A NOTE ON FREE CONVECTION*

J. A. BUSINGER

Dept. of Atmospheric Sciences, University of Washington, Seattle, Wash., U.S.A.

(Received 8 August, 1972)

Abstract. The limiting condition of $u_* \to 0$ in the unstable atmospheric boundary layer is usually referred to as 'free convection'. Some of the similarity laws that are proposed for this condition do not agree with experiment. A mechanism is proposed in this paper to show why the asymptotic free convection condition is never completely reached near the surface. It is shown that local turbulent shear production plays an important role in shaping the temperature profile even when the average velocity is zero.

1. Introduction

The condition of free convection in the atmospheric boundary layer is often defined as the condition where all the turbulent energy is generated by buoyancy forces and where the mean horizontal wind vanishes. This, of course, seldom occurs in the atmosphere, but the asymptotic behavior of the turbulent parameters defining the structure of the boundary layer when free convection is approached is of considerable interest. Surface layer similarity as originally proposed by Obukhov (1946, 1971) states that the mean temperature and wind fields depend only on the surface heat flux H, the surface stress τ_0, the buoyancy parameter g/T and the height from the ground z. This led to the well known velocity, temperature, and length scales:

$$u_* \equiv \left(\frac{\tau_0}{\varrho}\right)^{1/2}; \qquad \theta_* \equiv \frac{-H}{c_p \varrho u_*},$$

and

$$L \equiv -\frac{u_*^3 c_p \varrho T}{kHg}$$

(the Obukhov length), respectively, where
g = acceleration due to gravity
c_p = specific heat at constant pressure
H = heat flux density = $c_p \varrho \overline{w'\theta'}$
ϱ = density
T = temperature.
In free convection, $H > 0$ and $u_* \to 0$, and these scales lose their significance. In a recent paper, Wyngaard *et al.* (1971) point out that alternate velocity and temperature scales may be formulated by

$$u_f = \left(z \frac{g}{T} \frac{H}{c_p \varrho}\right)^{1/3} \quad \text{and} \quad \theta_f = \left\{\frac{T}{gz}\left(\frac{H}{c_p \varrho}\right)^2\right\}^{1/3}.$$

* Contribution No. 267, Dept. of Atmospheric Sciences, University of Washington.

They also point out that the dimensionless quantities scaled with these parameters must become constants. So they find, e.g.,

$$\frac{\sigma_\theta}{\theta_*} = \frac{\sigma_\theta}{\theta_f}\frac{\theta_f}{\theta_*} = \text{const.} \quad \zeta^{-1/3},$$

where $\zeta = -z/L$. This agrees well with observations. In fact, the asymptotic behavior commences for remarkably small values of ζ. On the other hand, quantities such as

$$\phi_h = \frac{kz}{\theta_*}\frac{\partial\bar{\theta}}{\partial z} \quad \text{and} \quad \phi_m = \frac{kz}{u_*}\frac{\partial\bar{u}}{\partial z}.$$

do not behave in the predicted way, i.e.,

$$\phi_h = \frac{kz}{\theta_f}\frac{\partial\bar{\theta}}{\partial z}\cdot\frac{\theta_f}{T_*} \propto \zeta^{-1/3} \tag{1}$$

and

$$\phi_m = \frac{kz}{u_f}\frac{\partial\bar{u}}{\partial z}\cdot\frac{u_f}{u_*} \propto \zeta^{1/3}. \tag{2}$$

Observations indicate powers of ζ to be $-\frac{1}{2}$ and $-\frac{1}{4}$, respectively (Businger *et al.*, 1971), for these two equations. Of course, it is possible that in these cases the observations did not include large enough values of ζ. In the following development, we try to shed some light on this problem.

2. Convection-Induced Stress*

In the limiting case when $\bar{u}=0$, also $\overline{u'w'}=0$ and therefore $u_*=0$. Still, if we make observations close enough to the ground, we will find fluctuations in u of sufficient duration that a stress exists locally for some time and therefore also a shear production of turbulent energy. Consider the case of a large uniform area with free convection extending over the entire planetary boundary layer up to a height h. Averaged over the horizontal area, the mean wind speed $\bar{u}=0$, consequently $u_*=0$ and all the available turbulent energy has been generated by the buoyancy force. Consider now the layer close to the surface over a relatively short period compared to the large-scale convection but over a relatively long period compared to the time it takes to develop a local wind profile. With this wind profile, we can observe a stress and a shear production of turbulent energy. This local temporary wind profile cannot be distinguished in characteristics from a true mean wind profile. This local flow leads us to believe that there is, averaged over the entire horizontal area, a mean shear production of turbulence which is not related to a mean wind but to the convective circulation in the boundary layer. It is reasonable to assume that the characteristic friction velocity w_* generated in this way is proportional to the convective scaling velocity u_{f0},

$$u_{f0} = \left(\frac{g}{T}\overline{w'\theta'}h\right)^{1/3}, \tag{3}$$

* A somewhat similar idea was formulated by Kraichman (1962), 'Turbulent thermal convection at arbitrary Prandtl Number', *Phys. of Fluids* **5**, 1374–1389.

where h is the height of the boundary layer. Deardorff (1970) finds that u_{f0} is the significant velocity scale in his numerical convection model. The coefficient of proportionality between w_* and u_{f0} will depend on the roughness of the surface in a similar way as u_* is related to the geostrophic wind u_g, with the difference that in this case the height h is an independent parameter, in contrast with Tennekes' (1970) opinion and in the sense that Deardorff (1970) uses it. We postulate now that

$$w_* = u_{f0} f\left(\frac{h}{z_0}\right), \tag{4}$$

where $f(h/z_0)$ is a not yet specified function which decreases with increasing h/z_0. It is assumed that the scale of the convective motion is small enough to neglect the effect of the Earth's rotation.

The shear production of the turbulent energy will now be proportional to w_*^3/z in analogy to the neutral case: u_*^3/kz. And the ratio of the buoyant production over the shear production is proportional to

$$\frac{g}{T}\frac{\overline{w'\theta'}z}{w_*^3} = a\left\{f\left(\frac{h}{z_0}\right)\right\}^{-3}\cdot\frac{z}{h}. \tag{5}$$

The interesting although expected result is that this ratio is not dependent on the magnitude of the heat flux. It also suggests that there is a lower limit to the Obukhov length, provided this length is defined as

$$L_{\min} = -\frac{w_*^3 T}{kg\overline{w'\theta'}} = -\frac{w_*^3 c_p \varrho T}{kgH}. \tag{6}$$

In this case

$$L_{\min} = -\frac{h\left\{f\left(\frac{h}{z_0}\right)\right\}^3}{a}, \tag{7}$$

which suggests that lower values of $L_{\min}$ may be observed over smooth than over rough terrain.

3. Consequences for Free Convection Similarity

If the previous arguments hold, the basic quantities that describe the structure of the free convection atmosphere are H, w_*, g/T and z, which is a set rather similar to the case when $u_* > 0$. Thus we have the following scaling temperature, scaling velocity and length:

$$\theta_*' = \frac{-H}{c_p \varrho w_*}; \quad w_*; \quad L_{\min}. \tag{8}$$

The argument that for large $z/-L_{\min} = \zeta$, the relative effect of w_* becomes small remains valid just as the relative effect of u_* becomes small when ζ is large. These conditions may be considered 'local free convection' as Tennekes (1970) suggested.

For local free convection, the scales u_f and θ_f remain important. This may be the reason why σ_w and σ_θ scale well with u_f and θ_f, respectively, because they are essentially local parameters.

On the other hand, the mean profiles are strongly linked to the structure near the surface because they reflect the integration from the surface to the height of observation. So for true free convection we have

$$\phi_h \to \phi'_h \equiv \frac{kz}{\theta'_*} \frac{\partial \bar{\theta}}{\partial z} \tag{9}$$

and

$$\phi_m \to \phi'_m \equiv \frac{kz}{w_*} \frac{\partial \bar{u}}{\partial z} \to 0 \tag{10}$$

because $\partial \bar{u}/\partial z \to 0$ when properly averaged.

The specific form of (9) may be expected to be the same as in the diabatic case, i.e.,

$$\phi'_h \propto \zeta'^{-1/2}.$$

See Elliott (1966) and Businger *et al.* (1971). And there is no longer a compelling reason why the $-\frac{1}{3}$ power law should be valid.

4. Conclusions

The experimental information needed to test the validity of these arguments is not easy to obtain. It will be necessary to determine the height h of the boundary layer throughout the experiments; to have conditions that closely approximate the free convection regime and to have at least two sites with very different roughnesses. Also, averages obtained with fixed-point observations become doubtful under truly free convection conditions. It will be necessary to have horizontally moving probes similar to the laboratory set up of Deardorff and Willis (1967) in their convection chamber.

References

Businger, J. A., Wyngaard, J. C., Izumi, Y., and Bradley, E. F.: 1971, 'Flux Profile Relationships in the Atmospheric Surface Layer', *J. Atmospheric Sci.* **28**, 181–189.

Deardorff, J. W.: 1970, 'Convective Velocity and Temperature Scales for the Unstable Planetary Boundary Layer and for Rayleigh Convection', *J. Atmospheric Sci.* **8**, 1211–1213.

Deardorff, J. W. and Willis, G.: 1967, 'Investigations of Turbulent Convection Between Horizontal Plates', *J. Fluid Mech.* **28**, 675–704.

Elliott, W. P.: 1966, 'Daytime Temperature Profiles', *J. Atmospheric Sci.* **23**, 678–681.

Obukhov, A. M.: 1946, 'Turbulence in an Atmosphere with a Non-Uniform Temperature', *Trudy Inst. Teoret. Geofis. Akad. Nauk S.S.S.R.*, No. 1. (Translation in: *Boundary-layer Meteorol.* **2**, 7–29).

Tennekes, H.: 1970, 'Free Convection in the Turbulent Ekman Layer of the Atmosphere', *J. Atmospheric Sci.* **27**, 1027–1034.

Wyngaard, J. C., Coté, O. R., and Izumi, Y.: 1971, 'Local Free Convection, Similarity, and the Budgets of Shear Stress and Heat Flux', *J. Atmospheric Sci.* **28**, 1171–1182.