Arch. Met. Geoph. Biokl., Ser. A, 29, 301–325 (1980)

ARCHIV
FÜR METEOROLOGIE
GEOPHYSIK UND
BIOKLIMATOLOGIE

556.556.4

Landesanstalt für Umweltschutz Baden-Württemberg, Karlsruhe, Federal Republic of German, and Great Lakes Environmental Research Laboratory, Ann Arbor, Michigan, U.S.A.

Free Surface Oscillations in Lake Constance with an Interpretation of the "Wonder of the Rising Water" at Konstanz in 1549[1]

E. Hollan, D. B. Rao, and **E. Bäuerle**

With 14 Figures

Received November 27, 1979

Summary

The structures and periods of the lowest four normal modes of Lake Constance and the fundamental mode of the Bay of Konstanz are determined by an application of a simplified version of the two-dimensional theory developed by Rao and Schwab [12]. The effect of the earth's rotation is neglected, because the medium size of the Lake implies only a minor influence on the oscillations. The calculations of the lowest four modes are verified by spectral analyses of simultaneous measurements of water level fluctuations at up to 9 stations on the shore. In particular, the water wonder at Konstanz in 1549 described by Schulthaiss [14] is explained in terms of a possible resonant excitation of the fundamental mode of the Bay of Konstanz. For the analysis of this event a translation of Schulthaiss' notes, which give the oldest known observations on seiches, is included in the text together with some historical comments.

Zusammenfassung

Die freien Schwingungen des Bodensees mit einer Interpretation des „Wasserwunders" von Konstanz im Jahre 1549

Die Struktur und Perioden der vier niedrigsten Eigenschwingungen des Bodensees und die Grundschwingung der Bucht von Konstanz wurden durch Anwendung einer vereinfachten Version der von Rao und Schwab [12] entwickelten zweidimensionalen Theorie bestimmt. Der Effekt der Erdrotation wurde vernachlässigt, da der Einfluß auf die freien Schwingungen des Sees durch die mittlere Größe des Seebeckens gering ist. Die Berech-

[1] Great Lakes Environmental Research Laboratory Contribution # 198.

0066-6416/80/0029/0301/$ 05.00

nungen der niedrigsten vier Eigenschwingungen werden durch Spektralanalysen von gleichzeitigen Messungen der Wasserspiegelschwankungen an 9 Uferstationen verifiziert. Im besonderen wird das Wasserwunder des Jahres 1549 in Konstanz, das von Schulthaiss [14] beschrieben wurde, als eine mögliche resonante Anregung der Grundschwingung in der Bucht von Konstanz erklärt. Zur Analyse dieses Ereignisses ist eine Übersetzung des Berichts über das „Wasserwunder“ zusammen mit einigen historischen Bemerkungen beigefügt. Die Mitteilungen von Schulthaiss stellen die ältesten derzeit bekannten Beobachtungen von Seiches dar.

1. Introduction

Although the free surface oscillations in Lake Constance generally do not exceed an amplitude of 5 cm [8], the phenomenon contributes considerably to flood damage of the flat nearshore areas, when maximum amplitudes of 20 to 30 cm coincide with high mean water levels. This condition occurs relatively often, since there is a high annual range of the water level. The mean value of this water level fluctuation is 1.5 m during the period 1877–1964 [7]. The low grounds around the lake, although intensively cultivated, remain unprotected against inundations. For this reason it is of interest to predict the spatially variable contributions of the free surface oscillations to the total maximum water level set-up.

The solution of the problem requires the application of a two-dimensional theory because of the irregular, though oblong shape of the lake (see map in Fig. 1) and was left to the present time, since efficient two-dimensional methods for natural water bodies have been developed in full generality only recently (see, for example, [11] and [12]). In the following investigation the most important gravitational modes of the lake are calculated by the theory of Rao and Schwab. The verification of the results is performed by spectral analysis of water level fluctuations measured by Mühleisen and Kurth [8] at up to 10 gauges between 1967 and 1974 and a set of measurements made by the Swiss Federal Institute of Environmental Protection, National Hydrological Bureau, in 1972 at Kreuzlingen. Valuable information is also provided by the historical observations by Forel [3] and Christoph Schulthaiss [14]. The latter report is the oldest known description of free surface oscillations in a lake. Because this report documents carefully one of the strongest cases of higher order oscillations ever observed in the lake, the event will be examined in more detail.

On the basis of Mühleisen and Kurth's [8] measurements, which included wind and air pressure observations on the shore, Hamblin and Hollan [4] tried to explain the excitation of the forced oscillations through a normal mode expansion technique. The normal modes in that investigation were calculated using a finite element method. However, the computed normal mode properties were verified against observations only in a qualitative manner for some of the modes. The theoretical approach adopted in the present investigation is based on a finite difference formulation of the eigenvalue problem. A systematic verification analysis using water level data was carried out to validate the theoretical calculations.

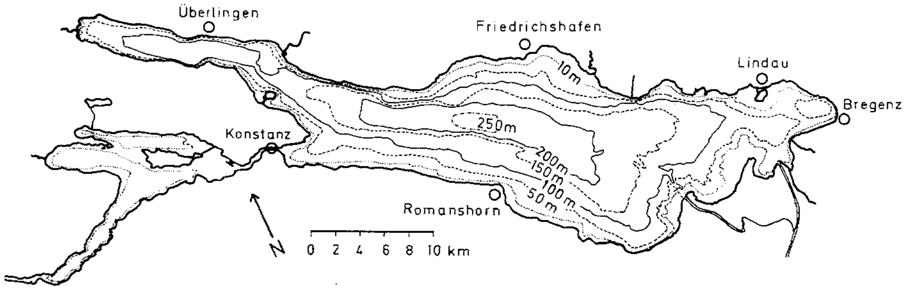

Fig. 1. Bathymetry of Lake Constance

2. Method of Calculation

The governing equations and the method of their solution are summarized only briefly in order to elucidate the simplified version of the general theory by Rao and Schwab [12] which was applied successfully to the Great Lakes of North America [13, 15]. Since Lake Constance is considerably smaller, though deeper than the Great Lakes, the influence of the Coriolis force on barotropic motions is less important. The radius of deformation c/f, where c is the phase speed of long gravity waves, and f is the Coriolis parameter, is about 291 km for the mean depth of h = 100 m of the lake. As the largest horizontal scale of the lake is only 62 km, which is much less than the radius of deformation, Coriolis force can be neglected in the calculations. The governing equations of small amplitude, free, quasi-static oscillations in a homogeneous lake may be written in terms of a Cartesian coordinate system as

$$\frac{\partial \tilde{\mathbf{M}}}{\partial t} = -gH\nabla\tilde{\eta} \qquad (1)$$

$$\frac{\partial \tilde{\eta}}{\partial t} + \nabla\cdot\mathbf{M} = 0.$$

The dependent variables in eq. (1) are the free surface displacement $\tilde{\eta}$ from the equilibrium position and the transport vector $\mathbf{M} \equiv H\tilde{\mathbf{V}}$. $\tilde{\mathbf{V}}$ is the velocity vector and $H(x, y)$ is the equilibrium depth of the lake. The gradient symbol in eq. (1) is defined with respect to the horizontal coordinates x and y.

Since the shallow open parts at the east end of the lake where the two main tributaries enter, and the outflow through the city of Konstanz (see Fig. 1) are very narrow, the lake may be considered completely enclosed. Hence the adiabatic boundary condition

$$\tilde{\mathbf{M}} \cdot \tilde{n} = 0 \tag{2}$$

has to be satisfied by the solutions of (1) on the coastline. $\tilde{n}$ is a unit vector normal to the coast.
The normal modes are assumed expressible in the following form appropriate for a closed basin:

$$\tilde{\mathbf{M}} = \mathbf{M}(x, y) \sin \sigma t \quad \tilde{\eta} = \eta(x, y) \cos \sigma t. \tag{3}$$

$\mathbf{M}, \eta$ are the space dependent parts of $\tilde{\mathbf{M}}$, $\tilde{\eta}$ and σ is the oscillation frequency. Substitute eq. (3) into (1) and eliminate the transport vector $\tilde{\mathbf{M}}$. This results in a self-adjoint elliptic equation

$$\nabla \cdot H \nabla \eta = -\lambda \eta \qquad \lambda \equiv \sigma^2/g \tag{4}$$

with the boundary condition $H \dfrac{\partial \eta}{\partial n} = 0$ in the boundary. In order to solve the problem for an irregular lake, it is necessary to discretize the continuous operatior $\nabla \cdot H \nabla$ using a finite difference grid covering the lake. The resulting finite difference equations constitute an algebraic eigen value problem for eigen values λ and eigen functions η. Each eigen function $\eta(x, y)$ represents the free surface amplitude distribution of the normal mode associated with a frequency σ which is related to λ as shown in eq. (4).
The basin of Lake Constance was approximated by grid squares of 1.4 km side fitted to the geometry of the lake (Fig. 2). The depths were read from the bathymetric chart of Lake Constance from 1893 [18] updated by the Cartographical Survey of the State of Baden-Württemberg [2] at the mouth of the main tributaries at the east end across from Lindau. The mean depth calculated from the numerical grid exceeds the value derived from the depth chart, 100 m, by 2.9%. To compare the calculated frequencies with observed frequencies this difference in the mean water level has to be accounted for and will be discussed later.
The open circles of the grid give the places where the height field is defined. The components M and N of the transport vector, $\mathbf{M}$, can be calculated from adjacent η's at the points marked by —•— and ⫶, respectively. At these latter points the depths were read from the charts. In total 235 points exist, where the height field is defined. The calculations give for each mode a frequency and the amplitude distribution in percentage of the maximum range encountered on the grid for each particular mode.

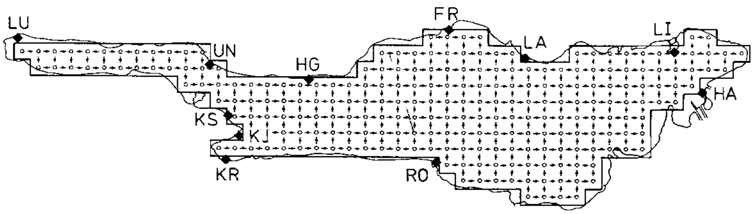

Fig. 2. Location of water level recording stations and numerical grid on Lake Constance with grid spacing of 1.4 km. Code letters refer to the recording stations listed in Table 1. Symbols of the grid are explained in the text

3. The First 4 Normal Modes Compared with Spectra of Observed Water Level Fluctuations

In this section, we will examine the properties (period and structure) of the lowest four longitudinal modes of Lake Constance. The spatial resolution provided by the measurement programme allows for a reasonable analysis of the structures of the lowest four modes. The network of observational stations is too wide for an investigation of higher modes. However, a partial verification of the periods of higher order modes is possible because the measurements at individual stations can provide sufficient resolution in time. An example of one of these higher modes is given in section 5. From Mühleisen and Kurth's [8] measurements 6 cases have been selected for the analysis of the low order oscillations. Four of these cases consist of a simultaneous set of up to 9 records, whereas in two cases only a single record is available. The dates of these events are listed in the second column of Table 2, where the observed and calculated periods of the first 4 normal modes are compared. The stations, at which these measurements were obtained, are included in the grid representation of the lake in Fig. 2 and designated by letter labels, while their names are listed in Table 1.
The spectra were calculated by the method of Blackman and Tukey [1], for which a programme of the Institute of Marine Research at the University of Kiel was used. In order to provide a machine-processable form of the data the original continuous records had to be converted into series of equally spaced measurements. A time interval of Δt = 1 or 2 min was chosen depending on the quality of the data for this purpose. Before the series were subjected to the spectral analysis, linear trend was eliminated from the data in each case. Hence the effects of long term changes of the level (< 1 cph) on the spectral density are removed to a large extent. For reduction of truncation errors in the spectra caused by the finite lengths of the series the spectral window of v. Hann was applied. In order to provide high resolution the lag window of the autocovariance function has been

Table 1. *Code Letters Used for the Water Level Recording Stations Around Lake Constance* (see Fig. 2)

Station name	Code letters
Ludwigshafen (B.W.)	LU
Unteruhldingen (B.W.)	UN
Konstanz-Staad (B.W.)	KS
Konstanz-Jakobsbad (B.W.)	KJ
Kreuzlingen (Switzerland)	KR
Hagnau (B.W.)	HG
Romanshorn (Switzerland)	RO
Friedrichshafen (B.W.)	FR
Lindau (B.)	LI
Hard (Austria)	HA

Abbreviations in brackets mean B.W.: State of Baden-Württemberg, B.: Free State of Bavaria.

chosen as great as possible. This condition results in a low statistical stability of the spectral density estimates in terms of Blackman's and Tukey's consideration for Gaussian signals. However, the reliability of the calculations may be checked by comparing the results of the spectral analyses for different events of oscillations as shown, for instance, in the case of the normalized amplitudes of the fundamental modes on June 25 and August 23 in 1967 given in Table 4. The spectra were computed with the same parameters, except for a small difference between the lengths of the actual series in both cases (960 and 846). The close agreement of the amplitudes shows that the estimates are indeed satisfactorily stable.

The spectra over the frequency range of 0.5 to 10 cph for the two cases selected for structure analysis are plotted in Fig. 3 and 4. As the analysis of the structures and the eigen-frequencies requires only the consideration of relative changes of power density, the absolute scaling of the spectra is not necessary. Hence, for convenience, the spectra of each case from various stations are shown arranged vertically one below another. The corresponding stations, at which the measurements were obtained, are designated in the diagrams by the same letter labels as in the grid representation in Fig. 2. Guided by the calculated eigen-frequencies we identified at first the prominent spectral peaks of the normal modes at several stations. Then we sought for minor peaks at or close to these frequencies in the spectra of the remaining stations and marked finally all detectable peaks by best-fit vertical lines in the diagrams, thus defining observed eigen-frequencies. The numbers

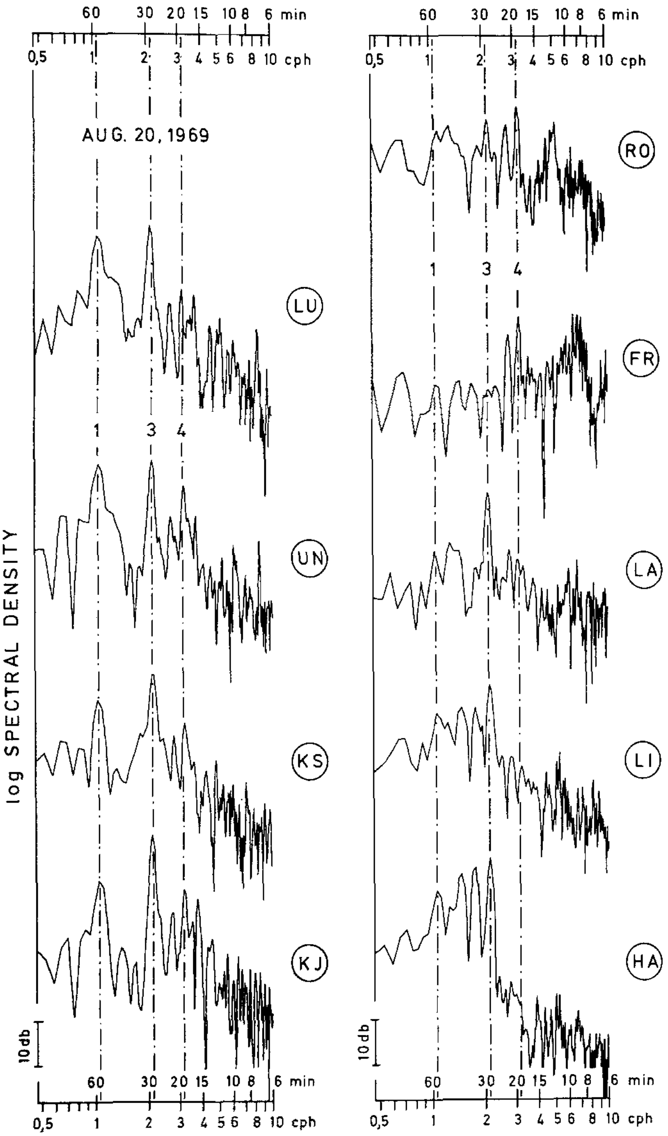

Fig. 3. Power spectra for 9 stations from August 20th, 1969. Each vertical line is a best fit to peaks occurring near this period in several spectra. The numbers indicate the order of the mode. Stations are denoted by the code letters listed in Table 1

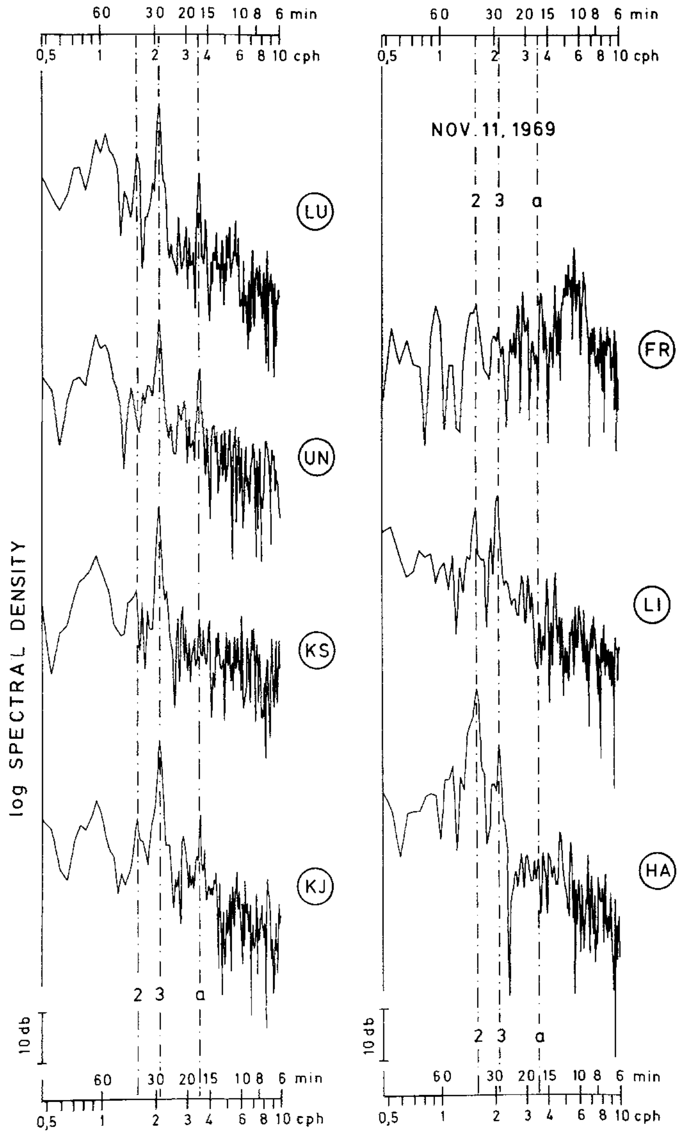

Fig. 4. Power spectra for 7 stations from November 11th, 1969. See Fig. 3 for further explanations. The fifth longitudinal mode referred to in section 4 is marked by the letter, *a*

added to these lines refer to the order of the corresponding normal mode. The eigen-periods obtained by this procedure are listed in Table 2 and compared with the calculated values. Generally the observed periods are higher than those of the calculated oscillations. This difference may be ascribed to frictional influences, which are not considered in the theory, and to the variations of the mean water level, when the different events were recorded. An estimation of this latter effect in terms of Merian's formula is included in the last column of the Table. It shows an approach of observed and calculated periods, except for the 4th mode, when these

Table 2. *Comparison of Calculated and Observed Periods of the Lowest 4 Normal Modes of Lake Constance*

Mode	Case	Period (min)	Depth correction ΔH (m)	Corrected period (min)[1]
1	calculated	53.87	−2.9	54.63
1	25. 6. 1967	55.64	−1.33	56.00
1	23. 8. 1967	55.57	−0.99	55.84
1	20. 8. 1969	56.89	−0.56	57.05
2	calculated	35.96	−2.9	36.47
2	11. 11. 1969	37.72	+ 0.54	37.61
3	calculated	27.03	−2.9	27.41
3	20. 8. 1969	28.44	−0.56	28.52
3	9. 11. 1969	27.63	+ 0.54	27.55
3	11. 11. 1969	28.28	+ 0.54	28.20
4	calculated	19.84	−2.9	20.12
4	13. 8. 1969	19.13	−0.35	19.16
4	20. 8. 1969	18.62	−0.56	18.87

[1] Corrected to 100 m chart depth.

values are corrected for the same mean water level corresponding to the bathymetric chart of 1893. The deviation of the 4th eigen-period from this behaviour may be related to the resolution of the spectral analysis, the discrete approximation of the basin by the size of the numerical grid, which causes greater inaccuracy as the spatial scales of the modes decrease and the fact that modes of even parity are usually not strongly excited, thereby making their period estimates a little uncertain. However, as seen from Table 2, the agreement between the observed and theoretical periods is very good with a maximum error of only about 6% for the fourth mode. One of the authors (Bäuerle) calculated recently the longitudinal modes of low order by the channel theory described by Platzman and Rao [10]. Table 3 shows the eigen-periods of the lowest four modes obtained from

Table 3. *Periods of the Lowest 4 Normal Modes Obtained from the Two-Dimensional Theories of Rao and Hamblin, as well as from the Channel Theory* (period in min)

Mode Order	Channel results (Bäuerle)	Two-dimensional results (Hamblin, 1978)	(Rao, Table 2)
1	54.0	53.4	53.9
2	39.1	35.7	36.0
3	26.2	27.2	27.0
4	18.7	19.4	19.8

the channel theory as well as from Hamblin and Hollan's [4] and our two-dimensional calculations. The differences between the results of the latter two theories are small. Hamblin and Hollan's periods show slightly lower values – except for the 3rd mode. When we look at the periods of the channel solutions, the neglect of the second dimension becomes apparent for the higher modes.

In order to verify the theoretical structures the observed power densities are converted to the corresponding amplitude densities. The amplitude distribution of each calculated normal mode was determined as a percentage of the maximum encountered for that mode. Hence the same representation of relative changes is worked out for the observed amplitudes. For this purpose one station was selected, at which the measured amplitude was normalized to the calculated value of the corresponding grid square. The amplitudes observed at the remaining stations were then multiplied by this factor, thus yielding the desired relative amplitude distribution of the real oscillation. The results of this evaluation and the calculated values are compared for each mode in Table 4, while the complete theoretical structures are shown in Fig. 6 and 7. In the following individual notes, the computed structures of the first four longitudinal modes and the results of the verification analysis are discussed. Local details of the structures will be described with the help of names of nearby situated villages or towns on the shore. The sites of these places are illustrated in a separate map of place-names in Fig. 5.

3.1 First Longitudinal Mode (53.87 min)

The maximum range of the fundamental mode occurs at the west end of the lake. As Fig. 6 shows, the amplitudes throughout Lake Überlingen are higher than at the east end, where the range amounts to slightly less than 50% of that at the opposite end. The nodal line is lightly curved to the west and ends at Helmsdorf on the northern shore and Kesswil on the southern shore (see also Fig. 5).

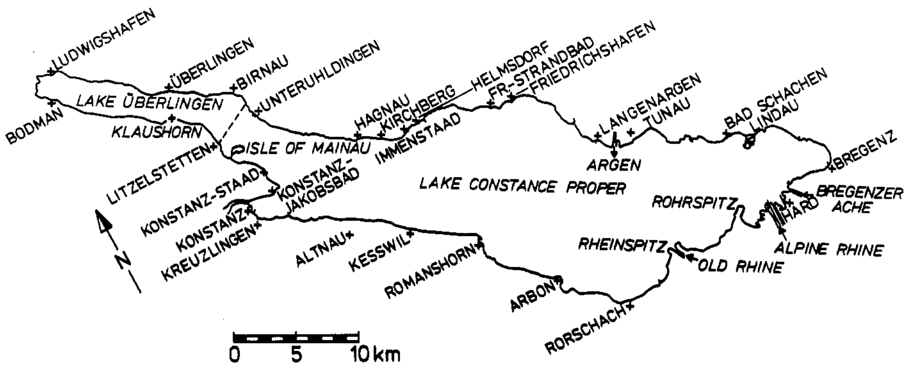

Fig. 5. Map of place-names used for the description of the lowest four normal modes

The calculated structure agrees well with the observations listed in Table 4. In particular, the two cases of June 25th and August 23rd in 1967 verify the calculation satisfactorily. Great differences appear in the case of August 20th 1969, and result from the lower spectral resolution as well as the reduced absolute intensity of the oscillation compared with the two previous cases. There is a higher difference between calculated and observed ranges at the station Hard across from Lindau. The discrepancy is probably linked with the fact that the recorder was placed inside a small lake with a narrow inlet from the main basin. Since descretization by grid squares of 1.4 km does not resolve these details, the local influence of the enclosed shallow water body on the normal modes is not included in the calculation. This shortcoming with respect to the amplitudes at Hard applies also for higher normal modes, as is evident in some cases listed in Table 4.

The earlier descriptions of the fundamental mode by Forel [3] agree to a large extent with our results. The position of the node, however, is not shifted so far to the west as he concluded on the basis of his measurements at Kirchberg and the lower depths in the western part of the lake. As to the period, Forel obtained a value of 55.8 min, which is nearly the same as in the two cases from 1967 (Table 2).

Calculation of Hamblin and Hollan [4] agrees essentially with our results. Due to the inclusion of earth's rotation in their calculations, the node appears transformed into an anticlockwise amphidromic system. However, in view of the relative unimportance of earth's rotation, the phase propagation associated with this amphidromic point is confined to a very narrow band near the nodal line and cannot be tested from the available observations.

Table 4. *Comparison of Calculated and Observed Elevation Ranges in % for the Lowest 4 Normal Modes of Lake Constance*

Mode order	Case	LU	UN	KS	KJ	HG	RO	FR	LA	LI	HA
1	calculated	100	52	34	(28)	16	9	9	17	40	(42)
1	25. 6. 1967	100	48	30	–	13	4	–	10	46	–
1	23. 8. 1967	100	47	–	–	14	7	6	12	45	–
1	20. 8. 1969	100	58	40	36	–	N	10	8	53	73
2	calculated	35	3	12	20	21	18	20	7	73	76
2	11. 11. 1969	41	N	12	18	–	–	22	–	73	161
3	calculated	51	21	37	58	33	7	5	19	33	36
3	20. 8. 1969	51	25	30	43	–	6	4	14	43	65
3	11. 11. 1969	51	20	31	42	–	–	5	–	38	19
4	calculated	*	*	*	*	*	10	*	*	*	*
4	20. 8. 1969	12	14	10	12	–	10	24	3	N	N

Underlined values represent amplitudes adjusted among one another.
–– means: not measured; N stands for: not distinguishable in the spectrum (low value);
*: values not exactly available because of weak structure in the isopleth-representation (comparison given qualitatively in Fig. 7). For the same reason the calculated values of the fundamental mode are given in brackets at stations KJ and HA.

3.2 Second Longitudinal Mode (35.96 min)

The structure of the second mode shows maximum range at the east end of the lake (Fig. 6). The eastern node is aligned between Tunau just east of the mouth of the Argen river and the mouth of the Old Rhine on the southern shore, while the western node is situated at the Isle of Mainau. From the distribution of the amplitudes in the central and western part of the lake it is clearly discernible that the central crest is shifted to the west. As indicated above in the case of the fundamental mode, this peculiarity is caused by the lower depths in that part of the lake. As observations of Mühleisen and Kurth [8] show, the second mode is usually not strongly excited. We were able to detect it only in one case measured on November 11th, 1969, when it prevailed in the water level fluctuations at the east end of the lake. The comparison of the computed and observed amplitudes in Table 4 exhibits satisfactory agreement except for the station Hard, where the measured value far exceeded the theoretical value as a result of local configurations of the bathymetry, which are not resolved by the numerical grid. At the station of Unteruhldingen the predicted western node is verified by an undistinguishable low amplitude denoted by N in Table 4.
The second mode was observed by Forel [3] at Kirchberg in the vicinity of its central crest, where only minor contributions of the fundamental mode

occur. On the basis of nine cases measured during January 1891, he obtained a period of 39.3 min, which is greater than our observed value, 37.72 min, by 4.2%. This difference may be attributed to the generally low mean water level in January. Unfortunately, there are no recent measurements available with necessary resolution in time and space to investigate the properties of the second mode in greater detail. Forel did not explain the oscillation, because he had no other method of analysis at his disposal than by visual inspection of the records. At his two other observational stations in Konstanz and Bodman the fundamental and third mode were prevailing and did not allow for an isolation of possible weak contributions of the second mode.

Hamblin and Hollan's [4] calculation yields only a rough picture of the structure. The positions of the nodes are well reproduced in terms of two anticlockwise amphidromies, while the amplitude distribution is presented with low resolution. At the east end a less high relative range appears than is confirmed by our analysis.

3.3 Third Longitudinal Mode (27.03 min)

The structure of this mode is characterized by high ranges at both ends (Fig. 6). The value at Bregenz exceeds that of Ludwigshafen by more than 20%. In the Bay of Konstanz unrealistically high amplitudes occur because of the rough approximation of the bay by only two grid squares. This results in a fictitious amplification of the oscillation resulting in a 30% difference between the calculated and observed range at Konstanz-Jakobsbad as shown in Table 4.

The western node is situated 1 km to the west of the line between Birnau and Litzelstetten, whereas the central node crosses the lake just west of the line between Friedrichshafen-Strandbad and Romanshorn. The eastern node appears far at the east end between Bad Schachen near Lindau and the head of the peninsular Rohrspitz on the southern shore. The spectral analysis of the two cases observed on August 20th and November 11th in 1969 yields satisfactory results compared with the calculation, if we disregard the differences in the Bay of Konstanz (see Table 4).

Forel [3] explained the third mode wrongly as the second mode. He concluded this from the usual relation between the eigen-periods of the fundamental mode and the higher ones according to Merian's formula. The misinterpretation resulted from the insufficient horizontal resolution of his measurements and from the assumption that there is an analogy to Lake Geneva, where the first and second eigen-periods accidentally satisfy Merian's relation.

For the third mode, there is good agreement between our calculated structure and that of Hamblin and Hollan. The positions of the nodes are

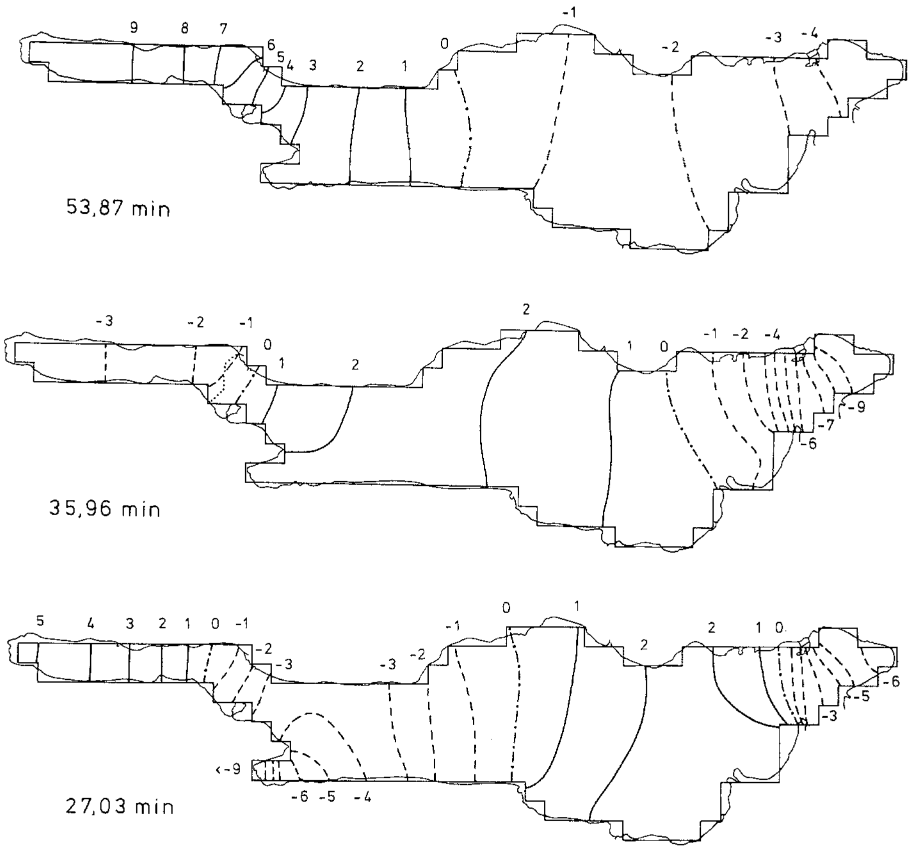

Fig. 6. Calculated amplitude distribution and period of the lowest Lake Constance mode (53.87 min), the second Lake Constance mode (35.96 min) and the third Lake Constance mode (27.03 min). Amplitudes are represented by co-range lines in steps of 10% of the maximum elevation

quite well reproduced, but the ranges at the ends of the lake differ by 20% just in the opposite sense. In the Bay of Konstanz Hamblin and Hollan used a finer resolution, thereby eliminating the fictitious amplification.

3.4 Fourth Longitudinal Mode (19.84 min)

The structure of this mode exhibits high ranges in the Bay of Konstanz and the bay east of the mouth of the Old Rhine (Fig. 7) due to the coarse approximation by the numerical grid. Except for these local differences the structure is in qualitative agreement with the observations. The verification is less precise, since the calculated amplitude distribution is displayed with low resolution throughout the main part of the lake.

In order to illustrate the qualitative fit of the calculated ranges to those observed on August 20th, 1969, the latter have been entered into the diagram in Fig. 7 by italic numbers. As noted in Table 4, the observed values have been normalized to the calculated amplitude at Romanshorn, where a line of equal range coincides with a measurement station.
At Lindau and Hard no peak occurs in the spectra at the eigen-period of the fourth mode. This result corresponds well with the nearby position of the eastern node. The next node to the west is aligned transverse to the lake between Arbon and Langenargen, where the observed low amplitude verifies the close proximity of the station to the nodal line.

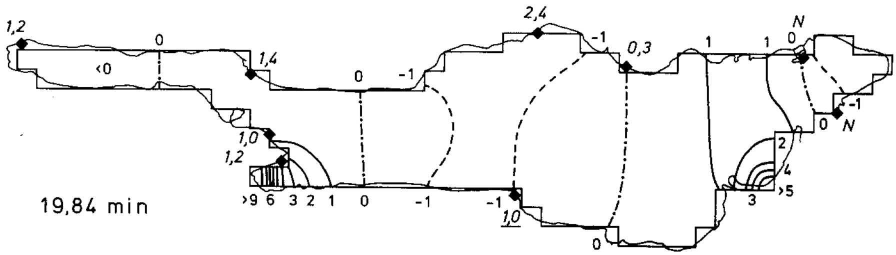

Fig. 7. The fourth Lake Constance mode (explanation of numbers in Fig. 6, observed elevations are entered by italic letters at the measurement stations shown in black squares, see Table 4 and the text for further explanation)

The remaining two western nodes cross the lake at the lines Altnau-Hagnau and Klaushorn-Überlingen. As expected, there is only a moderate increase of the observed range, when we proceed from the station Konstanz-Staad into the Bay of Konstanz. At the station of Konstanz-Jakobsbad the value is only 20% higher than in Konstanz-Staad compared with an increase of about 200% in the calculation.
The fourth mode has been detected neither by Forel nor by Mühleisen and Kurth. Hamblin and Hollan describe the structure by four anti-clockwise amphidromies just at the same positions of the nodes in our solution. Further details are scarcely reproduced by their calculations because of a strong cooscillation in the bay across from Lindau. This singular resonant structure is probably artificial since the bathymetry of the bay is very irregular and has been approximated by only one finite element.

4. Interpretation of the Wonder of the Rising Water at Konstanz by the Fundamental Mode of the Bay of Konstanz

The report on the water wonder at Konstanz given by the chronicler Christoph Schulthaiss [14] has been referred too by Forel [3], Thorade [16], Keller [6], Neumann and Pierson [9], Mühleisen and Kurth [8] as well as Hamblin and Hollan [4]. In the latter investigation an attempt was made to explain the phenomenon using a two-dimensional hydrodynamic model. The calculations indicated the possibility of a resonant coupling between a higher order normal mode and the fundamental mode of the Bay of Konstanz. However, no attempt was made to calculate the bay period precisely. In the present study, the period of the Bay of Konstanz was determined following the procedure used by Rao et al. [13] in their study of the Green Bay normal modes. Before we discuss these results, a translation of the Old German text is given below in order to explain the event Christoph Schulthaiss observed at Konstanz in 1549. In order to understand the report, some old localities mentioned in this report need to be referenced. For this purpose a map of the town from 1700 is reproduced in Fig. 8 showing the locations denoted in the translation by numbers *1* through *5*.

The old text reads:

"The wonder of the rising water"

"In the morning of this day, February 23 in 1549, the surface of the lake rose and fell by about one ell (59 cm). At high water the level rose up to the corner of the hospital *1*, at low water it fell so far that the water was swirling around the bases of the piles of the fishermen's jetty *3*. As soon as it had sunk this far it came surging back as if the waves had been driven by the wind (though there was no wind). This happened four or five times in about an hour as I saw myself. This continued until after noon, but decreased as time went on. The same thing happened further down in the Rhine. Several people from Paradis *5* wanted to raise their fish traps and found the Rhine was flowing on this day upstream towards the town and the Rhine bridge *4*, whereas it normally flows away from them. It also flowed backwards and forwards at the same time as the lake at the landing-place *2* and the fishermen's jetty *3*. This caused great astonishment among the people, since there was nobody who had ever before heard of such a thing happening."

We converted the observed wave-height of one ell into cm according to the definition of the old unit mentioned by Jänichen [5]. He reported that the unit of a short ell was in use in Konstanz in 1534. It was defined by the following relation between three linear measures:

$$1 \text{ Rute} = 6 \text{ ells minus } 1 \text{ Zoll} \tag{5}$$

where 1 Rute equals 3.5141 m and 1 Zoll represents the length of an inch, which is not precisely known for that time. If we assume the actual definition of an inch, i. e., 1 Zoll = 2.54 cm, the length of an ell results in 58.98 cm. Since the unit of 1 Zoll had the same small magnitude in former times, it is apparent from eq. (5) that the derived value of an ell depends only slightly on this quantity. For instance, if we introduce 1 badischer Zoll = 3 cm used in Konstanz until 1877 one ell amounts to 59.07 cm, which differs negligibly from the previous value.

It is obvious from Schulthaiss' description that an extraordinary seiche-like motion occurred in the Bay of Konstanz. The maximum amplitude of the oscillation amounts to about 30 cm, which represents one of the extreme values in Lake Constance. The period value is estimated less precisely to be between 12 and 15 min, but may also come to twice that value, since it is not clear whether Schulthaiss counted the number of complete or the number of half oscillations.

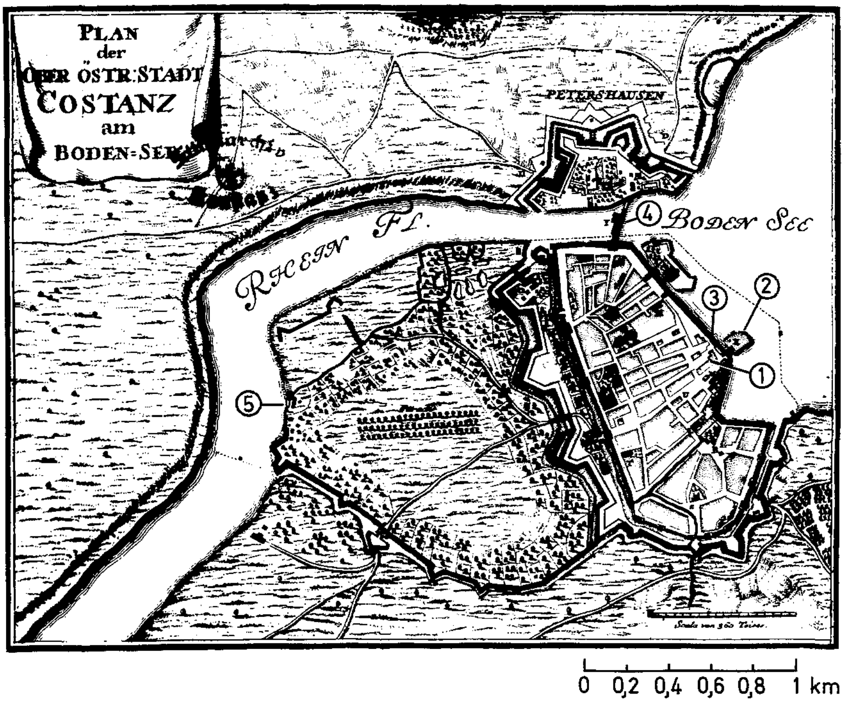

Fig. 8. Map of the city of Konstanz and surroundings ca. 1700. Locations denoted by numbers: *1* Heilig Geist hospital, *2* landing-place, *3* site of the fishermen's jetty, *4* Rhine bridge, *5* site of the former village of Paradis. The scale given in toise has been converted to km using the following definitions of former French linear measures quoted by Weisbach [17]: 1 toise = 6 old feet, 1 old foot = 0.324839 m (reproduction of the map by courtesy of the Archives of the city of Konstanz)

A possible explanation for the large oscillation in the bay is a resonant excitation of a natural mode of the bay – possibly, the lowest one – by one of the higher modes of Lake Constance. In order to substantiate this hypothesis, it is necessary to determine if the Lake has a natural mode (or several modes) with a period in the range of 12–15 min and also if the bay has a natural period in the same range. Table 5 shows the values of some of the higher normal modes of the Lake Constance system that are obtained from the computations. Since the calculations were made using a 235 × 235 matrix, yielding 235 eigen-values, it is reasonable to assume that the period estimates of the lowest 10 modes may be well determined. Since the structures of these modes tend to be complicated, we made no attempt to classify the oscillations as being longitudinal, or transverse or mixed but simply arranged the periods in a descending order and numbered the modes sequentially from 5 to 10. It is clear from Table 5 that the Lake Constance system does indeed have some modes with periods in the range of 12–15 min.

Table 5. *Calculated Periods of Higher Normal Modes of Lake Constance* (period in min)

Mode	5	6	7	8	9	10
Calculated period	17.86	16.13	14.83	13.96	12.06	11.65
Period corrected to 100 m chart depth:	18.11	16.36	15.04	14.16	12.23	11.81

Consider now the question of determining the natural period of the fundamental or the lowest mode of the Bay of Konstanz. An important problem in determining the periods of oscillation of a bay is the location of the mouth of the bay, where the boundary condition of zero height fluctuation has to be prescribed. Since the Bay of Konstanz is physically a part of the Lake Constance system, it is not obvious a priori where the mouth of the bay is located. However, by first solving for the oscillations of the entire Lake Constance system including the Bay of Konstanz, as we have done here, it is possible to locate the approximate position of the mouth of the bay.

The bay mode is characterized by an oscillation whose amplitude is essentially confined to the bay with very little amplitude over the remaining part of the lake. After the bay mode is thus identified, it is possible to carry out a further calculation for the period of oscillation using a finer grid over the domain spanning the bay and its mouth. Such a procedure has been used successfully by Rao et al. [13] in determining the oscillations of Green Bay in Lake Michigan and by Schwab and Rao [15] for the oscillations of Saginaw Bay and Georgian Bay in Lake Huron.

Fig. 9 shows the structure of the mode with a period of 17.86 min. This period is the fifth lowest in sequence and a possibility exists that this may be interpreted as the fifth longitudinal mode. However, the structure of the mode shows that the oscillation is essentially confined to the Bay of Konstanz with a curved nodal line close to the edge of the bay with very little amplitude elsewhere in the lake. Hence, this mode has the appearance of the fundamental mode of the bay but computed on a grid of 1.4 km interval. The 1.4 km grid, however, provides a rather poor resolution of the bay. Hence, to compute the period of oscillation of the bay more precisely, a grid of 0.5 km is used to cover the domain between the head of the bay and

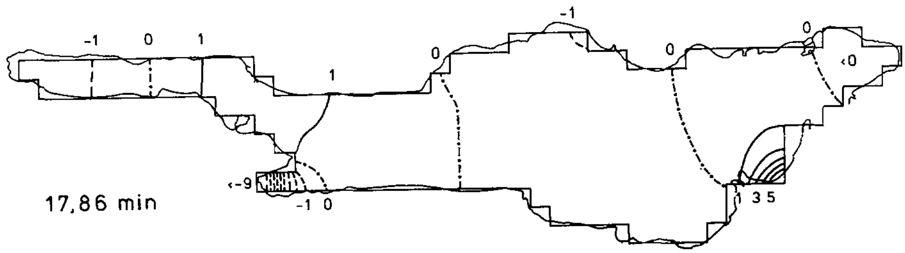

Fig. 9. The fifth Lake Constance mode showing inverse resonant structure in the Bay of Konstanz (explanation of numbers in Fig. 6)

its mouth as determined from the coarse grid calcualtion. The geometry of the bay on the 0.5 km grid is shown in Fig. 10. In this figure, there are two nodal lines. The inner one is obtained from the 1.4 km grid model. In order to examine the effect of changing the location of the nodal line on the

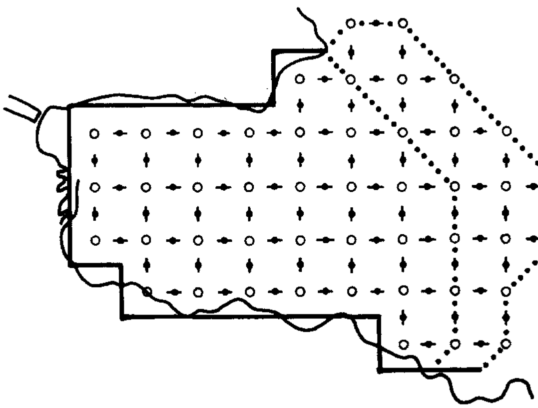

Fig. 10. Numerical grid on the Bay of Konstanz with grid spacing of 0.5 km. The two approximations of the mouth applied in the calculation are represented by dotted lines. Symbols of the grid have the same meaning as in Fig. 2

period of oscillation, an outer nodal line is also considered. The latter is arrived at by interpolating between the locations of this line as obtained from the 1.4 km grid model and a 2 km grid model which was used in some earlier calculations.

The period of oscillation of the lowest mode of the Bay of Konstanz using the inner nodal line is 12.8 min. Using the outer nodal line, it is found to be 13.6 min. The structure of the lowest mode using the inner nodal line is shown in Fig. 11. The outher nodal line does not change the modal structure in any essential way. The inner and outer nodal lines considered here represent respectively the inner and outer limits within which the real mouth of the bay can be expected to lie. Hence the actual period of the bay must be between the computed values of 12.8 and 13.7 min, and this conclusion is consistent with the observation of Schulthaiss [14]. Since Lake Constance has at least three modes with periods in the range of 12–15 mins, a resonant excitation of the Bay of Konstanz mode by one (or more) of the higher lake modes is a very viable hypothesis. Such higher order normal modes could have been excited by a small scale localized meteorological disturbance.

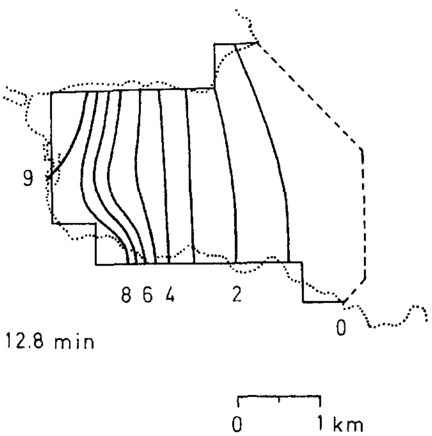

Fig. 11. Fundamental mode of the Bay of Konstanz obtained with the inner nodal line (explanation of numbers in Fig. 6)

Although Schulthaiss' description indicates that he was aware of the characteristics of wave motions, we have to consider an alternative interpreation. The German text raises the possibility that he counted in units of half oscillation in describing the event. In this case the period range of the event is between 24 and 30 min. An explanation in terms of a resonant oscillation will then have to involve the third longitudinal mode (eigenperiod 27 min). The verification analysis given in Table 4 yielded a weaker structure for the third mode in the Bay of Konstanz than was predicted (Fig. 6; 27.03 min). From the increase of the relative amplitudes between the stations Konstanz-Staad and Konstanz-Jakobsbad in Table 4 we may estimate the elevation in the inner bay to be about the same as at the western end at Ludwigshafen. This condition, however, implies that the oscillation must have been excited strongly throughout the lake, which was not confirmed by the observations.
We now consider some recent observations of water level fluctuations at Kreuzlingen near Konstanz in order to determine the observed period of the

cooscillation mode from spectral analysis. At this station a gauge of the same resolution as Forel's instrument (Sarasin's limnograph) was run for the period from November 10th through December 18th in 1972 by the Swiss Federal Institute of Environmental Protection (Schweizerisches Bundesamt

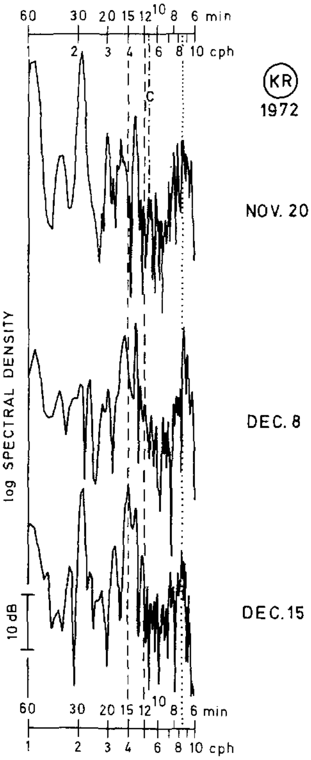

Fig. 12. Power spectra for the station of Kreuzlingen from November 20th, December 8th and 15th in 1972. The range of periods of Schulthaiss' [14] observations is marked by dashed vertical lines. *C* denotes the period of 11.5 min in the spectrum from November 20th. The peaks marked by the dotted line indicate an unknown higher order mode

für Umweltschutz, Landeshydrologie). 11 Sections of the record were subjected to spectral analysis and 8 of them exhibited distinct peaks in the range of periods between 12 and 15 min.

Three examples of these spectra are assembled in Fig. 12. For convenience we marked the limits of the interesting range of periods by dashed vertical lines. The central peak in this interval is located at 13.4 min in the cases of November 20th and December 8th. It is slightly shifted to 13.5 min on

December 15th, when the mean water level of the lake had decreased by about 20 cm. These spectral estimates are in good agreement with the computed value corresponding to the outer nodal line in Fig. 10. An interesting spectral evidence of the existence of the higher lake oscillation is indicated in a simultaneous measurement at Lindau on November 20th. This spectrum is represented in Fig. 14. The peak at the period marked by the letter S is approximately 13.2 min. Its amplitude is considerably smaller than in the bay as would be expected.

As Forel [3] pointed out, an oscillation with a period of about 15 min is typical for the lake at Konstanz. From the spectra of the records on December 8th and 15th we also obtain peaks at 16.4 min and 15.0 min, respectively. Probably these oscillations represent signals from the normal modes of Lake Constance labelled mode numbers 6 and 7 in Table 5. Since both these eigen-periods are close to the period of the bay, they will also tend to exhibit a strong resonance in the bay. It is possible that these modes could have made secondary contributions in the event of "water wonder".

The repeated occurrence of these two oscillations in the few measurements from the gauge at Kreuzlingen indicates that the resonance to the bay is often present in the water level fluctuations, though scarcely visible to the naked eye. The event in 1549 represents obviously one of the rare cases, when the conditions of a meteorological disturbance must have been most favorable for excitation of the appropriate higher order modes in Lake Constance with a resulting resonant amplification of these modes in the Bay of Konstanz.

5. Example of a Higher Order Normal Mode

The higher order modal solutions are more strongly influenced by the grid discretization. The amplitude distribution of these modes are, in general, very complicated and the computed structure may not be a satisfactory approximation to the true solution. However, there are some higher modes which appear to exhibit a coherent and simple structure. Even though no observations are available to verify the computed structures of these higher modes, an example of one such higher order mode with a coherent structure is shown in Fig. 13. This particular mode has an eigen-period of 11.49 min and consists of 6 longitudinal nodes and one in transverse direction as shown in Fig. 13. From the position of the latter nodal line at the mouth of the Bay of Rorschach it appears that the oscillation is characterized by a cross-oscillation in the broad central part of the basin. Since the prominent feature of this mode is the large amplitudes at Rorschach on the south side and Langenargen on the north side of the eastern part of the lake, it appears reasonable to consider this mode as a cross-channel oscillation between these two regions.

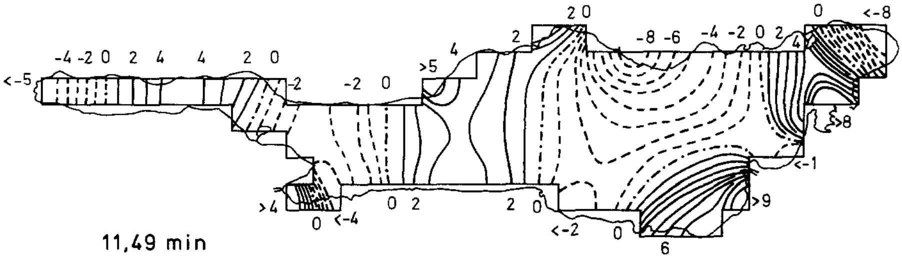

Fig. 13. Cross-oscillation in the central part of Lake Constance between Friedrichshafen and Rorschach obtained with 2.0 km grid spacing (explanation of numbers in Fig. 6)

It spectral evidence is apparent in a single observation at Lindau in the eastern part of the lake, where the range of elevation is generally higher than in the western half of the basin. The record was obtained simultaneously with that from Kreuzlingen on November 20th, 1972, which spectrum is shown in Fig. 12. In the spectrum from Lindau which is presented in Fig. 14 we realize a stronger contribution of energy in the range of

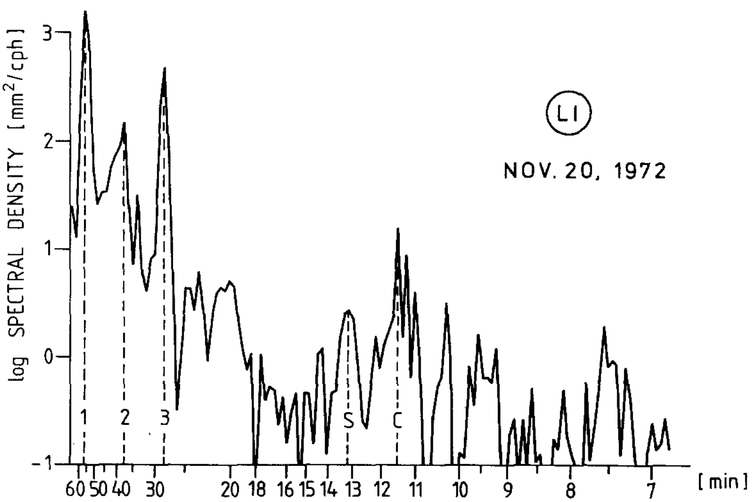

Fig. 14. Power spectrum for the station of Lindau from November 20th, 1972. Numbers denote the peaks of low order modes, whereas the peak in the range of periods observed by Schulthaiss [14] is marked by the letter S and that of the 11.5-min-oscillation by the the letter C

periods around 11.5 min, while in the same range at Kreuzlingen there is only a weak peak. It is marked in Fig. 12 by the letter *C*. The exact period resulting from the observation at Lindau amounts to 11.51 min (vertical line in Fig. 14 labelled by *C*). This value is precise to 1.1% due to the spectral resolution. In view of the low time resolution of the German records this agreement between calculation and measurement is remarkable.

Acknowledgements

The calculation of the normal modes was carried out at the Great Lakes Environmental Research Laboratory of the NOAA, Ann Arbor, Michigan, U.S.A., during a working visit of the first author in November 1975. The assistance of D. J. Schwab in making the calculations is greatly appreciated. We are very grateful to Prof. R. Mühleisen at the Institute of Astronomy of the University of Tübingen, who prepared the ground for the verification analysis by his measurement programme from 1967 through 1974, which was sponsored by the German Research Association. He provided the water level data from the German shore and informed us on necessary details as well as special circumstances of the measurements, which became important in the course of the analysis. We record our thanks also to the Swiss Federal Institute of Environmental Protection which ran the high precision gauge at Kreuzlingen simultaneously with the German stations in 1972. This collaboration was initiated by Prof. C. H. Mortimer, who also recommended later the calculation of the normal modes in the U.S.A.
The translation of Christoph Schulthaiss' observation was made by A. G. Kemball, MSc, then member of the scientific staff of the Institute of Environmental Protection, Karlsruhe, using an authentic transcription of the German original text and some historical comments, both kindly provided by Dr. H. Maurer, director of the archives of the city of Konstanz.

References

1. Blackman, R. B., Tukey, J. W.: The Measurement of Power Spectra, 190 p. New York: Dover Publications 1958.
2. Cartographical Survey of the State of Baden-Württemberg: Umgebung des Bodensees, Blatt 1 und 2, Topographische Karte 1 : 50 000, Stuttgart (1968).
3. Forel, F. A.: Die Schwankungen des Bodensees. Schr. d. Vereins f. Gesch. d. Bodensees *22*, 49–77 (1893).
4. Hamblin, P. F., Hollan, E.: On the Gravitational Seiches of Lake Constance and Their Generation. Schweiz. Z. Hydrol. *40*, 119–154 (1978).
5. Jänichen, H.: Wirtschaft und Verkehr. In: Der Landkreis Konstanz, amtliche Kreisbeschreibung (Staatliche Archivverwaltung Baden-Württemberg, Hrsg.), Vol. I, p. 361–404 (1968).
6. Keller, A.: Das „Wasserwunder" von Konstanz 1549. Schr. d. Vereins f. Gesch. d. Bodensees *72*, 173–179 (1953).
7. Kiefer, F.: Die Wasserstände des Bodensees seit 1871. Schr. d. Vereins f. Gesch. d. Bodensees *83*, 1–31 (1965).

8. Mühleisen, R., Kurth, W.: Experimental Investigations on the Seiches of Lake Constance. Schweiz. Z. Hydrol. *40*, 155–168 (1978).
9. Neumann, G., Pierson, W. J.: Principles of Physical Oceanography, 545 p. Prentice Hall: Englewood Cliffs 1966.
10. Platzman, G. W., Rao, D. B.: The Free Oscillations of Lake Erie. Studies on Oceanography, p. 359–382, University of Tokyo Press, 1964.
11. Platzman, G. W.: Two-dimensional Free Oscillations in Natural Basins. J. Phys. Oceanogr. *2*, 117–138 (1972).
12. Rao, D. B., Schwab, D. J.: Two-dimensional Normal Modes in Arbitrary Enclosed Basins on a Rotating Earth: Application to Lakes Ontario and Superior. Phil. Trans. R. Soc. Lond. A *281*, 63–96 (1976).
13. Rao, D. B., Mortimer, C. H., Schwab, D. J.: Surface Normal Modes of Lake Michigan: Calculations Compared With Spectra of Observed Water Level Fluctuations. J. Phys. Oceanogr. *6*, 575–588 (1976).
14. Schulthaiss, C.: Wunder anloffen des wassers. Collectaneen, Vol. VI, p. $80^1/_2$–81, Stadtarchiv Konstanz, A I 8 (1549).
15. Schwab, D. J., Rao, D. B.: Gravitational Oscillations of Lake Huron, Saginaw Bay, Georgian Bay and the North Channel. J. Geophys. Res. *82*, 2105–2116 (1977).
16. Thorade, H.: Probleme der Wasserwellen, 219 p. Hamburg: Henri Grand 1931.
17. Weisbach, J.: Der Ingenieur. Sammlung von Tafeln, Formeln und Regeln der Arithmetik, der theoretischen und praktischen Geometrie sowie der Mechanik und des Ingenieurwesens. 3. Aufl., 862 pp. Braunschweig: F. Vieweg & Sohn 1860.
18. Zeppelin, Graf E.: Die hydrographischen Verhältnisse des Bodensees. Bodensee-Forschungen, Teil III. Schr. d. Vereins f. Gesch. d. Bodensees *22* (Suppl.), 61–103 (1893).

Authors' addresses: Dr. E. Hollan, Landesanstalt für Umweltschutz Baden-Württemberg, Institut für Wasser- und Abfallwirtschaft, Griesbachstraße 3, D-7500 Karlsruhe 21, Federal Republic of Germany; Dr. D. B. Rao, Great Lakes Environmental Research Laboratory, National Oceanic and Atmospheric Administration, 2300 Washtenaw Avenue, Ann Arbor, MI 48104, U.S.A.; Dipl.-Ozeanogr. E. Bäuerle, Institut für Meereskunde an der Universität Kiel, Düsternbrooker Weg 20, D-2300 Kiel 1, Federal Republic of Germany.