can also be standardized directly in *columns* (system (2)). All the 100 measurements are added, per per-

*The matrices can be built up in other ways: thus individuals might be measured for their Spearman, *g, p, o, w, f, c,* etc. in standard terms, and then correlated amongst themselves with these factor estimates as population.

†It seems worth while as an example to pursue an investigation for such measurements. I hope to publish such data in due course.

son, the mean and standard deviation for each person is calculated, and his crude scores reduced to standard ones. Such measurements can be correlated,* for persons as variables and the 100 physical attributes as population. The measurements are all considered entirely with respect to a person, without any quantitative reference to any other person. Different units of length can be employed for each person without making the slightest difference to the correlations. But it is obvious that all persons will correlate positively with each other (unless they are grossly deformed); indeed, the correlations will often be nearly perfect. It may be held at once that a general factor will be found for such data, whatever other orthogonal or other common factors may emerge. Again, however, I shall discuss what meaning to attach to such factors later.

If the measurements are *first* standardized as in system (1), and then correlated as in system (2), we have an example of system (3). All the attributes are now given equal weight, so to speak, a nose being no longer than a leg in standard terms. So far as any general factor of system (1) is concerned, this system (3) will show no counterpart to it: but any factors that emerge should not be independent of the subsidiary ones coming from system (1). System (3), however, can contribute information not already supplied by the previous systems; and it may often be convenient to use because its measurements for a person are factor saturations and not regression estimates.

An example of system (4) is supplied if the crude measurements are first standardized as for system (2), and *then* correlated as in system (1). If there is only one general factor (plus specific factors) in system (2), then no factors should emerge from system (4). Otherwise factors will usually appear, and will correspond in some way with any subsidiary common factors of system (2). Again, however, it is well to realize that it might sometimes be highly pertinent and fitting to measure attributes primarily relative to an individual as in system (2), and thereupon seek to factorize the attributes as variables with respect to a population of persons.

Thus four correlation matrices accrue in the above example. The first is for physical attributes as variables, for a population of persons; the second is for physical persons as variables,† for a popula-

*I purposely begin by calling attention to the standardization, which is, of course, automatically effected in the usual product-moment correlation coefficient

$$r = \frac{\Sigma x y}{n \sigma_x \sigma_y} .$$

†As nearly whole as the 100 measurements allow: a large population of such measurements would allow a still closer approximation to the whole physical person.

tion of attributes, measured relative to each person himself;* the third is also for persons as variables, but the attributes have first been reduced so that all are *statistically* alike; the fourth is for physical attributes as variables, for a population of persons, but the attributes have all been gauged relative to each individual separately. It will not be denied that the matrices usually would contain significant correlations, and that factors would be discovered upon submitting these to factor analysis. It may also be granted that these factors, in the above example, with certain exceptions,† would be reflections of each other; that is, factors in system (1) would have a counterpart to some extent in each of the other systems (with the exceptions just mentioned).

Nevertheless very different methodological approaches are involved.

Two Methodological Approaches.

System (2) in the above example begins from the standpoint of the separate physical person. It neglects each person's *absolute* measurements, his absolute height etc. relative either to a population of persons or to an absolute physical standard (the metre or yard). Otherwise it can embrace in its population every physical measurement of a person, relative to himself.

It begins from the standpoint of the *whole* body-proportions of the person, and it can embrace all possible relations of the physical person in this respect. In a sense it can engulf the "total situation", for it takes all the physical measurements into account: but it always neglects the absolute features of all these physical measurements.

I shall show in due course that system (2) can in the same way embrace the relational aspects of a person in cognitive and orectic spheres no less than in this one of physical measurements. If we examine a person's moods, for instance, every perceivable mood-condition can enter into the population to be analyzed by way of system (2): but again we would in no way be concerned with the absolute quantities of any of the moods.

System (4) follows logically from (2), beginning from the standpoint of the proportions of body-parts relative to each individual himself. Each body-measurement is judged or gauged purely with respect

*Different units of measurement could be used for each person, so long as the same unit is used for all the measurements of the one person. The units could be inches for one person, centimetres for another, feet for another etc.

†Such as, that if only *one* factor covered the matrix for system (1), no broad factor could appear in system (3) in the above example.

to the individual *of which it is part.* In a sense the emphasis is still on the 'total' person, the "total situation".

System (1) begins with absolute measurements, in the sense of measurement in terms of a yard or metre. But it reduces all such measurements to standard terms, and the supposed absoluteness of the variates is merely a matter of relativity again — each person's height in standard terms is a measure made in terms of and relative to the heights of a population of persons, and similarly for each separate attribute.

It is true that system (1) could encompass the 'total' person in so far as all possible attributes (height etc.) are used as variables. But it completely neglects the relativity of such attributes, relative to each person, and it perpetrates a puzzling reduction of all the separate attributes to statistical terms of equal significance, irrespective of the relative significance of the attribute in the person. One cannot but have the suspicion that in the above example the system is dealing with severed heads, arms, noses, legs etc., piled into separate heaps, and that it will be impossible to find our way back to the whole person again.

I have already mentioned that although system (1) appears to begin with absolute variates, it does so only in a sense 'relative to a population of persons'. The system can certainly tell us if, and how, the various attributes vary proportionately in a population of persons. But it can tell us little or nothing about the whole body-build of any individual person. It supplies information of a *general* kind. There would seem to be only one set of conditions under which system (1) can give us information about the whole individual as such, namely, when all the variables are approximations to one and the same aspect or feature of individuals. This is the case, for instance, when mental tests are variables. The variates for each mental test can be legitimately reduced to standard terms; that the variables vary proportionately can be determined; but the information so supplied is pertinent to the *whole* individual, as Line* has shown. System (1) can supply undistorted information about mental test variates, because each represents, approximately, the same whole cognitive person in operation; each is an approximate measurement of *process,* measured in units of time. But unless some such common significance is attached to each variable, or *may* be so attached to each, system (1) cannot do other than distort the original facts, *so far as the individual*

*Line, W. *loc. cit.*

*is concerned,** as it undoubtedly does in the above physical example, and as it does, I shall later show, in work with temperament and character traits.

It is interesting to draw a comparison between systems (1) and (3), and (2) and (4), in terms of gestalt and the so-called atomistic theories in psychology. Gestalt theory scarcely needs a definition if it is identified with that of the Wertheimer-Koffka-Köhler school of thought. In particular this school rebels against the viewpoint that a whole is a sum of parts; the whole, rather, has primacy over any parts.†

Returning to our example of the physical attributes of persons, system (1) begins from the 'atomistic' standpoint. It begins with parts, and its factors also represent only parts of the individual. Thus, as is mentioned later, one factor may be specific to trunk measurements, and another to head measurements etc. Even the factors already known in system (1), such as *g, p, o, w,* etc., are also parts in the same sense.

It has hitherto been a criticism of system (1) that, having taken the person to pieces and measured his *g, p, o,* etc. separately, it could not proceed to put the person together again. The same criticism would apply, no less, to the physical variables or any factors arising from them. Now by applying system (3) to such part measures or factors, i.e. using the part measures as variates for a person, and thereupon correlating persons, we can certainly put these measures together again, with respect to each person. But having done so, system (3) by no means builds up the whole person again. Its factors can only be distorted, unreal, or potential, with respect to any individual, because it assumes the parts to be equally weighted for the individual at the outset — all the parts are given equal significance relative to the individual, when in fact they have not such equality. Beginning with parts in system (1), then, cannot lead to a *whole* person by way of system (3).

System (2), on the other hand does not begin with such equally weighted parts — instead, each is given its proper significance *relative to the whole of which it is part, and which governs these parts.* System (2) here seems to correspond with the gestalt standpoint in psychology; system (1) and (3) correspond no less with the atomistic standpoint. System (2) encompasses the "total situation"; system (1) does not really do so. System (1) begins by distorting the original

*So far as the population of persons is concerned it perpetrates no such distortion.

†Peterman, B. *The Gestalt Theory.* (Kegan Paul, London, 1932). (I refer to the "principle of the primacy of the whole over its parts".)

parts relative to the individual; system (2) effects no such distortion.

Nevertheless, as I have indicated, system (1) can always be used to supply information about the way in which any variables, for part or 'whole' aspects of a person, depend upon one another, i.e. it will show whether the variables are related, in the sense of varying proportionately in a population of populations, and with respect to that population. In this way, however, the system supplies information of interest to *general,* and not to *individual,* psychology, except when 'whole' aspects of persons are involved. As I have indicated, when the variables are for 'whole' aspects of persons, as is approximately the case for cognitive tests as variables, then system (1) not only supplies information of value to *general,* but also of essential interest to *individual* psychology. If, with Line, we consider the measurements supplied by mental tests (cognitive abilities, or functions or processes in general) to be for the 'whole' person in action, so that they have gestalt implications at the outset, then system (1) can supply information which in no way is a distortion of the original data—except in so far as any experimental abstraction from reality is artificial.

There can be no doubt that any data can be examined by either system (1) or (2). But there are circumstances under which it is most pertinent and proper to use either the one or the other, for otherwise the original data will be distorted.

Further Observations.

At risk of being tedious I add some further observations about the above example for physical attributes, and about the four standardization systems in general.

Any factor issuing from system (2) can have its nature completely estimated.* A factor so described is an estimate of the most typical (hypothetical) person for the given relational system of attributes or measurements. The factor is represented by a population of *measurements* (in standard terms) to which all persons of the factor are approximating in some degree, and the factor saturation of a person is an index of how closely he approximates to this 'typical' person.† The factor still refers to a 'total' person, except that it neglects the 'absolute' nature of the attribute, i.e. the relation of each to a

*See Stephenson, W. *A new application of correlation to averages.* Brit. J. Educ. Psych. VI. 1936. p. 43.

†I shall indicate in due course that factors in system (2) need not be analyzed orthogonally, and that thereby the " 'total' situation" is retained.

population of persons. This omission from the 'whole' person* is not so grievous a matter in system (2), as is the neglect of the relational features of the attributes in system (1).

Factors in system (1) are an indication that the separate variables entering into it vary proportionately in the population of persons. The person with the longest legs will tend also to have the longest arms, the widest chest, and the biggest feet etc. The essential nature of a factor in the latter case is represented by a population of *persons*, each person in which has his own constant factor amount (g, in standard terms), which tends to be the same in all the variables entering into the factor.† But this factor amount, which is estimated from a regression equation,‡ cannot be said to give us any description of the individual person. In the first place it is more than likely that any factor in system (1) will only be for *some* and not for *all* the physical measurements — one factor may be specifically connected with the trunk measurements, and another with head measurements, etc. This indicates at once that the factors are not encompassing the 'total' physical person. Secondly, the factor estimate for a *person* in system (1) only has a meaning relative to all the other persons involved; the estimate is relative to the population of persons.

System (3) concerns itself with a population of physical measurements, the same in name as for system (2). But the actual variates are all in standard terms — a nose is on the average no longer than a leg in standard terms, and system (3) begins with such measurements. A factor will no doubt emerge. *But its nature can scarcely be that of any 'real' physical person, because we have begun by mixing up the original measurements, so to speak, with those of other people.* We could therefore scarcely hope to return from the *general* (represented by the factor in system (3)) to the *particular* (the individual physical person).

System (4), on the other hand, does not perpetrate such an admixture in its original variates. Its factors again are an indication that the physical features entering into a factor vary proportionately, just as for system (1).

System (1), of course, is that used exclusively in the past in factor analysis. It supplies the basis for scientific work on empirically

*The word 'whole' must be taken here to mean relational together with 'absolute' aspects of the physical measurements.

†This is merely a literal description of what a theorem such as the Spearman two-factor (a common g and specific s factors) means.

‡In the Spearman two-factor theorem the amount is:

$$\underline{g = r_{ag} \cdot m} \pm .6745\sqrt{1 - r_{ag}^2}\ .$$

observed or objectively determined *individual differences.** Although it begins with the premisses of individual differences, it only contemplates measuring any individual for those differences which enter into a factor. When a factor appears, such as Spearman g, there is some guarantee that all persons are alike in respect of certain individual differences. In this respect the system is primarily of interest to General rather than to Individual psychology.

System (2) does not begin from the standpoint of individual differences of any kind, but with empirical discoveries of a qualitative kind. Individual differences are in no way essential to it. But it *gives rise* to a kind of individual differences which has never before been attended to by psychometrists, namely, individual differences *in type.*† So far as system (2) is concerned, however, all persons might be of a type i.e. approximating to the same "typical" person, and they could also be identical in degree of factor saturation for the type. It just happens that in fact they are not. That different types emerge, and different degrees of each, is a matter for objective determination. Preliminary results show, as we might expect, that for many populations of traits or other data, all persons are *not* of a type.‡ The *distributions* of degrees of type for a population of persons is also a matter for determination from the observed facts: for eidetic type studies, for instance, the distribution of factor saturations may be markedly skewed. Only the facts can tell.

Conclusion.

I cannot flatter myself that I have made everything in the above pages unequivocable, or even understandable. The main contributions, however, are clear enough.

1. The matrix of persons (row) and attributes (column) used by factorists can be employed in more ways than the one used almost exclusively in past factor analysis and psychometry generally. Four ways have been described and illustrated.

2. Two of these four (1, 3) have atomistic foundations unless they deal with *process* measurements. The other two (2, 4) have relativistic and gestalt implications.

3. The criterion for use of system (1) would seem to be that the variables are equally significant for the individual; when they are

*The words refer here, and always in the sequel, to differences with respect to a population of persons.

†The word 'type' is here and throughout this paper used in a special sense, so far as I know never before used. It is the subject of a paper to follow this one.

‡See examples in my various introductory papers.

not, then system (2) is the appropriate system to begin one's investigation with, from the standpoint of the individual.

4. There can be no doubt, I think, that system (2) deals most faithfully with the original data in the case of the physical measurements, from the standpoint of the individual.

5. System (4) supplies a means for investigating whether and how separate* attributes vary proportionately. It would seem to be the gestalt counterpart of system (1), but system (1) is more obviously applicable to studies of this *general* kind, i.e. the varying proportionalities of attributes for a population of persons.

I would conclude by reiterating that there is no question in the above of suggesting that systems (2) and (4) should *replace* systems (1) and (3) in psychometry. It is obvious that all four can be used, each in its appropriate place. But system (2) can deal with 'total' persons, whereas system (1) only does so for process measurement.

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Note added July 10th, 1936.

In the above paper I have been concerned with certain particular and general applications of the four factor systems. Only a brief definition of each system was given, a full account being reserved for the main work upon which I have been engaged for some time, which is to be published shortly. The above account seemed to be sufficient for the present purpose, which was to draw attention to other possibilities in factor analysis, and to other applications, than those widely known to most workers. But the brevity of the definitions, and certain special features of the example by which I have illustrated them, may lead to misunderstandings which the fuller treatment will obviate. Meanwhile, the following further notes may forestall misinterpretations, at least.

The important systems are (1) and (2), the other two being of minor immediate interest. I have clearly stated that systems (1) and (2) can be quite independent, but it will be better to state explicitly that I regard the two as, by very definition, statistically independent of one another in general. It will be helpful, in this connection, to distinguish between two different forms that system (2), or (1), can take.

System (2) in general is not to be regarded as the direct obverse or mere transpose of data already analyzable by way of system (1); nor is it the case that these two systems are merely two complementary ways of analyzing one and the same original matrix of data, the results being no less complementary or deducable one from the other. Such complementary conditions, and the possibility of direct analysis either in rows or in columns, is only possible under the special case of universality of *unit*. One must first define what units are involved in either the rows or the columns. It is not sufficient to put scores into a matrix and then say without more ado that they can be analyzed either by rows or by columns. Of course they *could* be; but unless something is told us about the units involved, such two-way analysis will usually be a futile and useless proceeding. It is precisely the purpose of systems (3) and (4) to ensure, under the conditions of non-universality of unit in either rows or columns respectively, a general reduction to comparable units, so that analysis would then be sensible and possibly useful in general. In the very special case, however, when one and the same unit

*The word again refers only to the mode of thinking about the attributes. Systems (1) and (4) proceed to show whether the attributes vary proportionately, and when any two so vary neither can be regarded as isolated or distinct.

of measurement is involved in *both* columns and rows by definition (as could be the case in the example of the physical attributes described above, if an *inch* is used throughout as the unit of measurement, for all persons and all attributes), direct analysis by either rows or columns is sensible and permissible. In this case the same original data, without any reduction of any kind, can be analyzed either by system (1) or by system (2).

In the general case, however, the data is by very definition distinct for analysis by way of system (1) or (2) respectively. In system (1) it is not in the least essential to have one and the same unit for all *attributes* or *tests*; it is merely essential that the unit for any one attribute should be one and the same for the whole population of persons. In system (2), likewise, it is not essential to have one and the same unit for all *persons*; it is merely essential that the unit for any one person should be the same for the whole population of attributes involved for that person. The units for system (1) moreover, are generally by definition or fact different from any of those involved in system (2), so far as we know, or need know.

It so happens that I consider this non-universality of unit, in either rows or columns respectively, to be a matter of the very first importance. In the study of personality, for instance, matters of the greatest possible interest depend upon this fact, that analysis can proceed without a universal unit of measurement.

The burden of the above paper, however, amongst others, was that of showing that even when we begin with special data of the kind used in the example, for the 100 physical attributes, and analyze it by both system (1) and system (2), the two different standardizations effected (when one correlates the rows, or the columns), effect a radical alteration in the original data, and therefore make the two subsequent factor analyses really incomparable. The two are only dependent in so far as both use the same crude data; but the different standardizations effect profound changes in this crude data. As I have already said, analysis by way of system (2) could proceed even if *different* units are used for each *person* respectively; but in that case direct obverse analysis of that same data by way of system (1) would be a futile and meaningless proceeding. The same is true, of course, if the original data is for unique or different units per *attribute* (but the same for all persons, per attribute); in such a case subsequent analysis of the same data as it stands, by way of system (2), would be futile and meaningless. But even if one and the same unit is used throughout, for all persons and all attributes, the respective standardizations to which the data is submitted (in effect) when either rows or columns are correlated, radically change the original data. In effect, therefore, one is not dealing with the same data in the two systems, for factor analysis only concerns standardized material, and not crude data. It would have been better to have stated this explicitly or more emphatically.

In the above paper I wrote (largely so as to allow me to press on with the essential argument) that the various factors arising in the different systems might, to some extent, be "reflections" of one another. Had space permitted, or intention inclined me to it, I should have elaborated this considerably. Actually I go on to show that the factors will in fact be very different in the various systems, both as to essential nature and as to interpretation, and this is generally the case. They are really only superficial "reflections", as matters of interpretation.

I had intended expanding on matters of the above kind in my fuller study of system (2), for there are still other interesting considerations involved. In general however, in system (2) I am concerned to adumbrate a system which will be the very poles apart from system (1). System (1) involves a population of *persons*, but there need only be *one* attribute for investigation; system (2) involves a population of attributes, but there might only be a *single* person involved. (I shall supply examples in due course of factor analysis, performed upon myself alone as variable.) I therefore demanded quantification which could be confined to a single person, uniquely if need be, and system (2) supplies a means for dealing with such quantifications. Similarly, in system (1) there is an unlimited population of persons from which to draw one's samples, and sampling error theory is pertinent. But in system (2) there may be a restricted population of attributes, so few that one's 'sample' constitutes the whole universe of these attributes; I have therefore to contemplate other than sampling error theory.

The statistical differences between systems (1) and (2), then, are radical and fundamental. Usually, by very definition we begin with data the units for which are expressly defined with respect to system (1), or with respect to system (2), and there is no necessary connection at all between them. The psychological applications of these systems are no less profoundly different.

In the above paper I have tried to indicate very briefly some of these psychological applications. System (1) has its counterpart in the 'atomistic' standpoint in psychology, whereas system (2) encompasses a gestalt standpoint. I shall, of course, have much more to say about this in future work, but sufficient has been said, I think, to point towards this important conclusion. I may be pardoned, however, if I elaborate a little on two matters touched upon very briefly in the above paper.

Factor analysis by way of system (1), as I shall show in detail elsewhere, is concerned primarily with *tendencies*, i.e. with the way in which attributes vary one with another in a population of persons. It begins from the standpoint of observed *individual differences* (relative to a population of persons), and arrives at information about abilities, processes, unitary traits, primary abilities, or in general, fundamental tendencies. Each such tendency or process is an explanation for the observed facts, namely, that such and such variables vary proportionately and uniquely. The factors are by definition universal. And it seems to be a fact that there is only a limited number of such fundamental tendencies in the human being: Spearman found only five or so; Thurstone specifies seven; the Thorndike Unitary Traits Committee hoped to find from 1 to 20. The implication is that these few fundamental tendencies account for, or explain, or are the cause of, all human conduct. Right through history there has been a search for such tendencies, or else a talk about them, from the classical humours of Hippocrates or Galen, to the classical Introversion-Extraversion of Jung, and now the same kind of thought is involved in the search for unitary traits. System (1) is the method for investigating such fundamental tendencies. I do not as yet deny its efficacy in this respect, but I draw attention to the fact that the system is limited to such investigations.

System (2) accomplishes something very different indeed. It can supply no direct information about fundamental tendencies of the kind just referred to. Instead it deals with whole personalities, so-called 'irrelevancies', in general with 'total situations' in what I take to be the gestalt meaning of these words. It deals with attributes relative to each individual, and to himself alone, and each attribute has its full and proper context (in so far as this is reasonably possible in any analysis) in the whole of which it is part. This is excellently illustrated, I think, by application of system (2) to the 100 physical measurements described above. Like all analysis, system (2) has to parcel out its applications. It can only deal with whole *aspects* of persons, with the physical whole, the mood-condition whole, the cognitive whole, and so forth. All measurement is relative to the individual, and to himself alone. It has no necessary concern *at all* with individual differences of the system (1) *a priori* or *a posteriori* kind — instead it gives rise to its own kind of individual differences, those of type and type saturation or loading. Its factors are not universal necessarily; they *may* be, but only as a matter of *a posteriori* objective determinations.

In the physical example I postulated 100 attributes. To cover the physical whole more completely, or more determinately, this population might be increased to 1000, or to 10,000 if need be. But even 20 of these attributes might define the physical whole of all persons sufficiently approximately. It would have to be shown that whatever population is selected from amongst all the possible physical attributes of a person, the factor systems only altered in determinacy, and not in essential nature. In such a case we could legitimately say that we are dealing with a real physical whole, a 'total situation'. It seems too obvious to mention that a whole situation can never be encompassed if by this we mean, as Thurstone apparently means, the infinite universe of all possible attributes and all possible persons. In system (2), as in system (1), we can only deal with a finite population of either persons or attributes. Even so, system (1) is fundamentally 'atomistic' (fortunately, for it is its special merit that it allows an approach to the study of fundamental tendencies), whereas system (2) deals with 'total situations' in the above gestalt sense. In system (1) each attribute is stripped from its total context, each is standardized in terms of a population of persons (which destroys all possibility of returning to the individual wholes of the person). The 'total situation' can *never* be realized in system (1), even if the whole universe of attributes are used as variables. It *may* be retained intact in system (2), with more or less determinacy, for as few as ten, or even less, attributes.

I have said that there are possibly only a few fundamental tendencies in the human being, and therefore only a few unitary traits are born out of system (1) analysis. But there are possibly millions of *types*, each a common factor, (common that is to several or many persons, but not necessarily to all), in system (2) analysis. Type psychology comes into its own by way of system (2). Almost everything that we factorists have denied about type psychology has been beside the point: types exist in great number, and one person can be of one type and not at all of another; types are *not* extremes of otherwise normal distributions, if by type we mean the interpretation given to a common factor in system (2) analysis.