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A Comparison of Local Approximation Methods for the Analysis of Meteorological Data

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With 6 Figures

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Summary

A number of local interpolation methods suitable for statistical and graphical meteorological applications, including weighted averages, least-squares polynomials and successive corrections, were compared in this study. Daily observations of temperature and geopotential height recorded in two years by 71 European stations were used as data sources; the results of the interpolations were checked against the observed values in 19 selected stations, the data of which were assumed as missing. The methods were tested in points with different conditions of data density and isotropy.

The best results were obtained with algorithms based on simple weighted averages, in one or more steps; in these methods a proper choice of the influence radii appears to be the key factor in determining the accuracy of the approximation. In particular, a successive correction (SC) method using a weighted average with a large radius as a first guess and two correction steps proved to give the highest accuracy with a reasonable computational time. The SC method was also used to generate grid-point fields of geopotential height in a six-month winter period, and the main statistical properties of this sample were compared with those of the corresponding fields analysed by the European Centre for Medium Range Weather Forecasts. The mean field, the standard deviations, and the principal components that explain the most part of the total variance were correctly reproduced in the sample generated by the SC method.

Zusammenfassung

Ein Vergleich lokaler Näherungsverfahren zur Analyse meteorologischer Daten

In dieser Untersuchung werden einige lokale Interpolationsmethoden, wie gewichtete Mittel, Polynome aufgrund der kleinsten Quadrate oder eine schrittweise Korrektur, die für statistische und graphische meteorologische Anwendungen geeignet sind, miteinander verglichen. Die täglichen Beobachtungen der Temperatur und der geopotentiellen Höhe von 2 Jahren und 71 europäischen Stationen wurden als Datenmaterial

herangezogen. Die Ergebnisse der Interpolationen wurden mit den Messungen von 19 ausgesuchten Stationen verglichen, deren Daten als fehlend angenommen worden waren. Die Methoden wurden in Punkten mit verschiedener Datendichte und Datenisotropie getestet.

Die besten Ergebnisse lieferten die gewichteten Mittel – bei dieser Methode erwies sich die Wahl geeigneter Einflußradien als entscheidend für die Genauigkeit. Besonders die Methode der stufenweisen Korrektur mit einem gewichteten Mittel bei einem großen Radius als ersten und zwei Korrekturschritten erwies sich bei angemessener Rechenzeit als die genaueste. Diese Methode wurde auch herangezogen, um an Gitterpunkten die geopotentiellen Höhen für eine sechsmonatige Winterperiode zu berechnen. Die statistischen Eigenschaften dieser Felder wurden mit denen des ECMWF (Europäisches Zentrum für Mittelfristvorhersagen) verglichen. Dabei wurde das mittlere Feld, die Standardabweichung und mit Einschränkungen die totale Varianz durch die oben erwähnte Methode korrekt reproduziert.

1. Introduction

At present, most studies concerning the objective analysis of meteorological fields are oriented to the development of sophisticated data assimilation schemes, which have the purpose of providing the initial fields for numerical weather prediction (NWP) models. The operational analysis schemes used in meteorological centres are today mainly based on statistical interpolation methods that were introduced by Eliassen [9] and Gandin [11]. In these methods, a first guess field (usually the forecast obtained from a previous analysis) is corrected by linear combinations of the deviations of observed data from the first guess. Models of the correlations of the observational error and of the first guess error are needed in order to determine the coefficients of these linear combinations in such a way that the theoretical root-mean-square (r.m.s.) errors of the resulting fields are minimized.
Besides the statistical information included in the correlation models, physical constraints (typically, the hydrostatic and the geostrophic balances) are included in these schemes in order to provide a physically consistent description of the three-dimensional structure of the atmosphere. A review of the different methods used for meteorological analysis, including a wide description of the statistical interpolation technique, is given by Gustavsson [16]; examples of statistical interpolation schemes can be found in Schlatter [24], Schlatter et al. [25], Bergman [1], Lorenc [19]; the structure of the correlation functions used in these schemes is widely discussed by Buell [4], Julian and Thiebaux [17], Thiebaux [27, 28].
Statistical interpolation schemes including physical constraints are very complex and require a lot of computational time; this is true also when they are used for applications different from NWP, for example diagnostic studies over limited areas. In general, the main requirement for an analysis scheme used for dynamical studies or numerical integrations is a high accuracy rather than a low computational cost.

However, there are many meteorological or climatological investigations that need fast methods for data analysis. The main field of application of these fast methods is the statistical analysis of large amounts of sparse data, in which they can be used both to estimate missing data and to check the reliability of the available ones. If the spatial distribution of the observing stations is very irregular, an interpolation onto a regular grid can be necessary before carrying out subsequent analyses, for example the computation of empirical orthogonal functions (see Karl et al. [18]). Besides, a local interpolation can be used to check the spatial coherence of the data as a preliminary step of more sophisticated analysis schemes; finally, fast approximation methods are needed for a number of meteorological graphical applications.

A wide range of local approximation techniques can be used for these purposes. Unfortunately, apart from early applications of least-squares polynomial fitting (see Panofsky [23], Gilchrist and Cressman [12], Corby [7], Endlich and Mancuso [10]), very little information is available about the performances of these techniques for meteorological purposes. A notable exception is the work by Goodin et al. [14, 15], who compared methods based on distance-weighted averages with a second-degree polynomial fitting. However, the main conclusions of their work was based on a test made with simulated data, in which an hemispheric surface was assumed as "true" field. The form of this surface may have influenced their results, causing an overestimation of the performances of the second-degree polynomial technique. Besides, Glahn [13] pointed out that some of the methods were not properly used.

The purpose of our study was to compare the performances of a number of local approximation methods applied to actual meteorological data. Daily observations of temperature and geopotential height at 850 mb recorded for two years by 71 European stations were used; the results of the interpolations were checked against the observed values in 19 selected stations, the data of which were assumed as missing. The methods were tested in points with different conditions of data density and isotropy.

Since the principal applications of these techniques are in the field of statistical analysis, an interpolation of a subset of the available data onto a regular grid was also performed, in order to test if the main statistical properties of this sample of grid point fields were comparable to those derived from the operational analyses of the European Centre for Medium Range Weather Forecasts.

2. Mathematical Algorithms

Let $C(x, y)$ be the two-dimensional field of a meteorological variable, and let us assume that the values of C are known only on a discrete set of N "stations" $S_k \equiv (x_k, y_k)$ in the spatial domain D

$$C_k = C(x_k, y_k) = C(S_k) .$$

In this study, the problem of estimating the value C_j assumed by C on an arbitrary point $P_j \equiv (x_j, y_j)$ inside D is solved by means of local approximation techniques. These techniques give an estimate of C_j using only the data coming from stations S_k in an appropriate neighbourhood of P_j. From a more general point of view, we can use the term "local approximation method" for an algorithm that evaluates C_j as a weighted average of the values of N functions $f_k(x, y)$, each one fitting the field C in a neighbourhood I_k of a station S_k

$$C_j = \sum_{k=1}^{N} w_{jk} f_k (x_j, y_j) \Big/ \sum_{k=1}^{N} w_{jk} \tag{1}$$

where the weights w_{jk} are different from zero only inside a neighbourhood I_j of P_j. Usually, the weights depend on the distance

$$r_{jk} = |P_j - S_k|$$

(but not only on it), and are zero when r_{jk} is greater than an influence radius R_j.

When this general scheme is adopted, the solution of the approximation problem will be dependent on the choice of four elements: 1) the neighbourhood I_j; 2) the neighbourhood I_k; 3) the mathematical form of the functions f_k; 4) the functional dependence of the weights w_{jk} on the distance r_{jk} and/or other parameters.

Two sets of solutions are particularly simple. In the first one, the functions are constant ($f_k = C_k$) and the weights w_{jk} depend only on the distance between P_j and S_k

$$C_j = \sum_{k=1}^{N} w(r_{jk})\, C_k \Big/ \sum_{k=1}^{N} w(r_{jk}) \,. \tag{2}$$

In this case the problem is reduced to the determination of the influence radius R_j and of an appropriate functional form for $w(r_{jk})$. This scheme was tested in our study with different choices for these two factors.

Other simple solutions can be obtained fitting only one function f_j to the observed data inside I_j by means of the least-squares technique

$$C_j = f_j (x_j, y_j) \,, \tag{3}$$

where the function f_j is determined by the minimization of the mean square error

$$E^2 = \sum_{k=1}^{N} w_{jk} (C_k - f_j (x_k, y_k))^2 \Big/ \sum_{k=1}^{N} w_{jk} \,. \tag{4}$$

Also in this case, the weights w_{jk} depend on the distance r_{jk} and are zero outside I_j. In our work, first and second degree polynomials were used as approximating functions, while the influence radius was determined locally so that a suitable number of stations was included in I_j.

Algorithms combining polynomial fitting and weighted averages as in eq. (1) were developed by Maude [20] and McLain [21, 22]. In Maude's method, each f_k is a second degree polynomial that is fitted exactly to the observed values on S_k and on the 5 nearest stations. So, I_k can be defined as a circle having as radius the distance of the fifth stations, while I_j includes all the stations for which $P_j \in I_k$. C_j is obtained as a weighted average of the values assumed on P_j by all the polynomials defined in a circle including P_j.

McLain suggested a triangulation of the whole spatial domain, using the stations as vertices of the triangles. As in Maude's algorithm, a second degree polynomial is fitted in the neighbourhood of each station, but more than 5 data and the least-squares technique are used to define the polynomials. C_j is evaluated as a weighted average of the three functions f_k corresponding to the vertices of the triangle in which P_j is included.

Following the basic ideas suggested by these works, we tested an interpolation scheme in which each f_k is a first or second-degree polynomial fitted by the least-squares technique to the data included in a circular neighbourhood of each station. The size of I_k was chosen so that a fixed number of stations had a non-zero weight in the determination of f_k. Also each I_j is a circular neighbourhood, but its radius R_j was determined independently from the size of the neighbourhood of the stations. So the value C_j is given by eq. (1), where a Gaussian dependence on the distance was assumed for the weights and the radius of I_j was chosen according to the results obtained using constant functions $f_k = C_k$.

All the methods described above are single-step methods. The last algorithm tested in our work is a multiple-step method based on successive corrections of a "first guess" field. Let $C_j^{(1)}$ and $C_k^{(1)}$ be the values assumed by the first guess field on the point P_j and on the station S_k. The first correction step can be written as

$$C_j^{(2)} = C_j^{(1)} + \alpha^{(1)} \sum_{k=1}^{N} w_{jk}^{(1)} \, (C_k - C_k^{(1)}) \Big/ \sum_{k=1}^{N} w_{jk}^{(1)} , \tag{5}$$

where $\alpha^{(1)}$ $(0 < \alpha^{(1)} \leqslant 1)$ is a coefficient depending on the relative weight assigned to the first guess. If the approximation given by eq. (5) is computed also for each station S_k, eq. (5) can be used iteratively to compute (at the m-th step)

$$C_j^{(m+1)} = C_j^{(m)} + \alpha^{(m)} \sum_{k=1}^{N} w_{jk}^{(m)} \, (C_k - C_k^{(m)}) \Big/ \sum_{k=1}^{N} w_{jk}^{(m)} . \tag{5a}$$

Usually both the influence radius R_j and the coefficient $\alpha^{(m)}$ are decreased at each step.

The successive correction method was introduced in meteorology by Bergthorsson and Döös [2] and Cressman [8]. Bergthorsson and Döss used a weighted average of the climatology and the forecast from a previous analysis as a first guess for the 500 mb height field, while Cressman's first guess fields for different variables and levels were essentially based on the 12-hour forecast.

Since none of the approximation methods previously discussed uses external information in addition to the data reported by the stations, we decided to test a simple version of the successive correction method in which the first guess is simply a weighted average of these data computed with a large influence radius.

3. Data Set and Criteria of Comparison

The data set used for the comparison of the local interpolation schemes is composed by daily observations of temperature and geopotential height at 850 mb, extracted from the upper-air soundings of 71 European stations; it covers a two year period, from 1 January 1980 to 31 December 1981. These data were chosen because of their high signal-to-noise ratio, their uniform reliability and the fact that the order of magnitude of the random measurement errors is known, their root-mean-square (r.m.s.) values being about 1 °C for temperature and 10 m for geopotential height. All these factors are important when, as in this case, one wants to compare the results of the approximations with direct observations.

To perform the comparison among the different algorithms, a subset of 19 stations (divided into three groups) was chosen. The test of each method consisted in three runs of the corresponding FORTRAN program; in each run, the data from a group of stations were eliminated from the sample and the approximations were computed on the corresponding points.

The choice of the 19 stations was made in the following way. For each station $S_k = (x_k, y_k)$ in the total sample, the 10 nearest stations were considered ($S_{lk}, l = 1, \ldots, 10$). An index of the asymmetry of the geographical distribution of these stations around S_k was computed

$$AS_k = 0.1 \left| \sum_{l=1}^{10} (S_{lk} - S_k) \,/\, |S_{lk} - S_k| \right| . \tag{6}$$

Then, 19 stations were selected so that they could represent the whole resulting range of variability of AS, and were divided into three groups so

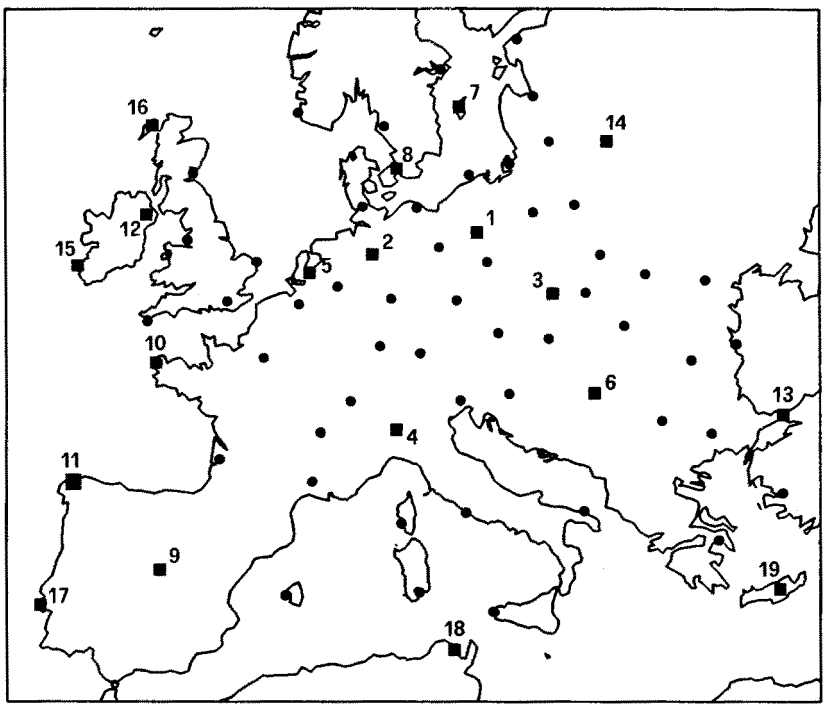

Fig. 1. European stations used as data sources; the numbered squares indicate the stations chosen for the verification of the approximation methods

that the stations in a given group had the greatest possible distance from each other.

Fig. 1 shows the location of the 71 meteorological stations, and in particular of the 19 stations selected for the comparison. Their AS indices are listed in Table 1. The stations can be grouped into 4 classes according to this index:

$$\begin{aligned} \text{I class:} \quad & 0. \leqslant AS < 0.25 \\ \text{II class:} \quad & 0.25 \leqslant AS < 0.50 \\ \text{III class:} \quad & 0.50 \leqslant AS < 0.75 \\ \text{IV class:} \quad & 0.75 \leqslant AS < 1\,. \end{aligned}$$

Table 1 also shows how many of the 71 stations belong to each class.

In the following sections, the skill of the different methods will be evaluated comparing the r.m.s. errors computed for each selected station from the time series of the observed and the approximated values. For the most significant tests, the results will be shown in Tables 2a/2b where, for each AS class, the range of variability of the r.m.s. error among the selected stations will be indicated.

Table 1. *Percentages of Stations in the Four Asymmetry Classes and Asymmetry Indices of the Selected Stations*

AS class (% of stations)	Selected stations	AS index
I	1	0.077
(37%)	2	0.115
	3	0.149
	4	0.188
	5	0.217
II	6	0.278
(28%)	7	0.334
	8	0.392
	9	0.425
	10	0.468
III	11	0.535
(25%)	12	0.555
	13	0.629
	14	0.666
	15	0.733
IV	16	0.830
(10%)	17	0.833
	18	0.871
	19	0.934

4. Description of the Tests

In this section, a detailed description of the way in which the methods described in section 2 were applied to our data sample will be given. In order to avoid confusion, in the following we shall use (as in section 2) the symbol P_j to indicate the geographical points corresponding to the stations on which the results of the approximations were computed, whereas we shall indicate as S_k all the other stations used as data sources. We shall also use the symbol $\underline{r}_{jk}$ for the vector pointing from P_j to a surrounding station S_k, while $r_{jk} = |\underline{r}_{jk}|$ will be, as usual, the distance between P_j and S_k.

a) *Distance-Weighted Averages*

The simple method described by eq. (2) was tested with different choices for the weights and the influence radius R_j. In a first series of tests, constant

values were assumed for the influence radius, and two weighting functions were chosen

$$w_1(r_{jk}) = \begin{cases} R_j^2/r_{jk}^2 & 0 < r_{jk} \leqslant R_j/2 \\ 16\,(1 - r_{jk}/R_j)^2 & R_j/2 < r_{jk} \leqslant R_j \\ 0 & r_{jk} > R_j \end{cases} \tag{7a}$$

$$w_2(r_{jk}) = \begin{cases} \exp(-4\, r_{jk}^2/R_j^2) & 0 \leqslant r_{jk} \leqslant R_j/2 \\ 4\, e^{-1}\,(1 - r_{jk}/R_j)^2 & R_j/2 < r_{jk} \leqslant R_j \\ 0 & r_{jk} > R_j \end{cases} \tag{7b}$$

The weighting functions are continuous and derivable in $r_{jk} = R_j/2$ and $r_{jk} = R_j$: this fact ensures a smooth variation of the approximated field when a new station crosses the boundary of the influence circle. Both have the same dependence on distance when $r_{jk} > R_j/2$, but they give different results close to the stations. As can be seen in Goodin et al. [14, 15], the eq. (7a) produces an approximated field that tends to the observed value C_k when the point P_j approaches a station S_k; besides, the first derivatives of the field tend to zero. Conversely, there is no exact fit of the data using eq. (7b).

With regard to the length of the influence radius three values of 500, 600 and 700 km were used, according to the results of Stephens and Stitt [26]. They found that the optimum influence radius for the first step of a correction method with a constant value as a first guess ranges between d and $2d$, where d (the average separation between 2 stations) is the square root of the ratio between the total area of the domain and the number of stations.

Six tests were so performed, using all the possible combinations of the selected weights and radii. Comparisons of the results obtained with the same radius showed no significant difference between the two weighting functions, probably because very few control points were located close to the stations used as data sources. On the other hand, the best influence radius was different from point to point.

So, two further tests were performed, using the two weights and an influence radius dependent on the coordinates (x_j, y_j) of the point in a polar stereographic projection. A simple expression for $R_j = f(x_j, y_j)$ was deduced in the following way:

– the domain was divided into three areas, each one with nearly uniform density of stations;

– a value $R = 1.6\, d$ was computed for each area, again according to the work of Stephens and Stitt [26];

– a first order polynomial was fitted to these three values, assumed as representative of the centroids of the subdomains.

We obtained

$$R_j = 676. - .106\, x_j - .210\, y_j\ , \tag{8}$$

where x_j and y_j are respectively measured in the eastward and northward direction, the origin is the point (10°E, 45°N) and all the distances are expressed in kilometers.

The two tests gave better results than the previous ones, and again showed very little differences between the two weights. In some cases, the first function (eq. (7a)) showed a slightly better performance; however, this type of weight was not suitable for most of the other approximation methods because of the exact fit over the stations, so in all the following tests the weights (eq. (7b)) were used.

The results of the weighted average method with $R(x_j, y_j)$ given by eq. (8) and Gaussian weights (eq. (7b)) are indicated as WA in Tables 2a/2b. Since the differences between the r.m.s. errors obtained with the two weighting functions were usually as small as 0.1°C or 1 m, these results are representative of the performances of both the weights.

a) *Averages with Direction-Dependent Weights*

In a successive test, the weights were modified in order to take into account the asymmetric distribution of the stations S_k around the points P_j. The correction factor was derived by Boone and Samuelson [3], with small modifications. If $\underline{v}_{jk} = \underline{r}_{jk}/r_{jk}$ is the unit vector pointing from P_j to S_k, and $\overline{\underline{v}}_j$ is a weighted mean of these vectors inside I_j

$$\overline{\underline{v}}_j = \Sigma_k\ w\,(r_{jk})\,\underline{v}_{jk} / \Sigma_k\ w\,(r_{jk}) \tag{9}$$

then the direction-dependent weights can be defined as

$$W^*(\underline{r}_{jk}) = w\,(r_{jk})\,(1 - c \cdot \underline{v}_{jk} \cdot \overline{\underline{v}}_j) \tag{10}$$

The value of the constant c ($\leqslant 1$) can be chosen according to the importance that one wants to give to the directional term. We chose $c = 0.5$, so that in a highly asymmetric case ($|\overline{\underline{v}}_j| \sim 1$) the actual weights can range between $0.5 \cdot w(r_{jk})$ (for stations along the maximum-density direction) and $1.5 \cdot w(r_{jk})$. Clearly, if the station distribution is completely symmetric ($\overline{\underline{v}}_j = \underline{0}$) the directional factor is 1 for all the stations.

This test was performed using the Gaussian weights $w(r_{jk})$ given in eq. (7b) and the geographically dependent influence radius R_j defined by eq. (8); its results are indicated as DWA in Tables 2a/2b.

c) *Least-Squares Polynomials*

The distance-weighted least-squares technique summarized by eqs. (3) and (4) was applied to fit first and second degree polynomials in the neighbourhood I_j of each point P_j. Around each point, the stations were ordered with

increasing distance, and the influence radius R_j was defined as the distance of the sixth station (for the first order) and of the eleventh station (for the second order polynomials); in this way, respectively 5 and 10 stations contributed to the definition of the polynomials. The Gaussian weights (eq. (7b)) were used for these two tests, the results of which are indicated as LS1 and LS2 in Tables 2a/2b.

d) *Weighted Averages of Least-Squares Polynomials*

As already described in section 2, the weighted-average and the least-squares technique can be combined as in eq. (1), where the functions f_k are polynomials fitted in the neighbourhood I_k of each station S_k, and the values of these polynomials on P_j are then averaged according to the distance r_{jk}. Again we used first and second degree polynomials as fitting functions, and each neighbourhood I_k was defined so to include respectively 5 and 10 stations in addition to the one in the centre of I_k. On the other hand, for the weighted averages the influence radius was defined according to eq. (8).

The Gaussian weights (eq. (7b)) were used both in the computation of the polynomials and in the averages of their values, with different influence radii.

The results of these two tests are indicated as WLS1 and WLS2 in Tables 2a/2b.

Table 2. a) *Range of Variability of 850 mb Temperature r.m.s. Errors for the Eight Local Approximation Methods Tested in This Study.* The results are subdivided according to the classes defined in the text. b) As in a), but for 850 mb geopotential height

AS class	WA	DWA	LS1	LS2	WLS1	WLS2	SC	DSC
a) 850 mb temperature (°C)								
I	1.1–3.1	1.0–2.8	1.0–2.4	1.2–2.8	1.1–3.1	1.1–2.8	1.0–3.0	1.0–2.8
II	1.3–3.2	1.3–3.2	1.4–3.1	1.3–3.8	1.4–3.6	1.3–3.8	1.3–3.1	1.3–2.9
III	1.6–4.0	1.7–3.2	1.5–3.7	1.9–5.9	1.6–3.5	1.6–6.1	1.5–3.6	1.5–3.4
IV	2.4–3.7	2.5–3.7	3.1–5.4	5.8–14.8	2.8–4.8	5.0–11.4	2.5–3.2	2.3–3.3
Total	1.1–4.0	1.0–3.7	1.0–5.4	1.2–14.8	1.1–4.8	1.1–11.4	1.0–3.6	1.0–3.4
b) 850 mb height (m)								
I	8.–18.	7.–16.	8.–16.	7.–16.	7.–19.	8.–17.	6.–17.	6.–16.
II	12.–22.	10.–18.	9.–16.	9.–22.	9.–21.	8.–23.	9.–15.	9.–16.
III	15.–35.	13.–35.	11.–39.	10.–53.	9.–27.	9.–58.	11.–29.	11.–30.
IV	18.–36.	18.–39.	21.–53.	40.–127.	20.–37.	28.–86.	15.–27.	16.–29.
Total	8.–36.	7.–39.	8.–53.	7.–127.	7.–37.	8.–86.	6.–29.	6.–30.

e) *Successive Corrections*

The successive correction method (eqs. (5) and (5a)) was tested performing three weighted-average steps with the following influence radii:

first guess:	$R_j = 2000$ km
1st correction:	$R_j = 1000$ km
2nd correction:	$R_j = 500$ km

Other tests were carried out, with a lower (i.e. closer to 1) ratio between the influence radii in two successive steps or with radii of 2400, 1200 and 600 km, but their results were considerably worse.
Because of the smoothness both of the first guess and of the first correction, the smallest errors were obtained with $\alpha^{(1)} = \alpha^{(2)} = 1$. The Gaussian weights were used both in the simple distance-dependent form (eq. (7b)) and with the directional factor (eq. (10)). The results of these last two tests are indicated respectively as SC and DSC in Tables 2a/2b.

5. Discussion on the Results of Different Methods

As described in section 3, Tables 2a/2b summarize the results of our tests showing the range of r.m.s. errors among the 19 stations chosen as reference points; the minimum and maximum values for the whole group and for the 4 classes defined according to the asymmetry index are presented.
Three clear indications emerge from the comparison of these results:
– The methods based on the simple least-squares polynomial fitting (LS1 and LS2) give very high errors where the distribution of the stations as asymmetric (AS > 0.5); besides, the performance of this technique is worse using 2nd degree rather than first degree polynomials. When the values of the polynomials defined in the neighbourhood of each station are averaged to produce the final estimates (WLS1 and WLS2), the errors become much smaller, and the 1st degree polynomials give again better results. However, with the exception of WLS1 for the 850 mb height, these errors are still greater than those obtained with the algorithms based on simple weighted averages.
– The results obtained introducing a directional factor in the weights used both in the simple averages and in the successive correction method are contradictory. This factor seems to be more efficient in compensating for small asymmetries in the station distribution (AS classes I–II) than for large ones (AS classes III–IV). In the case of the geopotential height, the results of the directional weights in the last two classes are even slightly worse than those obtained with the simple dependency on the distance. Taking into account the increase in computational time (about a factor of 2) required by the directional weights, their introduction does not seem justified.

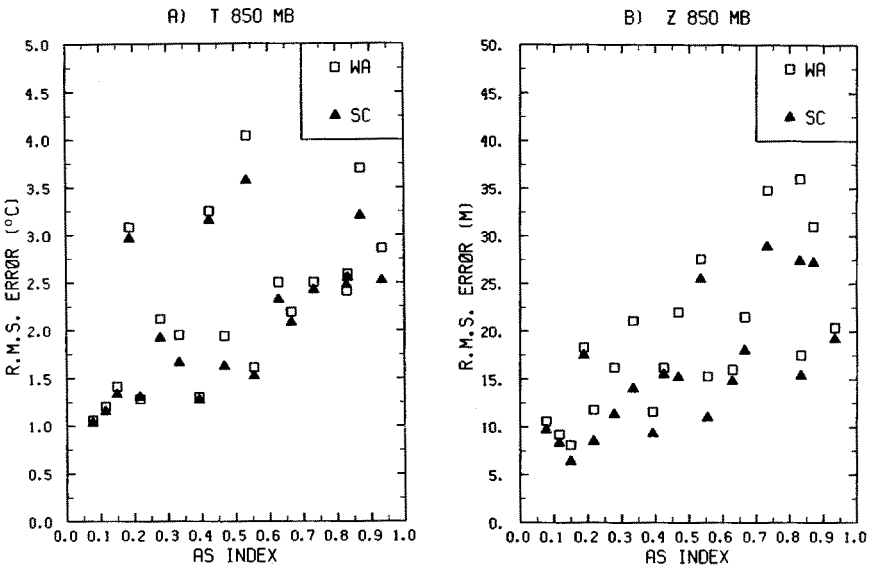

Fig. 2. a) R.m.s. errors of 850 mb temperature, plotted against the asymmetry index (AS) of the 19 stations selected for the verification. b) As in Fig. 2a, but for 850 mb geopotential height

– The successive correction method gives a constant improvement over the simple weighted-average technique, particularly relevant in the case of the geopotential height. This can be clearly seen in Fig. 2, where the r.m.s. errors obtained with the two techniques (using distance dependent weights) are plotted against the AS index of the 19 stations selected for the verification. At least for the geopotential, the significance of the improvement seems to justify the large increase in computational time required by the successive correction method (about a factor 6). In our opinion, despite this increase, the time required by this algorithm is acceptable for most applications.

In conclusion, a successive correction method based on three averaging steps with decreasing influence radii appears to be the most accurate among the local interpolation algorithms that we tested. However, if a very short computational time is required, a simple weighted average technique can also give good results, if the influence radius is properly defined taking into account the local density of stations. The errors of these methods show a clear dependence on the asymmetry of the station distribution; when this asymmetry is small (as for classes I and II, which include 65% of the total sample of 71 stations) the interpolation errors have an amplitude that, at least for the geopotential, is comparable with the observational accuracy.

6. A Comparison with ECMWF Analyses

In the previous section we have showed that local approximation methods based on one or more averaging steps, with a proper choice of the influence radii, can give r.m.s. errors that are acceptable if compared to the observational errors. As we said in the introduction, one must not expect the same accuracy that can be obtained by more sophisticated analyses based on multivariate statistical interpolation methods and on data sets with a wider and denser spatial coverage. But, since the principal applications of these fast and simple techniques are in the field of statistical analysis, we must check if at least the main statistical properties of a sample of fields analysed by means of these methods are comparable to the corresponding ones derived from more accurate methods.

To perform this test, we used the successive correction (SC) method to produce a sample of grid point fields of 850 mb height over the European area (from 36°N to 57°N and from 9°W to 27°E, resolution $3° \times 3°$, 104 points in total). All the available data from the 71 stations of Fig. 1 were used for this test, but the time period was limited to 184 days in October, November and December 1980 and 1981, in order to work on a more homogeneous sample. Data for the same variable, grid and period were also extracted from the analyses of the European Centre for Medium Range Weather Forecasts (ECMWF) and the statistical properties of the two sets were computed as follows.

For each sample a principal component analysis on the standardized fields was performed, and on the base of its results the value x_{jt} on the j-th grid point at time t was expressed as

$$x_{jt} = \mu_j + \sigma_j \cdot x'_{jt} \tag{11}$$

$$x'_{jt} = \sum_{i=1}^{104} c_{it} A_{ij} , \tag{11a}$$

where μ_j is the mean value on the j-th point in the 184-day period; σ_j is the corresponding standard deviation; x'_{jt} is the standardized value of x_{jt}; A_{ij} is the component of the i-th eigenvector of the temporal correlation matrix corresponding to the j-th grid point; c_{it} is the value of the i-th principal component (PC) at time t.

A first comparison can be done on the means and the standard deviations computed from the two data sets. These fields and their differences (SC-ECMWF) are shown in Fig. 3. One can see that the mean field is well reproduced by the successive correction method, the mean error being (in absolute value) less than 5 m over the most part of the selected area; the greatest error obviously occur over the seas where the values are extrapolated from the coastal regions.

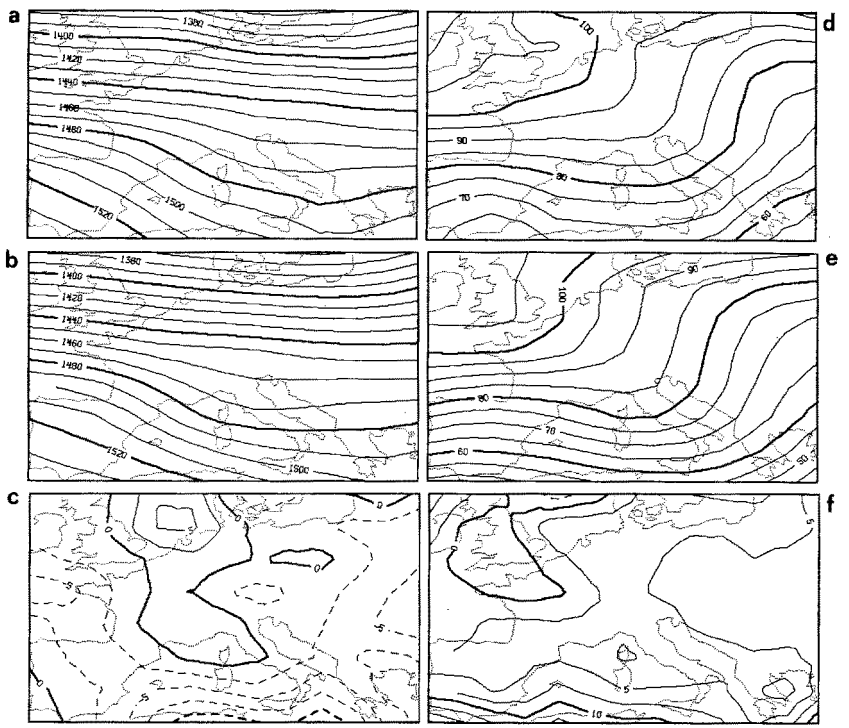

Fig. 3. Comparison of statistics for two samples of grid point fields of 850 mb height in the period October–December 1980/October–December 1981, the first one interpolated by the SC method from the data of 71 stations, the second one deduced from ECMWF analyses: a) mean field, SC; b) mean field, ECMWF; c) mean difference, SC-ECMWF; d) standard deviation, SC; e) standard deviation, ECMWF; f) standard deviation difference, SC-ECMWF

A more systematic difference between the two samples can be found in the standard deviation field. The successive correction method generates higher standard deviations over nearly whole Europe, and tends to reduce the differences between the northern and the southern regions. Even though the largest errors (~ 10 m) occur close to the southern border of the domain where very few data are available, an overestimation of the order of a few metres also occurs in areas where the observational network is dense. Despite these differences (which are on average less than 10% of the actual value) the standard deviation fields computed from the two samples show a clear resemblance, and also for this parameter the results of the SC method are satisfactory.

If we now examine the similarity of the standardized fields, we can see that

the increase in variance generated by the SC method has very little influence on the evaluation of the structure and the relative amplitude of the dominant modes of variation.
As in most of the meteorological applications of PC analysis, the standardized field can be approximated very well by its expansion on a limited number of eigenvectors. With a proper choice of this number (which is based on the distribution of the explained variance among the PCs), one can retain practically the whole significant part of the signal with a great reduction of the degrees of freedom in the spatial domain.

Table 3. *Percentages of Total Variance Explained by the First 10 PCs Derived from the Fields Generated by the Successive Correction Method (SC) and from the ECMWF Analyses.* Cumulative values in parentheses

PCs	SC	ECMWF
1	57.93 (57.93)	56.35 (56.35)
2	16.62 (74.55)	16.21 (72.56)
3	13.86 (88.41)	14.03 (86.59)
4	4.17 (92.58)	4.70 (91.29)
5	2.93 (95.51)	3.18 (94.47)
6	1.47 (96.98)	1.71 (96.18)
7	.93 (97.91)	.96 (97.14)
8	.58 (98.49)	.75 (97.89)
9	.32 (98.81)	.49 (98.38)
10	.25 (99.06)	.31 (98.69)

Table 3 shows the percentages of variance of the first 10 PCs deduced from the fields obtained by the SC method and from the ECMWF analyses. One can see that the distribution of variance among the two sets of PCs is very similar, and that in both cases a very good representation of the standardized field can be obtained by the first 6 components.

Obviously, it is not sufficient to compare the spatial structure of the first eigenvectors of the correlation matrix to test the similarity of the standardized field. First of all, Buell [5, 6] demonstrated that, over a limited area, the structure of the eigenvectors is strongly influenced by the boundaries and, consequently, by the shape of the domain; besides, also the differences between the time coefficients (i.e. the PCs) must be taken into account. So, a good measure of the similarity between the standardized fields can be obtained computing, for each grid point, the r.m.s. difference between the expansions of these fields on the leading eigenvectors deduced from the respective data.

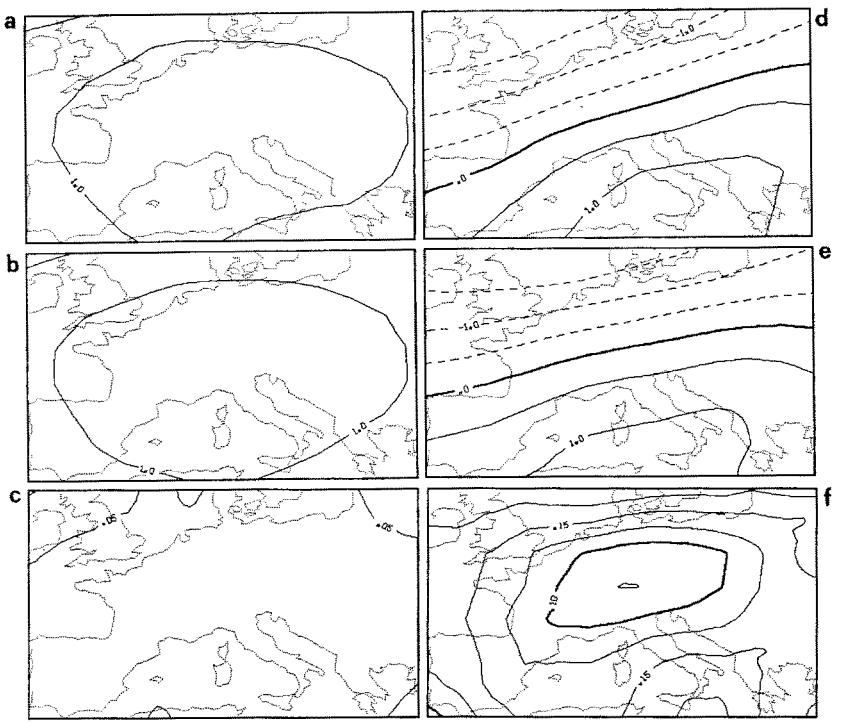

Fig. 4. a) 1st eigenvector of SC standardized fields; b) 1st eigenvector of ECMWF standardized fields; c) r.m.s. difference between the expansions of the standardized fields on the 1st eigenvector of SC and ECMWF samples; d) 2nd eigenvector of SC standardized fields; e) 2nd eigenvector of ECMWF standardized fields; f) r.m.s. difference between the expansions of the standardized fields on the first 2 eigenvectors of SC and ECMWF samples

If the superscripts S and E indicate respectively the SC and the ECMWF results, the r.m.s. difference between the standardized fields in the j-th grid point can be expressed as

$$\delta_{jN} = \left[\sum_{t=1}^{184} \left(\sum_{i=1}^{N} (c_{it}^{S} A_{ij}^{S} - c_{it}^{E} A_{ij}^{E}) \right)^2 \right]^{1/2}, \qquad (13)$$

where N is the number of selected eigenvectors and PCs.
We calculated δ_{jN} for each point and for $N = 1, \ldots, 6$. Since the r.m.s. amplitude of the standardized field is 1, the field generated by the successive correction method can be considered as a good approximation of that

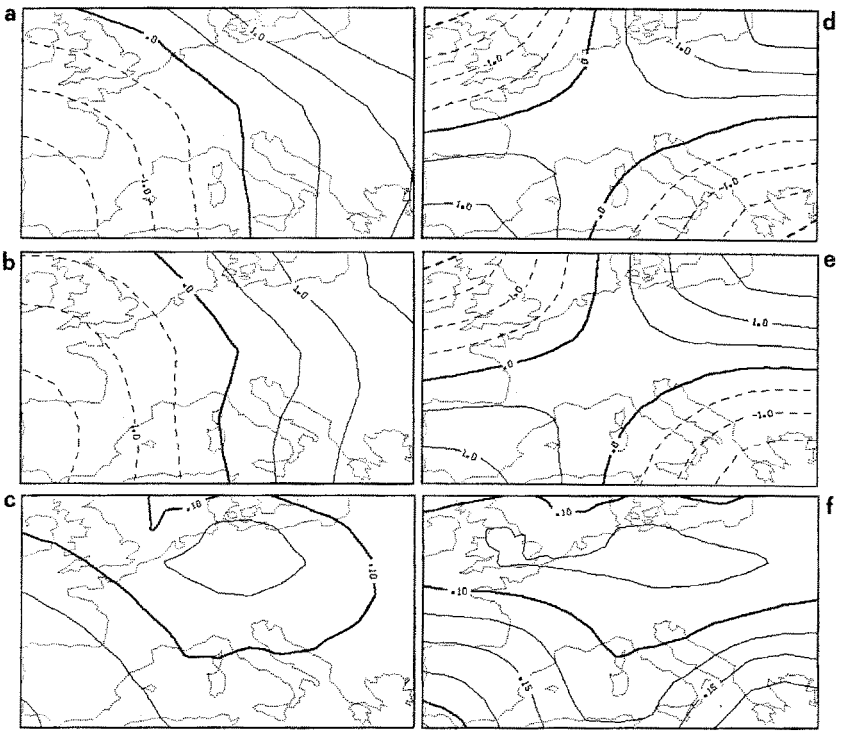

Fig. 5. a) 3rd eigenvector of SC standardized fields; b) 3rd eigenvector of ECMWF standardized fields; c) r.m.s. difference between the expansions of the standardized fields on the first 3 eigenvectors of SC and ECMWF samples; d) 4th eigenvector of SC standardized fields; e) 4th eigenvector of ECMWF standardized fields; f) r.m.s. difference between the expansions of the standardized fields on the first 4 eigenvectors of SC and ECMWF samples

derived from the ECMWF analyses if the r.m.s. difference does not exceed 0.2.

The results of this comparison are shown in Figs. 4, 5, 6 where the first 6 eigenvectors of each set and the distribution of δ_{jN} are plotted. One can see that the eigenvectors are nearly identical in the two samples, but the most important result is that δ_{j6} is below 0.2 over almost the whole area and is below 0.1 over Central Europe, where the radiosonde network is homogeneous.

We can so conclude that the statistically significant information contained in the sample of ECMWF analyses of 850 mb height is correctly reproduced in the grid point fields obtained by our simple version of the successive correction method; the most relevant deviation, in our opinion, is the

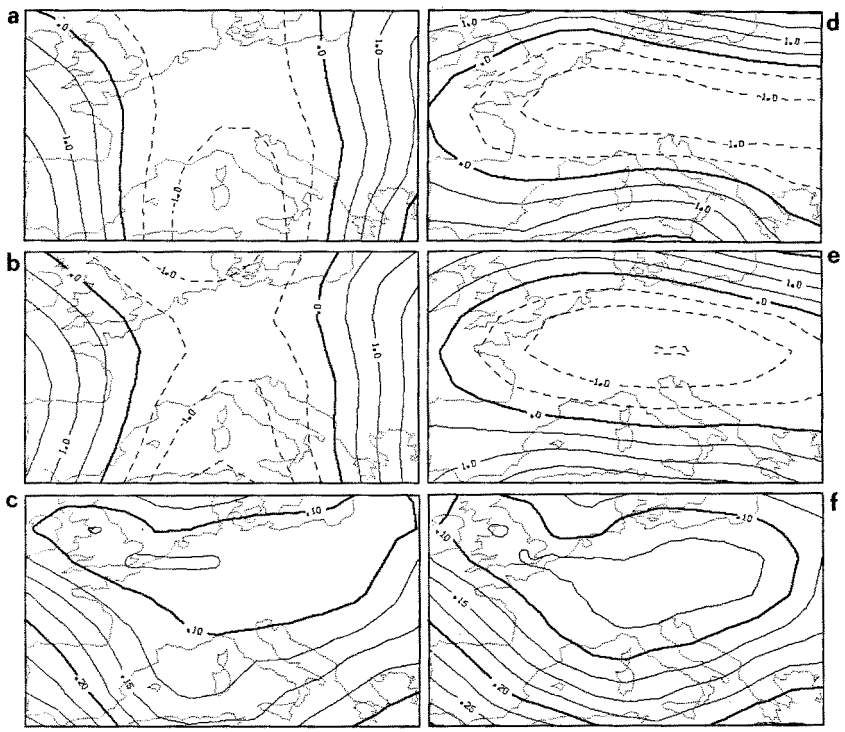

Fig. 6. a) 5th eigenvector of SC standardized fields; b) 5th eigenvector of ECMWF standardized fields; c) r.m.s. difference between the expansions of the standardized fields on the first 5 eigenvectors of SC and ECMWF samples; d) 6th eigenvector of SC standardized fields; e) 6th eigenvector of ECMWF standardized fields; f) r.m.s. difference between the expansions of the standardized fields on the first 6 eigenvectors of SC and ECMWF samples

increase in the standard deviation of the field, which, however, is less than 10% of the actual value over the most part of Europe. This means that this method can be satisfactorily used for the statistical and graphical applications indicated in the introduction, if the influence radii are properly chosen.

7. Conclusions

Several local approximation algorithms, including weighted averages, least-squares polynomials, a combination of these two techniques, and successive corrections, were tested in our work using data of temperature and geo-

potential height at 850 mb from 71 European radiosondes in a two-year period. The comparison of the r.m.s. differences between the observed data over 19 selected points and the values interpolated from the surrounding stations gave results that can be summarized as follows:

– The methods based on least-squares polynomials give much worse results than those based on weighted averages in one or more steps, particularly on points surrounded by an asymmetric distribution of observed data.

– For the methods based on weighted averages, a proper choice of the influence radius is much more important than the specification of the weighting function in determining the quality of the approximation. Using a single-step algorithm, the lowest errors were obtained with an influence radius dependent on the geographical coordinates, according to the density of stations in the area surrounding each point. Also the successive correction method proved to be very sensitive to the choice of the radii in the different steps, and particularly to the ratio between them. Conversely, no significant improvement was obtained using a directional weighting function depending not only on distance but also on the distribution of the surrounding stations.

– Among all the algorithms, a successive correction method based on a weighted average with a large influence radius as a first guess, two correction steps with decreasing influence radii, and weights having a Gaussian dependence on distance, proved to give the best results with a reasonable computational time. However, good results can be obtained also with a single-step weighted average using, as previously indicated, an influence radius dependent on the local density of station, with very low computational time.

– The results of these two methods show a clear dependence on the asymmetry of the local distribution of data, which is particularly evident for the geopotential height. When the asymmetry is small, the amplitude of the approximation errors is comparable to the observational accuracy; when the asymmetry is large the approximation error is intermediate between the observational error and the standard deviation of the variable in a seasonal sample.

A subsequent comparison between a sample of grid-point fields generated by the successive correction method from the radiosonde geopotential data in 184 winter days and corresponding ECMWF analyses showed that the successive correction algorithm provides very good estimates of the main statistical properties of the analysed field: namely, the mean field, the standard deviations, the leading eigenvalues and eigenvectors of the correlation matrix and the time dependent principal components that explain the most part (over 95%) of the variance of the sample. The errors in these quantities are small over the whole domain, with an obvious increase over areas with very few data; the only difference that has some relevance is an

increase in the standard deviation of the field (generally about or less than 10%) that tends to reduce the latitudinal gradient of the variance.
In conclusion, this simple version of the successive correction method, even with no external first guess, appears to be adequate for most of the statistical and graphical applications of meteorological data analysis. It is obvious that more sophisticated methods are required when a reliable physical and dynamical description of the three-dimensional structure of the atmosphere is required for diagnostic or prognostic studies.
From our results it is evident that a careful choice of the influence radii is fundamental to obtain the best results for any given data set. The magnitude of the radius in the first step must not exceed the distance at which the correlation between data over different points loses its significance, while the radius of the last correction must be of the order of the average distance between the data sources. Since a good value for the ratio of the radii in two successive corrections is about 2, these factors also determine the maximum number of significant corrections. However, the method can be easily implemented through a computer program in which, depending on the application, the actual number of steps and the corresponding influence radii can be varied in order to reach the most convenient comprise between accuracy and computational costs.

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