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FRACTURE TOUGHNESS CHARACTERISTICS OF LAMINATED COMPOSITES

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Parameters characterizing the resistance of laminated composites to interlaminar fracture are discussed. The properties of the specific interlaminar fracture work, i.e., the amount of work spent on the formation of a unit of new surface of interlaminar crack, were examined. Taking account of the anisotropy of the material, this work may be characterized using a matrix. Upon change in the direction of crack growth, the matrix elements are transformed similarly to the components of a symmetrical second rank tensor. An interpretation is offered for the matrix elements. The proposed theoretical model was in accord with our experimental results.

The properties of most composite materials have highly pronounced anisotropy [1]. This is true in regard to their fracture toughness. The load parameters and active generalized forces (stress rate coefficients, strain energy release rate and J integral) depend on the direction of crack propagation. Thus, the J integral for two cases of interlaminar crack growth in a unidirectional composite, namely, along and transverse to the fibers, differs significantly (Fig. 1). This extends entirely to the fracture toughness indices of the material independently of whether they are given in stress rate coefficients or other terms.

Let us examine the specific interlaminar fracture work for laminated composites, i.e., the work spent to form a unit of new surface of an interlaminar crack. Experiments to determine this term are relatively simple. Thus, relative to tear fracture (mode I), testing rectangular samples with a prior interlaminar notch for stripping is sufficient. The specific fracture work is defined as the relative work increment ΔW of the loading force to the increment of crack area ΔA. This term is necessary, for example, in predicting surface delamination in composite structures [2-6].

Let us study closed delamination in a composite plate upon compression in the plane of the plate. Let us assume for simplicity that the delamination boundary is smooth and that it is given in polar coordinates (Fig. 2). The specific interlaminar fracture work γ varies along the delamination perimeter as a function of polar angle φ. However, in its physical sense, this term should depend on the direction, in which delamination progresses at each point of the contour. It would seem that small increments in the delamination dimensions occur due to displacements of the contour points in the direction of the external normal n to the contour. For example, if the composite is reinforced unidirectionally by fibers along the x-axis, then work γ depends on angle θ, which is formed by the normal with the x-axis (see Fig. 2). When $\varphi = \theta = 0$, interlaminar fracture occurs as schematically represented in Fig. 1a, while fracture occurs as shown in Fig. 1b when $\varphi = \theta = \pi/2$. The specific interlaminar fracture work in the case of intermediate angles lies between the maximum (when $\theta = 0$) and minimal values (when $\theta = \pi/2$).

Hence, the specific fracture work is a type of tensor quantity. At first glance, this is not in accord with the usual concept of work as a scalar quantity. However, these values are taken for a fixed direction of crack propagation. The fracture areas in Figs. 1a and b are equal but the work spent on crack formation is quite different. The reason for this behavior lies in the complex fractographic pattern, which entails multiple cracking, damage to the matrix and matrix—fiber boundary, and pulling and tearing of the fibers.

We would expect that the specific fracture work is a type of tensor quantity even if it is limited by one type of fracture, in particular, interlaminar tear. Since the fracture work in the isotropic case is characterized by a single scalar term, it is reasonable to assume that it will be a second rank tensor in the anisotropic case. Until now neither theoretical nor experimental

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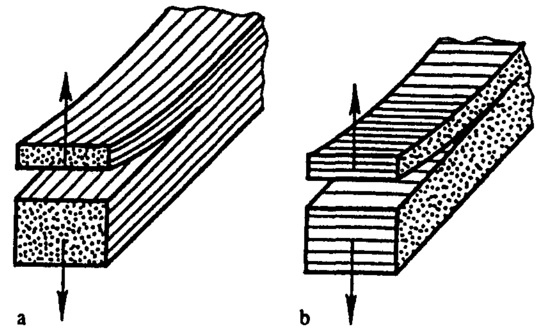

Fig. 1. Fracture in Mode I of a unidimensional fiber composite upon crack propagation along (a) and across the fibers (b).

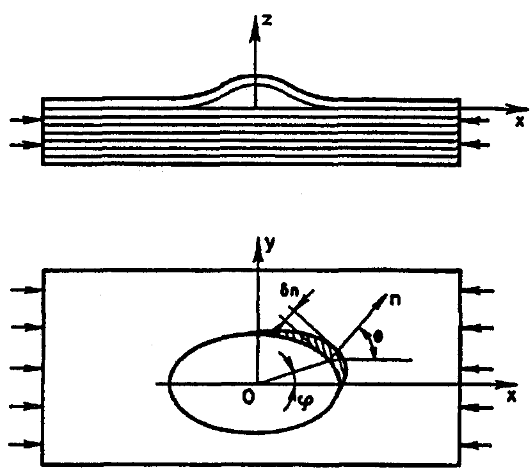

Fig. 2. Delamination in a laminated composite plate under compression.

data were available, which could support or disprove this hypothesis. The properties of the specific fracture work as a geometrical object may prove more complicated. This may be seen if we examine the properties of the corresponding load parameters. Thus, the load velocity coefficients for an orthotropic linear medium are related to the elastic constants of this medium by quite complicated equations. In turn, the elastic constants are components of some fourth rank tensor or are expressed through these components. Thus, the term γ given by the following matrix

$$\gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \tag{1}$$

will be designated only conditionally as the specific fracture work tensor, assuming that upon rotation of the crack front, matrix (1) is transformed according to the transformation rules of a second rank symmetrical tensor onto planes x_1, x_2. If the starting values are given along the major axes,

$$\gamma = \begin{pmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{pmatrix}, \tag{2}$$

the specific fracture work for a crack, whose front is inclined at angle θ, is defined as

$$\gamma_\theta = \gamma_1 \cos^2\theta + \gamma_2 \sin^2\theta; \qquad \eta_\theta = \frac{\gamma_1 - \gamma_2}{2} \sin 2\theta. \tag{3}$$

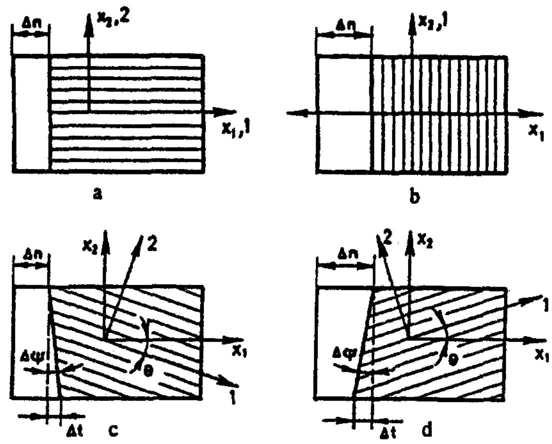

Fig. 3. Interlaminar fracture of a composite: a, b) along the major axes of tensor γ; c, d) in an oblique direction.

In this case, $\gamma_1 \geq \gamma_2$, while the nondiagonal values for angles θ and $\theta + \pi/2$ are designated as $\gamma_{12} = \gamma_{21} = -\eta_\theta$. The minus sign before η_θ is necessary for simplification of the sign law. There is nothing unusual in the introduction of this sign rule: we need only recall the analogous situation with moments of inertia in the tensor representation of such moments.

The question arises as to the physical significance of matrices (1) and (2). It is natural to assume that the Neumann principle is applicable to the specific fracture work (as to any other physical term). Thus, if the composite has orthogonal symmetry axes in plane x_1, x_2, the major values of tensor γ in this case coincide with the generally accepted values (Fig. 3a, b), while the value of γ_θ determined using the first formula in Eqs. (3) coincides with γ_1 when $\theta = 0$ and with γ_2 when $\theta = \pi/2$. In both cases, $\eta_\theta = 0$.

In order to clarify the physical significance of the nondiagonal elements of matrix (2) or, equivalently, the significance of η_θ in the second formula in Eqs. (3), let us examine an example. Let interlaminar fracture of a composite occur in a direction not coinciding with the symmetry axes. The crack front in this case will be displaced "obliquely," i.e., with deviation from the direction of the tear toward the directions with least resistance to fracture. For example, if a flat rectangular sample with "oblique" fiber orientation is subjected to tear in the direction of an axis, the fracture area will consist of two parts. One part will correspond to frontal displacement of the crack, while the second will correspond to rotation of the front toward the direction of least resistance at some angle $\Delta\psi$ (Fig. 3c, d). If angle θ measured from major axis 1 is positive, then the front rotation $\Delta\psi$ will also be positive. Counter-clockwise angles are usually considered positive (Fig. 3c). When $\theta < 0$, we thus obtain $\Delta\psi < 0$ (Fig. 3d).

It is more difficult to relate the newly formed areas with the total interlaminar fracture work. The work spent on the frontal displacement of the crack is clearly equal to $\gamma b\Delta n$, where b is the sample width and Δn is the frontal component of the displacement. The area $b\Delta t/2$ remains, where $\Delta t = b\tan\Delta\psi$, which is the displacement of the crack due to its deviation from the frontal direction. The corresponding fracture work is proportional to $\eta_\theta b\tan\Delta\psi/2$. This value will be positive both when $\theta > 0$ and $\theta < 0$ due to the simultaneous change in signs of η_θ and $\Delta\psi$. Thus, it is possible to explain the variable sign nature of the nondiagonal elements of matrix (1).

The value of the coefficient in the expression for work in the formation of the triangular fracture region remains not entirely clear. The case of very strong anisotropy may serve as the leading concept. Let $\gamma_1 >> \gamma_2$ and $\theta = \pi/4$.

Then, Eqs. (3) give $\gamma_\theta \approx \eta_\theta \approx \gamma_1/2$. Assuming that $\Delta W = \gamma_\theta b\Delta n + \eta_\theta b\Delta t$, we find that $\Delta W = (\gamma_1/2)\Delta A$. Here, $\Delta A = b(\Delta n + \Delta t/2)$, i.e., the area of the trapezoid corresponding to the newly formed delamination, while $\gamma_1/2$ has the significance of the specific fracture work in "oblique" tear. Then, we assume that

$$\Delta W = (\gamma_\theta\, \Delta n + |\,\eta_\theta\,|\, \Delta t/2)\, b. \tag{4}$$

Experiments were carried out to check the applicability of these assumptions. It would be simplest to check the first formula in Eq. (3). This requires only tear testing of a samples with major axes oriented differently to the sample axis and,

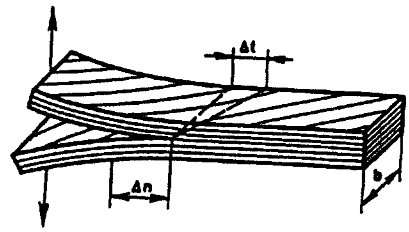

Fig. 4. Scheme for testing for "oblique" interlaminar tear in the opening mode.

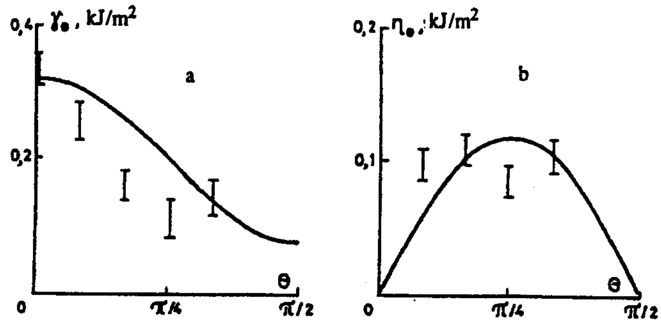

Fig. 5. Comparison of the experimental and calculated values of the elements of the specific fracture work tensor for a textile/epoxy composite: a) diagonal elements and b) nondiagonal elements.

thus, to the tear direction using a two-arm scheme (Fig. 4). The second formula in Figs. (3) along with Eq. (4) may be used to check the interpretation of the nondiagonal elements. More precisely, the following equation may be checked

$$\frac{\gamma_1 - \gamma_2}{2} \sin 2\theta = \frac{\Delta W}{b} - (\gamma_1 \cos^2 \theta + \gamma_2 \sin^2 \theta) \frac{2\Delta n}{\Delta t}.$$

Here, γ_1 and γ_2 are found by tear testing in the direction of the major axes, while ΔW, Δn, and Δt are found by tear testing also in the "oblique" direction.

Textile/epoxy composite samples were tested made from linen-woven fiberglass and epoxy resin. The samples were prepared by cutting from one plate at different angles $0 \leq \theta \leq \pi/2$, where θ is measured from the direction of the weft. Upon tear in the direction of the weft, the fracture work $\gamma_1 \approx 320$ J/m^2. In the direction of the base, $\gamma_2 \approx 80$ J/m^2. The dependence of the specific fracture work on θ is shown in Fig. 5. Four or five samples were tested for each angle. The scatter of the values found for γ_θ and η_θ is rather large. The scatter intervals are shown in Fig. 5. The solid lines were calculated using Eqs. (3). The agreement may be considered satisfactory if we consider that fracture in mode 1 could not be achieved in its pure form in the testing. The mixed nature of the fracture introduces an additional effect, which is not taken into account in Eqs. (3).

Analogous results for SVM unidirectional organic plastic are given in Fig. 6. Strips of this composite were adhered by metal bases, together with which they were subjected to interlaminar tear using a two-arm scheme. For the first major direction (along the fibers), we obtained the mean value $\gamma_1 = 1.45$ kJ/m^2. For the second major direction $\gamma_2 = 0.77$ kJ/m^2. Figure 6 indicates that the calculated and experimental results are in accord here better than in the case of the textile/epoxy composite.

In many cases, the crack propagates in the direction of the external normal to the initial contour despite the anisotropy of the specific fracture work. Let the delamination boundary S be smooth (see Fig. 2). We designate the element of length of the contour ds and the variation of the external normal to the contour δn and obtain

$$\delta A_f = -\oint_S \gamma_\theta(\theta)\, \delta n\, ds, \tag{5}$$

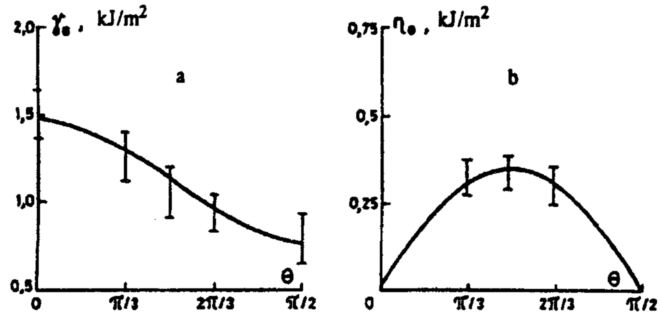

Fig. 6. Comparison of the experimental and calculated values of the specific fracture work tensor for a unidirectional Kevlar/epoxy composite: a) diagonal elements and b) nondiagonal elements.

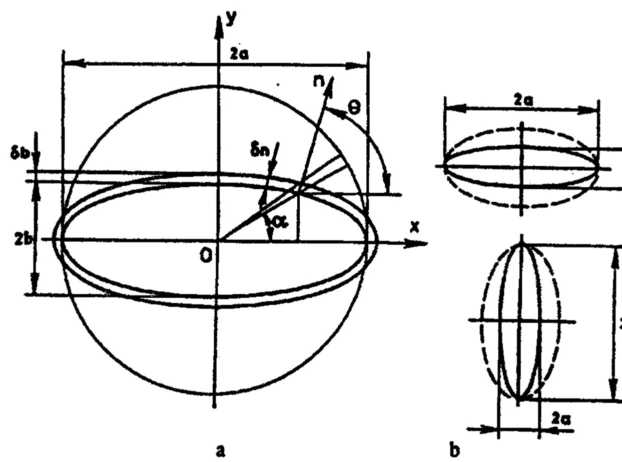

Fig. 7. Elliptical delaminations: a) variation of the dimensions and b) growth of elongated delaminations.

where $\gamma_\theta(\theta)$ is determined using Eqs. (3). The integration is carried out over the entire delamination contour, in particular, in polar coordinates:

$$\delta A_f = -\int_0^{2\pi} \gamma_\theta[\theta(\varphi)]\, r\delta r\, d\varphi, \tag{6}$$

where $\delta r(\varphi)$ is a variation of the polar radius corresponding to the increment of the normal in the direction $\theta(\varphi)$. In special cases, suitable curvilinear coordinates or another parametrical form should be used instead of polar coordinates for giving the delamination contour.

We shall show how to calculate the generalized resistance force for elliptical delamination in the composite. Let us examine delamination with semiaxes a and b having given the coordinates of its boundary through parametrical angle α, i.e., $x = \cos\alpha$, $y = \sin\alpha$ (Fig. 7a). Then, the elemental boundary length ds and slope θ of the external normal to the boundary is given by the following equations

$$ds = (a^2 \sin^2\alpha + b^2 \cos^2\alpha)^{1/2}, \qquad \mathrm{tg}\,\theta = k\,\mathrm{tg}\,\alpha. \tag{7}$$

Here, $k = a/b$. Let us take the equality

$$(a + \delta a)\cos(\alpha + \delta\alpha) \equiv x + \delta n \cos\theta;$$
$$(b + \delta b)\sin(\alpha + \delta\alpha) \equiv y + \delta n \sin\theta, \tag{8}$$

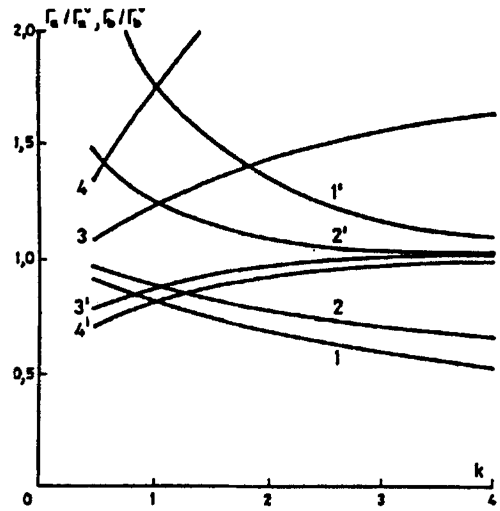

Fig. 8. Dimensionless generalized resistance forces for elliptical delaminations: 1-4) Γ_a, 1'-4') Γ_b at $\gamma_2/\gamma_1 = 0.25$ (1, 1'), 0.5 (2, 2'), 2 (3, 3'), 4 (4, 4').

where δa and δb are variations of the semiaxis lengths, $\delta\alpha$ is a variation of the parametrical angle corresponding to variation of the boundary in the direction of the external normal. We use Eqs. (8) to find that the variation of the length of the normal is

$$\delta n = \frac{b \cos^2 \alpha \, \delta a + a \sin^2 \alpha \, \delta b}{(a^2 \sin^2 \alpha + b^2 \cos^2 \alpha)^{1/2}}. \tag{9}$$

Let us substitute Eqs. (7) and (9) into general Eq. (5) for virtual fracture work and find the generalized resistance force

$$\Gamma_a = 4b \int_0^{\pi/2} \gamma[\theta(\alpha)] \cos^2 \alpha \, d\alpha;$$

$$\Gamma_b = 4a \int_0^{\pi/2} \gamma[\theta(\alpha)] \sin^2 \alpha \, d\alpha. \tag{10}$$

In particular if the axes of the ellipse are directed along the major axes of tensor γ_{jk}, i.e., $\gamma(\theta) = \gamma_1 \cos^2 \theta + \gamma_2 \sin^2 \theta$, Eqs. (10) take the form

$$\Gamma_a = 4b \int_0^{\pi/2} \frac{(\gamma_1 + \gamma_2 k^2 \operatorname{tg}^2 \alpha) \cos^2 \alpha \, d\alpha}{1 + k^2 \operatorname{tg}^2 \alpha};$$

$$\Gamma_b = 4a \int_0^{\pi/2} \frac{(\gamma_1 + \gamma_2 k^2 \operatorname{tg}^2 \alpha) \sin^2 \alpha \, d\alpha}{1 + k^2 \operatorname{tg}^2 \alpha}. \tag{11}$$

The integrals in Eqs. (11) are calculated elementarily:

$$\Gamma_a = \frac{\pi b \, [\gamma_1 (2k + 1) + \gamma_2 k^2]}{(k + 1)^2}; \quad \Gamma_b = \frac{\pi a \, [\gamma_1 + \gamma_2 k \, (k + 2)]}{(k + 1)^2}. \tag{12}$$

Let us examine several special cases. Let $\gamma_1 = \gamma_2 = \gamma$, i.e., the material is transverse-isotropic. Then, we arrive at the following formulas [2]

$$\Gamma_a = \pi\gamma \, b; \quad \Gamma_b = \pi\gamma \, a. \tag{13}$$

If $\gamma_1 \neq \gamma_2$ but the ellipse has a highly oblate form, approximate evaluations may readily by obtained using Eqs. (12). Thus, when a << b

$$\Gamma_a \approx \pi\gamma_1 b, \quad \Gamma_b \approx \pi\gamma_1 a. \tag{14}$$

This implies that when γ_1 and γ_2 have the same order, $\Gamma_a >> \Gamma_b$, while delamination grows (when all other conditions are equal) in the Ox-direction. The fracture work in this case is determined mainly by the resistance of the composite to splitting along this axis. On the other hand, when a >> b, we find the following approximations

$$\Gamma_a \approx \pi\gamma_2 b; \qquad \Gamma_b \approx \pi\gamma_2 a. \tag{15}$$

Resistance to crack propagation mainly depends on the specific fracture work along the Oy-axis, while delamination proceeds in the direction of this axis. On the whole, Eqs. (14) and (15) show that the delamination patterns, whose form is quite noncircular, have a tendency to grow toward a circular form (if, of course, the values of the specific fracture work along the two major axes have the same order). This conclusion is illustrated in Fig. 7b.

The results of our calculations using Eqs. (12) are shown in Fig. 8, in which Γ_a/Γ_a^0 and Γ_b/Γ_b^0 are given as functions of dimensionless parameters $k = a/b$ and γ_2/γ_1. Here, $\Gamma_a^0 = \pi\gamma_1 b$ and $\Gamma_b^0 = \pi\gamma_2 a$. Curves 1, 2, 3, and 4 were plotted for Γ_a/Γ_a^0 when $\gamma_2/\gamma_1 = 0.25$, 0.5, 2, and 4, while curves 1'—4' were plotted for Γ_b/Γ_b^0 at the same values of γ_2/γ_1. If $\gamma_2/\gamma_1 = 1$, then $\Gamma_a = \Gamma_a^0$ and $\Gamma_b = \Gamma_b^0$, which corresponds to Eqs. (13).

These results concern tear fracture (mode I). An analogous model may be developed for modes II and III, for example, for delamination by the action of shear forces. The situation is complicated in the case of fracture through mixed modes. If a crack propagates between two orthotropic layers with the same properties and the same orientation, the specific fracture work matrix will take the following form

$$\gamma = \begin{bmatrix} \begin{vmatrix} \gamma_I^{11} & \gamma_I^{12} \\ \gamma_I^{21} & \gamma_I^{22} \end{vmatrix} & & \\ & \begin{vmatrix} \gamma_{II}^{11} & \gamma_{II}^{12} \\ \gamma_{II}^{21} & \gamma_{II}^{22} \end{vmatrix} & \\ & & \begin{vmatrix} \gamma_{III}^{11} & \gamma_{III}^{12} \\ \gamma_{III}^{21} & \gamma_{III}^{22} \end{vmatrix} \end{bmatrix}. \tag{16}$$

The remaining elements of this matrix are zero and become non-zero if the properties of adjacent layers are different or the layers are differently oriented. Similar difficulties arise in calculating the energy release rates when the crack front moves along the boundary between two homogeneous elastic layers [7]. Additional difficulties arise in composite materials due to the complex fractographic pattern of interlaminar cracking. There is cause to expect that some of the zero vacancies in matrix (16) should be occupied for a complete description of fracture of the boundary of two layers with different properties.

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