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Quantification and RM

§ 1. Introduction *

Routley and Meyer [7] conjecture (with some misgivings) that the Anderson-Belnap system ***RQ*** (***R*** with first order quantifiers) is complete with respect to a certain extension of their semantics for ***R***. The Routley-Meyer conjecture has proven intractable. For reasons that will not be examined here, the distinctive three-placed accessibility relation of the Routley-Meyer semantics seems to get in the way.

But the semantics of the relevant logic ***RM*** can be formulated with a more orthodox two-placed accessibility relation (cf. [2]). So the purpose of this paper is to prove an appropriate version of the Routley-Meyer conjecture for ***RMQ*** in terms of a "binary" semantics.

§ 2. Syntax

The language of ***RMQ*** is quite ordinary, and is basically the language of ***RQ*** as formulated in [6], except that for convenience negation ($\sim$) is taken as primitive instead of the false sentential constant ***f***, and disjunction ($\vee$) is defined in the usual de Morgan manner from conjunction ($\wedge$) rather than taken as primitive. The other primitives are implication ($\rightarrow$) and the universal quantifier ($\forall$), with the existential quantifier ($\exists$) defined in the standard way.

Note well that a distinction is made between *real* variables (a, b, c, etc.) and *apparent* variables (x, y, z, etc.), with only the latter bindable by quantifiers. A *sentence* is a formula containing no free occurrences of apparent variables. We shall always deal explicitly with a denumerable language, but the results can routinely be extended to languages of larger cardinality subject to the usual assumptions about the symbols being well-ordered.

We shall have use in the sequel for the notion of the *linguistic extension $L+V$ of a language L by a set of real variables V*. We identify this with the set of sentences constructed from the symbols of L together with the elements of V taken as additional real variables.

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The proof theory of ***RMQ*** is like that of ***RQ*** except that the "mingle" axiom scheme

A0. $A \to (A \to A)$

is to be inserted at the front of the list of axiom schemes of [6, p. 99]. Also our choice of primitives allows for the deletion of the disjunction axioms A6–A8, and requires (cf. [1], § 14.3) the addition of axiom scheme

A17. $(A \to \sim B) \to (B \to \sim A)$

The reader who does not have [6] in hand is reminded that all closures of instances of axiom schemes are taken as axioms so as to dispense with generalization as a rule, the only rules then being *modus pones* and adjunction.

Relative to a given language, an ***RMQ*** theory is defined analogously to an ***RQ*** theory in [6] as any set of sentences containing all axioms of ***RMQ*** which is closed under *modus ponens* and adjunction. We often identify ***RMQ*** with the smallest ***RMQ*** theory. We say that a sentence A is *deducible* in an ***RMQ*** theory Δ from a set of sentences θ (in symbols, $\theta \vdash_{\Delta} A$ iff $A \in \Gamma$ for all ***RMQ*** theories Γ such that $\Delta \cup \theta \subseteq \Gamma$.

Adapting and extending more notions from [6], we say that an ***RMQ*** theory Δ is *prime* whenever $A \vee B \in \Delta$ implies $A \in \Delta$ or $B \in \Delta$, $\forall$*-complete* whenever $A(a) \in \Delta$ for all real variables a implies $\forall x A(x) \in \Delta$, $\exists$*-complete* whenever $\exists x A(x) \in \Delta$ implies $A(a) \in \Delta$ for some real variable a and *straight* (the term is Routley's) whenever Δ has all three of these properties. Note well that we have no need for the epithet "regular" used in [7] to single out theories containing all logical theorems. Since we have reverted to the earlier usage of [6], which builds this into the definition of "theory". That we need consider only regular theories in proving completeness is special to ***RM***, and does not extend to the systems ***R*** and ***E***.

It is convenient to introduce $A \supset B =_{df} (A \wedge t) \to B$.[1] The idea of this benthymematic implication" is due in principle to Anderson and Belnap, "ut Meyer has studied it extensively in this particular form (cf. esp. [5]), finding that it has the behaviour in ***R*** of intuitionistic implication. Indeed, the superintuitionistic system ***LC*** of Dummett may be translated into ***RM*** using that definition of implication, cf. [3].

The following was stated for ***RQ*** theories in [6].

[1] The sentential constant $\boldsymbol{t}$, intuitively the conjunction of all true sentences, can be added conservatively to ***RMQ*** with the axiom scheme $A \leftrightarrow (\boldsymbol{t} \to A)$, or else contextually defined (cf. [1], § 27.1.2, or [3]). Although officially $\boldsymbol{t}$ is not part of our language for ***RMQ***, we shall not hesitate to make use of it when convenient.

DEDUCTION THEORY (1ST VERSION). *Let Δ be an **RMQ** theory, θ a set of sentences, and A, B sentences. Then*

$$\theta, A \vdash_{\Delta} B \quad iff \quad \theta \vdash_{\Delta} A \supset B$$

Also the following was proven for ***RM*** theories in [2] and extends immediately to ***RMQ*** theories.

DEDUCTION THEOREM (2ND VERSION). *Let Δ, θ, A, B be as in the 1st Version. Then*

$$\theta, A \vdash_{\Delta} B \quad and \quad \theta, \sim B \vdash_{\Delta} \sim A \quad iff \quad \theta \vdash_{\Delta} A \rightarrow B.$$

§ 3. Semantics.

***RM** model structures* (*m. s.*) were introduced in [2] as structures (G, K, R), where K is a non-empty set, R is a (weak) linear on K, and G is the R-least member of K. For a *constant domain* m.s. we add a non-empty set D (the domain) so as to obtain a structure (G, K, R, D).

In the sequel, we adopt for convenience a "quasi-substitutional" interpretation of the quantifiers of the sort favoured by A. Robinson, R. Smullyan and others for classical first-order logic. This uses the notion of a *U-sentence* (U a non-empty set), which is exactly like a sentence except that actual elements of U have been substituted for some or all occurrences of real variables.[2]

***RMQ** Semantics.* An ***RMQ** model* will be a structure (G, K, R, D φ), where (G, K, R, D) is a constant domain m s. and φ (the *valuation*) is a function assigning each real variable a member of D and each atomic D-sentence, relative to a member of K, a non-empty subset of $\{\mathsf{T}, \mathsf{F}\}$[3]. The obvious "Rose-by-any-other-name" Requirement is made that if P is an atomic D-sentence and $P(\varphi)$ is the result of replacing each real variable a of P in every occurrence by $\varphi(a)$, then $\varphi(P, H) = \varphi(P(\varphi), H)$ for each $H \in K$. Finally for each atomic D-sentence P we require the

[2] Since on the abstract approach to language common to most recent logical work, "symbols" can just as well be shoes, and ships, and sealing wax (or prime numbers for that matter) as marks on paper, U-sentences are just sentences in the linguistic extension of the given language by the set of "real variables" U.

[3] The idea that sentences can be valued as simultaneously both true and false is admittedly rather odd. The reader wanting motivation should consul [2]. Incidentally, K. Pledger has suggested privately that the motivation in [2] is unduly pessimistic, since the Hereditary Condition has things getting more and more contradictory as time goes on *if* one regards HRH' as indicating that the evidential situation H temporally *precedes* the one H'. But Pledger suggests that the temporal order of the accessibility relation should optimistically be thought of in the reverse order. Thus one starts with a situation in which many sentences (for all one knows) are just as much true as false, and then one improves on this situation as time goes on by accumulating evidence that occasionally decides things one way or the other.

HEREDITARY CONDITION. *If HRH', then $\varphi(P, H) \subseteq \varphi(P, H')$.*

We regard φ as extended inductively to compound D-sentences according to the following rules:

($\sim$T) $\mathsf{T} \in \varphi(\sim A, H)$ iff $\mathsf{F} \in \varphi(A, H)$;
($\sim$F) $\mathsf{F} \in \varphi(\sim A, H)$ iff $\mathsf{T} \in \varphi(A, H)$;
($\wedge$T) $\mathsf{T} \in \varphi(A \wedge B, H)$ iff $\mathsf{T} \in \varphi(A, H)$ and $\mathsf{T} \in \varphi(B, H)$;
($\wedge$F) $\mathsf{F} \in \varphi(A \wedge B, H)$ iff $\mathsf{F} \in \varphi(A, H)$ or $\mathsf{F} \in \varphi(B, H)$;

($\rightarrow$T) $\mathsf{T} \in \varphi(A \rightarrow B, H)$ iff for all $H' \in K$ such that HRH', (i) $\mathsf{T} \in \varphi(A, H')$ only if $\mathsf{T} \in \varphi(B, H')$, and (ii) $\mathsf{F} \in \varphi(B, H')$ only if $\mathsf{F} \in \varphi(A, H')$;
($\rightarrow$F) $\mathsf{F} \in \varphi(A \rightarrow B, H)$ iff either (i) $\mathsf{T} \notin \varphi(A \rightarrow B, H)$ or (ii) $\mathsf{T} \in \varphi(A, H)$ and $\mathsf{F} \in \varphi(B, H)$;

($\forall$T) $\mathsf{T} \in \varphi(\forall x A(x), H)$ iff $\mathsf{T} \in \varphi(A(a), H)$ for all $a \in D$.
($\forall$F) $\mathsf{F} \in \varphi(\forall x A(x), H)$ iff $\mathsf{F} \in \varphi(A(a), H)$ for some $a \in D$.

The following may be verified along the lines of the inductive proof of the corresponding lemma in [2] (the new case $A = \forall x B(x)$ may be done by the reader in his mind's eye).

HEREDITARY LEMMA FOR ***RMQ***. *Let (G, K, R, D, φ) be an **RMQ** model. For any D-sentence A, if HRH', then $\varphi(A, H) \subseteq \varphi(A, H')$.*

§ 4. Soundness.

A sentence A is a *logical consequence* of a set of sentences Δ in ***RMQ*** iff for all ***RMQ*** models with first component G and last component φ, $\mathsf{T} \in \varphi(B, G)$, for all $B \in \Delta$, implies $\mathsf{T} \in \varphi(A, G)$. We write this in symbols as $\Delta \vDash_{RMQ} A$.

The reader may establish the

SOUNDNESS THEOREM FOR ***RMQ***. *If $\Delta \vdash_{RMQ} A$, then $\Delta \vDash_{RMQ} A$.*

In doing so he will find useful both the Hereditary Lemma and also a ma extending the "Rose Requirement" to compound sentences.

§ 5. Completeness of *RMQ*

We make a conscious adaptation of the argument of Gabbay in [4] (which the reader should have at hand) for the completeness of 2nd-order intuitionistic propositional calculus augmented with the propositional quantifier version of A15. Gabbay's argument makes frequent use of the Deduction Theorem for intuitionist implication which just does not hold tor ***R-Mingle*** implication. But it does hold for ***R-Mingle*** enthymematic fmplication (our Deduction Theorem in the 1st Version), and therein lies ihe secret of the adaptation. An analysis of Gabbay's argument reveals that only rarely is it important that the implication involved in an application of the Deduction Theorem is the primitive implication of the system, and then our Deduction Theorem in the 2nd Version can be made to serve instead.

We shall borrow terminology from [4], but we do not follow Gabbay in his use of the word "theory" since the word has already been used in this paper and in earlier papers on relevant logic in a more standard sense. We shall instead call an ordered pair (Δ, θ) of non-empty sets of ***RMQ*** sentences simply an ***RMQ*** pair.

An ***RMQ*** pair (Δ, θ) is *consistent* iff for no $A_1, \ldots, A_m \in \Delta$, $B_1, \ldots, B_n \in \theta$ do we have $\vdash_{RMQ} (A_1 \wedge \ldots \wedge A_m) \supset (B_1 \vee \ldots \vee B_n)$,[4] and *complete* in a certain language iff for each sentence A of that language, $A \in \Delta$ or $A \in \theta$.

An ***RMQ*** pair (Δ, θ) is *saturated* iff Δ is a prime, $\exists$-complete ***RMQ*** theory. An ***RMQ*** pair (Δ, θ) is of *constant domain* in a certain language iff whenever $(\Delta, \theta \cup \{\forall x B(x)\})$ is consistent, then for some real variable a of the language $(\Delta, \theta \cup \{\forall x B(x), B(a)\})$ is consistent. The reader may easily verify that an ***RMQ*** pair (Δ, θ) is saturated, consistent, complete and of constant domain iff (1) Δ is a straight ***RMQ*** theory which is "proper" (not all sentences are theorems) and (2) θ = set of sentences $- \Delta$.

An ***RMQ*** pair (Δ^*, θ^*) *extends* another (Δ, θ) iff $\Delta \subseteq \Delta^*$ and $\theta \subseteq \theta^*$. We now have a series of lemma *à la* Gabbay.

LEMMA 1. *Let (Δ, θ) be a consistent **RMQ** pair. Then (Δ, θ) can be extended to a complete saturated, and consistent **RMQ** pair (Δ^*, θ^*) of constant domain in a language with denumerably many new real variables.*

PROOF is basically the same as Gabbay's, wherein (Δ_0, θ_0) is set to be (Δ, θ) and $(\Delta_{n+1}, \theta_{n+1})$ is defined inductively from (Δ_n, θ_n), and finally (Δ^*, θ^*) is defined so $\Delta^* = \bigcup_{n\in\omega} \Delta_n$, $\theta^* = \bigcup_{n\in\omega} \theta_n$. However we shall change the inductive step of the construction somewhat.[5]

Thus let V be a set of $\aleph_0$ new real variables and let $B_1, B_2, \ldots$ be an enumeration of all the sentences in the linguistic extension of the given language by V.

Now if $(\Delta_n \cup \{B_{n+1}\}$ is consistent, construct Δ_{n+1} by adding B_{n+1} to Δ_n. At the same time if $B_{n+1} = \exists x F(x)$, add $F(a)$, where a is the first real variable (in some standard enumeration including those in V) such

[4] We could have alternatively defined *consistency* with $\rightarrow$ in place of $\supset$ if we had at the same time built into our definition of an ***RMQ*** *pair* (Δ, θ) that $t \in \Delta$, making further changes in the sequel as necessitated.

[5] N. D. Belnap, Jr. communicated to me in 1973 a result which is in effect the precise analogue for ***RQ*** of Gabbay's Lemma 1 (but of course completely independent), with consistency defined using $\rightarrow$ and *no* requirement made that $t \in \Delta$ (cf. note 4). The structure of the proof of our Lemma 1 borrows from Belnap, and also the proof of our Lemma 4, bringing out (I think) more clearly the similarities between the proofs.

that $(\Delta_n \cup \{\exists x F(x), F(a)\}, \theta_n)$ is consistent. Set $\theta_{n+1} = \theta_n$. On the other hand, if $(\Delta_n \cup \{B_{n+1}\}, \theta_n)$ is not consistent, construct θ_{n+1} by adding B_{n+1} to θ_n. Add as well if $B_{n+1} = \forall x F(x)$, $F(a)$, where a is the first real variable such that $(\Delta_n, \theta_n \cup \{\forall x F(x), F(a)\})$ is consistent. Set $\Delta_{n+1} = \Delta_n$.

Of course it has to be shown in each case that there is such a real variable a. Prior to arguing this we observe that either $(\Delta_n \cup \{B_{n+1}\}, \theta_n)$ or $(\Delta_n, \theta_n \cup \{B_{n+1}\})$ is consistent; otherwise (Δ_n, θ_n) would be inconsistent on account of

$$A \supset (B \vee C), (A \vee B) \supset C \vdash_{\mathbf{RMQ}} A \supset C.$$

Now a can be picked as the first member of V not in any of Δ_n, θ_n, B_{n+1}. Otherwise, on the basis of the prior observation it is easy to see that either (I) $(\Delta_n, \theta_n \cup \{\forall x F(x)\})$ is consistent while $(\Delta_n, \theta_n \cup \{\forall x F(x), F(a)\})$ is inconsistent, or (II) $(\Delta_n \cup \{\exists x F(x)\}, \theta_n)$ is consistent while $(\Delta_n \cup \{\exists x F(x), F(a)\}, \theta_n)$ is inconsistent. But both can be shown impossible.

We treat only (I), (II) being similar. If $(\Delta_n, \theta_n \cup \{\forall x F(x), F(a)\})$ is inconsistent, then some $A_1, \ldots, A_i \in \Delta_n$, $B_1, \ldots, B_j \in \theta_n$ are such that

$$\vdash_{\mathbf{RMQ}} (A_1 \wedge \ldots \wedge A_i) \supset (B_1 \vee \ldots \vee B_j \vee \forall x F(x) \vee F(a)).$$

But they by Lemma 5 of [6] and the Deduction Theorem (1st Version), together with the idempotence of $\vee$,

$$\vdash_{\mathbf{RMQ}} (A_1 \wedge \ldots \wedge A_i) \supset (B_1 \vee \ldots \vee B_j \vee \forall x F(x)),$$

contradicting the consistency of $(\Delta_n, \theta_n \cup \{\forall x F(x)\})$.

It is now routine that (Δ^*, θ^*) is consistent, complete, saturated, and of constant domain in the linguistic extension by the new constants.

Lemma 2. *Suppose that the* **RMQ** *pair (Δ, θ) is consistent and of constant domain in a certain language and that θ is finite. Let A, B be two sentences of this language so that $(\Delta', \theta') = (\Delta \cup \{A\}, \theta \cup \{B\})$ is consistent. Then (Δ', θ') is of constant domain.*

Proof is exactly like that of Gabbay's Lemma 2, but with enthymematic implication in the role of intuitionistic implication.

Lemma 3. *Let (Δ, θ) be a consistent, complete, saturated* **RMQ** *pair of constant domain in a certain language, and let $(A \to B) \in \theta$. Then at least one of $(\Delta \cup \{A\}, \{B\})$, $(\Delta \cup \{\sim B\}, \{\sim A\})$ is a consistent* **RMQ** *pair of constant domain in the same language.*

Proof. By the Deduction Theorem (2nd Version) at least one of $(\Delta \cup \{A\}, \{B\})$, $(\Delta \cup \{\sim B\}\{\sim A\})$ is consistent, since otherwise $\Delta \vdash_{\mathbf{RMQ}} A \to B$, contradicting the consistency of (Δ, θ).

If $(\Delta \cup \{A\}, \{B\})$ is consistent, the argument procedes as for Gabbay's Lemma 3, using enthymematic implication in the definition of the key sentence $F' = \forall y(A \supset (B \vee \forall x F(x) \vee F(y)))$. If $(\Delta \cup \{\sim B\}, \{\sim A\})$ is the consistent one, then there is a precisely parallel argument with $F'' = \forall y(\sim B \supset (\sim A \vee \forall x F(x) \vee F(y)))$ in place of F'.

LEMMA 4. *Let (Δ, θ) be a consistent, complete, saturated **RMQ** pair of constant domain in a certain language. Assume that $F = (A \rightarrow B) \in \theta$. Then there exists a consistent, complete, saturated **RMQ** pair (Δ_F, θ_F) of constant domain in the same language such that $\Delta \subseteq \Delta_F$ and either* (1) $A \in \Delta_F$, $B \in \theta_F$, or (2) $\sim B \in \Delta_F$, $\sim A \in \theta_F$.

PROOF. By Lemma 3 at least one of $(\Delta \cup \{A\}, \{B\})$, $(\Delta \cup \{\sim B\}, \{\sim A\})$ is consistent and of constant domain in the same language. Set one such to be (Δ_0^*, θ_0^*). Define $(\Delta_{n+1}^*, \theta_{n+1}^*)$ in precisely the same inductive fashion that $(\Delta_{n+1}, \theta_{n+1})$ was defined in Lemma 1 except no new real variables are added. In that earlier construction we were able to depend upon the new real variables so as always to be able to constantly add "witnesses" to existential sentences on the left and "hostile witnesses" to universal sentences on the right. This time we are stuck with the given language, and so must exploit the constant domain property instead.

We argue then that if (Δ_n^*, θ_n^*) is defined, consistent, and of constant domain, so is $(\Delta_{n+1}^*, \theta_{n+1}^*)$. It follows readily from (Δ_n^*, θ_n^*) being of constant domain that if $B_{n+1} = \forall x F(x)$, then $(\Delta_{n+1}^*, \theta_{n+1})$ is defined and consistent, The case when $B_{n+1} = \exists x F(x)$ is more interesting, but is argued precisely like Gabbay's Case (1d) in the proof of his Lemma 4, again putting enthymematic implication in to do the job of intuitionist implication.

That $(\Delta_{n+1}^*, \theta_{n+1}^*)$ continues to be of constant domain follows from Lemma 2.

Finally, set $\Delta_F = \bigcup_{n \in \omega} \Delta_n^*$, $\theta_F = \bigcup_{n \in \omega} \theta_n^*$. A routine argument shows (Δ_F, θ_F) to have all the desired properties of the lemma.

Before stating the last lemma we need a definition. Where Δ is a straight **RMQ** theory, the *canonical **RMQ** model determined by* Δ, $(G_\Delta, K_\Delta, R_\Delta, D_\Delta, \varphi_\Delta)$, is as follows: $G_\Delta = \Delta$; $K_\Delta = \{\Delta' : \Delta'$ *is a straight* ***RMQ*** *theory and* $\Delta \subseteq \Delta'\}$; $\Delta_1 R_\Delta \Delta_2$ iff $\Delta_1 \subseteq \Delta_2$; $D_\Delta = \{a$: *a is a real variable*$\}$; $\varphi_\Delta(a) = a$ for each real variable a; and (i) $\mathsf{T} \in \varphi_\Delta(P, \Delta')$ iff $P \in \Delta'$, and (ii) $\mathsf{F} \in \varphi_\Delta(P, \Delta')$ iff $\sim P \in \Delta'$. That $(G_\Delta, K_\Delta, R_\Delta)$ is an ***RM*** m.s. follows directly from *Lemma* 2 of [2].

LEMMA 5. *Let Δ be a straight **RMQ** theory. Then in the canonical **RMQ** model determined by Δ,* (i) $\mathsf{T} \in \varphi_\Delta(A, \Delta')$ *iff* $A \in \Delta'$, *and* (ii) $\mathsf{F} \in \varphi_\Delta(A, \Delta')$ *iff* $\sim A \in \Delta'$.

PROOF. By induction on the length of A, much as in [2], but the case $A = B \rightarrow C$ deserves some new attention, and of course there is the completely new but routine case $A = \forall x B(x)$.

As for the first, in [2] it was needed to show that for a *prime* ***RM*** theory Δ, that $B \to C \in \Delta$ iff for all *prime* ***RM*** theories $\Delta' \supseteq \Delta$, $B \in \Delta'$ only if $C \in \Delta'$, and $\sim C \in \Delta'$ only if $\sim B \in \Delta'$. The same thing now needs to be shown for *straight* ***RMQ*** theories. But the gut of this is just our Lemma 4.

PROOF OF COMPLETENESS OF ***RMQ***. Suppose not $\Delta \vdash_{RMQ} A$. Then $(\Delta, \{A\})$ is consistent. By Lemma 1 there is a consistent, complete, saturated ***RMQ*** pair (Δ^*, θ^*) of constant domain (albeit in a linguistic extension). By the observation preceding Lemma 1, Δ^* is a straight ***RMQ*** theory. Consider the canonical ***RMQ*** model determined by Δ^*. By Lemma 5 (since $\Delta \subseteq \Delta^*$ and $A \notin \Delta^*$), not $\Delta \models_{RMQ} A$.

COROLLARIES. *The usual corollaries, e.g. the compactness Theorem and the Löwenheim-Skolem Theorem, follow in the usual ways.*

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