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Intensional Isomorphism and Identity of Belief

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THE criterion that beliefs expressed by given sentences are identical if and only if the sentences are intensionally isomorphic is contained in Carnap's analysis of belief statements (in *Meaning and Necessity*, §§13–15). And it may be advantageous to separate this criterion from other features of Carnap's analysis, in order to examine it independently.

For our present purpose it will be sufficient to confine attention to a single language, which we may take to be Carnap's S_1 with various individual and predicator constants added to it as required,[1] and to consider L-equivalence and intensional isomorphism, only of designator matrices[2] within this one language and containing the same free variables. It will be recalled that Carnap's definition of 'intensionally isomorphic' depends on a definition of the semantical term 'L-true.'[3] The designator matrices A and B, containing the same free variables, are then said to be L-equivalent if and only if the closure of $A \equiv B$ is L-true.[4] And two designator matrices containing the same free variables are said to be intensionally isomorphic if one can be obtained from the other by a series of steps which consist

of (1) alphabetic changes of bound variable, (2) replacements of one individual constant by another which is L-equivalent to it, and (3) replacements of one predicator constant by another which is L-equivalent to it.[5]

To intensional isomorphism, in this sense, as criterion of identity of belief, there are objections which may be offered on the basis of Carnap's Principle of Tolerance, the principle namely that *every one is at liberty to build his own form of language as he will.*[6]

By the Principle of Tolerance, no one shall forbid us to introduce two completely synonymous predicator constants, or two completely synonymous individual constants, into a language (such as Carnap's S_1), if we choose to do so. Exactly this situation is evidently contemplated in Carnap's definition of 'intensionally isomorphic,' and published informal discussions of the definition have in fact sought out examples of synonymous constants, e.g., in the English language, to be used for purposes of illustration. It is true that formalized languages constructed by logicians rarely contain synonymous primitive constants, as it is clear that the inclusion of such synonyms among the primitive constants would not be consistent with the logician's usual demand for economy of primitives. But to object to a language on the ground of lack of economy is not to say that it is an inadmissible language, but only that it fails to serve a certain purpose. (And the same language which fails to serve one purpose may for that very reason better serve another.)

However, by the Principle of Tolerance, it is also possible to introduce into a language like S_1 two predicator constants (or two individual constants) which are L-equivalent but not synonymous. For example, let the individuals be the positive integers, and let P and Q be predicator constants, such that P*n* expresses that *n* is less than 3, and Q*n* expresses that there exist x, y, and z such that $x^n + y^n = z^n$. It is of course permissible to introduce P and Q as primitive constants, together perhaps with axioms containing them, such as may be suggested by their meanings.[7] For the sake of illustration let us suppose that Fermat's claim, to have had a proof of his (now so-called) Last Theorem, was correct. Then P and Q are L-equivalent, and it may even be possible to prove $(n)[Pn \equiv Qn]$ from the axioms. Yet it is evident that one might believe that $(En)[Qn \sim Pn]$ without believing that $(En)[Pn \sim Pn]$, since the proof of Fermat's Last Theorem, though it be possible, is certainly difficult to find (as the history of the matter shows).

Thus if intensional isomorphism is to serve as criterion of identity of belief, Carnap's definition requires the following amendment:

In (2) and (3) as given above, the condition of L-equivalence shall be replaced by that of synonymy.

Again by the Principle of Tolerance it is possible to introduce a predicator constant which shall be synonymous with a specified abstraction expression of the form $(\lambda x)[..x..]$; or to introduce an individual constant synonymous with a specified individual description of the form $(\iota x)[..x..]$.* And (unlike the case of synonymous primitive constants) it may be held that something like this actually occurs in formalized languages commonly constructed—namely those in which definitions are treated as introducing new notations into the object language,[8] rather than as metatheoretic abbreviations. But whether or not the process is called definition, it is clear by the Principle of Tolerance that nothing prevents us from introducing (say) a predicator constant R as synonymous with the abstraction expression $(\lambda x)[..x..]$, and taking $R \equiv (\lambda x)[..x..]$ as an axiom.[9] And if this is done, then R must be interchangeable with $(\lambda x)[..x..]$ in all contexts, including belief contexts, being synonymous with $(\lambda x)[..x..]$ by the very construction of the language—by definition, if we choose to call it that.

Thus we are led to a second amendment of Carnap's definition, as follows:

In addition to (1), (2), and (3), as given above, steps of the following kinds shall also be allowed: (4) *replacement of an abstraction expression by a synonymous predicator constant;* (5) *replacement of a predicator constant by a synonymous abstraction expression;* (6) *replacement of an individual description by a synonymous individual constant;* (7) *replacement of an individual constant by a synonymous individual description.*

For intensional isomorphism as modified by these two amendments of Carnap's definition, let us introduce the name 'synonymous isomorphism.' It is proposed that synonymous isomorphism, as thus defined for the language S_1, and as extended by more or less obvious analogy to many other languages,[10] should replace Carnap's intensional isomorphism as criterion of identity of belief.

In order to make this possible, it is necessary to provide a determination of synonymy as a part of the semantical basis of S_1, or of other language employed. This might be done directly, by means of *rules of synonymy* and *rules of non-synonymy*, or it might be done indirectly by means of *rules of sense*.[11] In either case there are certain obvious limitations upon the Principle of Tolerance which must be taken into account: for example, though we are at liberty in introducing a new constant to fix its meaning in any non-circular fashion that we please, and in particular to make it synonymous with any expression already at hand, we may not by arbitrary convention make the constant synonymous with an expression containing

* EDITORS' NOTE: Limitations of the linotype font have made it necessary to use the regular instead of the inverted iota.

that same constant; and having once fixed the meaning of a constant, we are not then free to make further arbitrary conventions about its meaning (in particular, the same constant may not be made synonymous with two different expressions unless one of these synonymies can be shown to be a consequence of the other).[12]

II

Since our proposal of synonymous isomorphism is almost opposite in tendency to a modification of intensional isomorphism which is proposed in a recent paper by Hilary Putnam,[13] and which seems to be at least partly supported by Carnap,[14] it becomes necessary to consider Putnam's proposal, and in fact to rebut it (in the sense of showing it to be superfluous) if our own is to be maintained. Both Putnam and Carnap rely heavily on a brief remark in a paper of Benson Mates,[15] in such a way that it will be sufficient for our purpose if Mates's remark (as interpreted by Putnam) can be refuted.

Mates introduces two sentences D and D′ which shall be particular sentences that are different but intensionally isomorphic. The two sentences D and D′ being not otherwise specified by Mates, let us choose them for the purpose of the present discussion as follows:

D. The seventh consulate of Marius lasted less than a fortnight.

D′. The seventh consulate of Marius lasted less than a period of fourteen days.

For the sake of the illustration, we suppose that the word 'fortnight,' in English, means a period of fourteen days and is synonymous with 'a period of fourteen days.'[16] And in order to secure the complete synonomy of D and D′, we have used in D′ the phrase 'less than a period of fourteen days' rather than the shorter and more natural 'less than fourteen days.'

The sentences D and D′, as chosen above, are then not intensionally isomorphic but synonymously isomorphic. They serve our present purpose the better for that very reason. In fact Mates, though directing his remark in the first instance against intensional isomorphism, concludes by saying that it is not affected if 'intensionally isomorphic' is replaced by 'synonymous' throughout. And in reproducing Mates's argument we shall replace his 'intensionally isomorphic' everywhere by 'synonymously isomorphic'—synonymous isomorphism being our proposed explicatum of synonymy.

Consider then, following Mates, the two sentences:

(14) Whoever believes that the seventh consulate of Marius lasted less than a fortnight believes that the seventh consulate of Marius lasted less than a fortnight.

(15) Whoever believes that the seventh consulate of Marius lasted less

than a fortnight believes that the seventh consulate of Marius lasted less than a period of fourteen days.

According to Mates, it is true that:

(16) Nobody doubts that whoever believes that the seventh consulate of Marius lasted less than a fortnight believes that the seventh consulate of Marius lasted less than a fortnight.

But is not true that:

(17) Nobody doubts that whoever believes that the seventh consulate of Marius lasted less than a fortnight believes that the seventh consulate of Marius lasted less than a period of fourteen days.

In fact a counter-example against (17) is evidently provided by philosophers who have considered the question of the criterion of identity of belief, and perhaps in particular by readers of this paper. For by considering this question of philosophical analysis, one is almost inevitably led, at least tentatively, to doubt that (15), or else to entertain an analogous doubt in the case of some other pair of synonymously isomorphic sentences (in place of D and D'). Even if this doubt is afterwards overcome by some counter-argument, the very possibility of entertaining the doubt that (15), without simultaneously doubting that (14), shows (14) and (15) to be non-interchangeable in belief contexts.[17] The historical facts as to who has doubted what or as to the truth of (16) are not really relevant here,[18] but only the *possibility* of doubting that (15) without doubting that (14). Since, according to Mates, (14) and (15) are synonymously isomorphic, the result is to discredit synonymous isomorphism as criterion of identity of belief.

It must be understood that those who are supposed to have doubted that (15) without doubting that (14) are supposed also to have had a sufficient knowledge of the English language so that the doubt was not, for example, a doubt about the meaning of the word 'fortnight' in English.

Nevertheless it is natural to suggest as a means of overcoming Mates's difficulty that it is after all not possible to doubt that (15) without doubting that (14); and that the doubt which has been or may have been sometimes entertained by philosophers in considering the question of the criterion of identity of belief is not the doubt that (15), but a doubt that does have reference to linguistic matters, namely the doubt that:

(18) Whoever satisfies in English the sentential matrix 'x believes that the seventh consulate of Marius lasted less than a fortnight' satisfies in English the sentential matrix 'x believes that the seventh consulate of Marius lasted less than a period of fourteen days.'[19]

If this suggestion can be supported, the difficulty urged by Mates dis-

appears, as (18) is clearly not synonymously isomorphic either to (14) or to:

(19) Whoever satisfies in English the sentential matrix 'x believes that the seventh consulate of Marius lasted less than a fortnight' satisfies in English the sentential matrix 'x believes that the seventh consulate of Marius lasted less than a fortnight.'[20]

Now the test of translation into another language, originally suggested by C. H. Langford, is often valuable in determining whether a statement under analysis is to be regarded as a statement about some sentence, linguistic expression, or word, or rather as about something which the sentence, expression, or word is being used to mean.[21] I have used this test elsewhere[22] to support the conclusion that the object of a belief shall be taken to be a proposition rather than a sentence, if certain important features of the ordinary usage of indirect discourse are to be preserved. But I say that the same test in the present connection leads to a conclusion of opposite kind—namely that the doubt whose existence or possibility Mates urges (as a difficulty in the analysis of belief statements) is a doubt about certain sentential matrices, and thus a doubt that (18) rather than a doubt that (15).[23]

Let us therefore translate (14), (15), and (18) into German.

The translation of (18) is:

(18') Wer auf Englisch die Satzmatrix 'x believes that the seventh consulate of Marius lasted less than a fortnight' erfüllt, erfüllt auf Englisch die Satzmatrix 'x believes that the seventh consulate of Marius lasted less than a period of fourteen days.'

As soon as we set out to translate (14) and (15), our attention is drawn to the fact that the German language has no single word which translates the word 'fortnight,' and that the literal translation of the word 'fortnight' from English into German is 'Zeitraum von vierzehn Tagen.'[24] In consequence, the German translations of (14) and (15) are identical, as follows:

(14')(15') Wer glaubt dass das siebente Konsulat des Marius weniger als einen Zeitraum von vierzehn Tagen gedauert habe, glaubt dass das siebente Konsulat des Marius weniger als einen Zeitraum von vierzehn Tagen gedauert habe.

Of course we must ask whether the absence of a one-word translation of 'fortnight' is a deficiency of the German language in the sense that there are therefore some things which can be expressed in English but cannot be expressed in German. But it would seem that it can hardly be so regarded—else we should be obliged to call it a deficiency of German also that there is no word to mean a period of fifty-four days and six hours, or that the Latin word 'ero' can be translated only by the three-word

phrase 'ich werde sein.' Indeed it should rather be said that the word 'fortnight' in English is not a necessity but a dispensable linguistic luxury.

Granted this, let us translate into German 'Mates doubts that (15) but does not doubt that (14).'[25] As the resulting German sentence is a direct self-contradiction, and as it cannot matter to the soundness of our reasoning whether we carry it out in English or in German, we must conclude that Mates (whatever he himself may tell us) does not really so doubt—and that he must have mistaken the doubt that (18) for the doubt that (15).

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NOTES

[1] We also suppose that Carnap's sign '≡' of identity of individuals is a predicator constant, and that when A and B are individual expressions, A ≡ B is to be understood merely as an abbreviation or alternate way of writing ≡AB. This modification of S_1 serves to simplify the discussion but is not otherwise essential to the conclusions we reach.

[2] I follow Carnap's terminology, in spite of my own preference for a somewhat different terminology—e.g., 'well-formed formula' instead of 'designator matrix.'

[3] The definition of 'L-true' need not be repeated here. But notice should be taken of two necessary corrections to the definition as it is developed in §§1–2 of Carnap's book.

In 2-2 the correction of Kemeny must be adopted (*Journal of Symbolic Logic*, 16:206 (1951)); i.e., in place of "every state-description" the restriction must be made to non-contradictory state-descriptions. Otherwise consequences will follow that are certainly not intended by Carnap, for instance that no two different atomic sentential matrices (and no two different predicator constants) can be L-equivalent.

In the rules of designation 1-1 and 1-2, the way in which the English language and certain phrases of the English language are mentioned, rather than used, is inadmissible—as may be seen by the fact that it forces the tacit use, in 1-3 and 1-4, of certain rules of designation *of the English language*, which, if stated, would have a quite different form from 1-1 and 1-2. For example, Carnap's rule of designation, " 's' is a symbolic translation [i.e., from English] of 'Walter Scott'," should be changed to a rule which mentions the man Walter Scott rather than the words 'Walter Scott'; perhaps it should be simply " 's' refers to Walter Scott," in order to justify the inference from 1-3 to 1-4.

These corrections are not directly relevant to the present paper, but our discussion presupposes that suitable corrections have been made.

[4] Carnap uses '≡' not only between sentential matrices as a sign of material equivalence, but also between other designator matrices as a sign of identity (in place of the usual '=').

[5] Because of the restriction to the single language S_1 and to designator matrices containing the same free variables, we have been able to give a simplified form to Carnap's definitions of 'L-equivalent' and 'intensionally isomorphic.'

[6] In this form, as applied to the construction of a new language and the determination of what its expressions shall mean, the Principle of Tolerance is hardly open to doubt. The attempt to apply the Principle of Tolerance to the transformation rules of a language after the meaning of the expressions of the language has already been determined (whether by explicit semantical rules or in some looser way) is another matter, and certainly doubtful, but is not at issue here. In fact Carnap (if he ever did) does not now maintain the Principle of Tolerance in this latter and more doubtful form (see §39 of his *Introduction to Semantics*).

[7] There is no condition to the effect that a predicator constant must express a simple

property, rather than such a comparatively complex property as that which is here expressed by Q. In fact some of Carnap's examples of predicator constants express properties which are evidently not especially simple. And it is moreover not clear how the distinction between a simple and a complex property could be made precise in any satisfactory way (except by making it relative to the choice of a particular language).

[8] This is the account of definition which is given, for example, by Hilbert and Bernays in *Grundlagen der Mathematik*. In constructing formalized languages, others (including myself) have often preferred to avoid definitions in this sense, which change the object language by adding new notations to it. But such avoidance is on the same ground of economy that underlies the avoidance of synonymous primitive constants, and need not be demanded when economy is not the objective.

[9] Compare Carnap, *The Logical Syntax of Language*, §22, 1(b).

[10] In particular to any of the languages considered in my paper, "A Formulation of the Logic of Sense and Denotation" (*Structure Method and Meaning*, pp. 3–24), and to languages obtained from these by adding constants of any types, with specified meanings.

It is necessary to explain that the statement on page 5 of that paper, that Alternative (0) "may be described roughly by saying that it makes the notion of sense correspond to Carnap's notion of intensional structure" is an error (unless "roughly" is understood in a very liberal sense). The intention of Alternative (0) is rather that two well-formed formulas shall have the same sense if and only if they are synonymously isomorphic.

[11] See my "The Need for Abstract Entities in Semantic Analysis," *Proceedings of the American Academy of Arts and Sciences*, 80(No. 1):100–12 (1951).

[12] Compare the "rules of definition," originally Aristotelian, which are often included in books on traditional logic.

[13] "Synonymity, and the Analysis of Belief Sentences," *Analysis*, 14(No. 5):114–22 (1954).

[14] In a forthcoming paper, "On Belief Sentences: Reply to Alonzo Church."

[15] "Synonymity," *University of California Publications in Philosophy*, vol. 25 (1950), see the lower half of page 215.

[16] To treat the English language as a language for which syntactical and semantical rules have been fully given is of course to make a supposition contrary to fact, but it is one which is very convenient for illustrative purposes and has in fact been adopted in informal discussion by Carnap, Mates, Putnam, and many others. Use of this device has the effect that it may be necessary in the course of the illustration just to invent a rule of English, either to fill a gap in the rules as found in existing grammars and dictionaries or to remove an equivocacy. In the present context, for instance, we have been obliged to decide arbitrarily (or on the basis of mere plausibility) that 'fortnight' is synonymous with 'a period of fourteen days' rather than with 'a period of two weeks'; existing English dictionaries either fail to decide this point or disagree among themselves, probably because universal familiarity with the multiplication table tends to obscure the fact that the two latter (quoted) phrases are not synonymous with each other.

[17] A context of doubting is of course a belief context, since to doubt is to withhold belief. And a criterion of identity of belief must also be a criterion of identity of doubt.

[18] Doubt being one of the fundamentals of philosophical method, it would be hard indeed to find a proposition that some philosopher might not be found to doubt.

[19] The two occurrences of the phrase 'in English' would usually be omitted, but strictly they are necessary; for the semantical relation of satisfaction (or fulfillment) is a ternary relation among an individual, a sentential matrix, and a language.

[20] The point is that *names* of two different sentences are not synonymous in any sense, and in particular not synonymously isomorphic, even though the sentences themselves be synonymously isomorphic.

[21] (Added August 4, 1954.) The existence of more than one language is not usually to be thought of as a fundamental ground of the conclusions reached by this method. Its role is rather as a useful device to separate those features of a statement which are

essential to its meaning from those which are merely accidental to its expression in a particular language, the former but not the latter being invariant under translation. And distinctions (e.g., of use and mention) which are established by this method it should be possible also to see more directly. The point is well illustrated by a paper of Wilfrid Sellars, "Putnam on Synonymity and Belief," forthcoming in *Analysis*, in which conclusions the same as or similar to those of Part II of this paper are reached by a more direct analysis. Professor Sellars's paper and mine were written independently, but I saw a copy of it by return mail when my own was submitted to *Philosophical Studies*.

[22] *Analysis*, 10(No. 5): 97–99 (1950).

[23] The object of the doubt must still be a proposition, but a proposition *about* certain sentential matrices.

[24] The shorter translation 'vierzehn Tage' would be more usual, but is not quite literal, as may be seen by considering the question of translating the phrase 'three fortnights' into German.

[25] Of course '(14)' and '(15)' are here used, not as names of the sentences which we have so numbered, but just as convenient abbreviations. The reader must imagine the full sentences written out in place of the '(14)' and '(15).' Indeed throughout the paper such parenthetic numerals are to be understood as *abbreviations* when preceded by the word 'that'—but elsewhere as *names* of their sentences.

Professor Copi Concerning Analysis

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THAT no analysis can take place without analytical means is, presumably, axiomatic. But if that is so, then the account of analytical philosophy given by Professor Irving M. Copi in *Philosophical Studies* (vol. 4, no. 6) can hardly stand.

Professor Copi states that the business of philosophical analysis is to give "theoretical definitions of philosophical concepts" and that this "amounts to affirming the correctness of the theory in whose terminology the definition is formulated" (pp. 89, 90). But he bars the way to the accomplishment of analysis in these terms by pronouncing against what he calls "short-cut alternative methods," namely, "the touchstone of clarity," the criterion of "epistemological priority," and "consonance with ordinary language" (pp. 90, 91).

Consider the term 'truth,' which is among examples given by Professor Copi of the proper subject of philosophical analysis. Let us suppose this term to be analyzed into a relation of correspondence. We shall then need to analyze 'correspondence,' and perhaps that brings us to likeness, which in turn may lead to what we refer to by 'unanalyzable.' But we are to be estopped here, for anything unanalyzable will have to stand as something