Inventory, channel coordination and bargaining in a manufacturer-retailer system

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Substantial research literature has been developed over the years on the subject of inventory. The more recent literature has examined the fundamental relationships between inventory control and price theory. A significant portion of this literature assumes the ultimate consumer demand as a constant and characterizes the relationship between a manufacturer and a retailer as a leader-follower problem. A primary assumption in these studies is that the manufacturer, as the leader, exerts almost complete control over the behavior of the retailer. However, in practice, the retailer does exert some control over the manufacturer. This paper develops a framework that integrates inventory control with constant demand and the economic relationship between consumer demand and retail price. Within this framework, the impact of order quantity, wholesale price and retail price on the behavior of both the manufacturer and the retailer is investigated. Furthermore, this paper explores the issues and conclusions that result from coordinating the relationship between the manufacturer and the retailer. Our analyses demonstrate that channel coordination can be achieved by utilizing well-known bargaining models. A numerical example is provided to illustrate our theoretical findings.

Keywords: Inventory, bargaining, channel coordination, cross-constrained game.

1. Introduction

There are three major research streams in the literature of channel coordination between a manufacturer and a retailer. The first stream of the research assumes a perfectly competitive market. The ultimate consumer demand for the product is considered as a constant and operating costs, such as ordering costs, production setup costs, and inventory holding costs, depend on only order quantity. Examples of this stream include, but are not limited to, Banerjee [1], Buffa and Miller [2], Chakravarty and Martin [3], Chiang et al. [7], Dolan [8], Hadley and Whitin [10], Kohli and Park [14], Lal and Staelin [15], Lee and Rosenblatt [16], Peterson and Silver [23], Sethi [24], Weng and Wong [26], and Weng [27]. The second stream of the research focuses on a monopolistic market. The consumer demand for the product is considered as a function of retail price and operating costs are assumed as constants. Examples of this stream are Jeuland and Shugan [12], McGuire and Staelin [21], Etgar [9], Zusman and Etgar [30], and Williamson [29]. The third stream of the research investigates the impact of quantity discounts on the efficiency of manufacturer-retailer transactions where the retailer is in a monopolistic position for the product and operating costs are considered as a function of order quantities and retail price. Quantity discounts are shown to be effective mechanisms to achieve system efficiencies. Examples of this stream are Li and Huang [17], Li et al. [18, 19], and Weng [28].

This paper addresses the issues and problems of channel coordination in a manufacturer-retailer system where the retailer is in a monopolistic position for the product, i.e., the ultimate consumer demand is a function of the retail price, and operating costs depend on both order quantities and retail price. Our analyses are similar to studies in the third stream because this paper also integrates the first two streams of research by considering inventory control and price theory in a single framework. However, there are differences between our research and studies in the third stream. The studies in the third stream focus on the impact of quantity discounts on the efficiency of transactions in the manufacturer-retailer system. Our analyses yield insights into the role that order quantity, wholesale price and retail price play in efficient channel coordination of the manufacturer-retailer system.

Efficient channel coordination plays a significant role in modern manufacturing and retailing systems. A variety of problems can be addressed with the methodology developed by this research. One version of coordination between the manufacturer and the retailer is illustrated by several large rent-a-car companies. Suppose one or more of these companies are owned by the big automobile manufacturers, e.g., Hertz is owned by Ford. Some rent-a-car companies eventually sell those cars that have been utilized in the business. The price of second-hand cars sold by the rent-a-car companies is dictated by the automobile manufacturers since they have a majority (or minority) ownership in the rent-a-car business. In the 1980's, cooperation between the manufacturer and the rent-a-car dealers led to setting used car prices quite low and the used car business of rent-a-car companies expanded very rapidly.

As an alternative example, the relationship between a state electrical power authority, e.g., NYS power authority, and a regional electrical power company, e.g., the Long Island Lighting Company (LILCO), fits the methodology developed in this paper. In this situation, the retailer (LILCO) sells electrical power to its customers, and it occupies a monopolistic position as it is the only available supplier of electricity to the region. For a variety of reasons, LILCO also purchases power from the NYS power authority to supplement its supply. Consequently, the power authority as the seller of supplemental power occupies the position of the "manufacturer". Coordination between manufacturer and retailer, in this case the NYS power authority and LILCO, can enhance the outcomes of both the system and its participants.

We start our analysis in section 2 by delineating the assumed relationships and decision variables of the manufacturer and the retailer, or a group of homogenous retailers. Consumer demand is explicitly expressed as a downward sloping function of the retail price. We retain the assumptions, made by Monahan [22], Li [20], Li and Huang [17], and Li et al. [18, 19], that the manufacturer follows a lot-for-lot policy, i.e., the manufacturer produces only the amount ordered by the retailer, the replenishment time is constant or negligible, and the production rate is infinite, so that the manufacturer's inventory is immediately transferred to the retailer. The retailer's inventory policy is assumed to be the widely used Economic Order Quantity (EOQ) model. The manufacturer's decision variable is the wholesale price to charge the retailer, and the retailer's decision variables are the retail price and the order quantity. Both the manufacturer and the retailer maximize their annual average profits.

The case where the manufacturer is the leader and the retailer is the follower is discussed in section 3. In game-theoretic terminology, this is called a two-stage non-coordinate game or a "sequential-moves" game (see, for example, Simaan and Cruz 25] and Charnes et al. [4]). The manufacturer first declares the wholesale price, the retailer, under the Economic Order Quantity, then decides on the retail price. The unique equilibrium point is obtained.

We address system coordination in section 4. We show that if both the manufacturer and the retailer employ only the system EOQ order quantity in their coordination, the order quantity, the manufacturer's annual profit and the system's annual profit are higher, while the retailer's annual profit is lower than those at non-coordination. We also show that the coordinated retail price and wholesale price are lower, the coordinated manufacturer, retailer, and system annual profits are higher than those at non-coordination.

When system profits are increased, we must consider how these additional profits will be divided. The division of the additional profits between the manufacturer and the retailer are examined in section 5. One famous bargaining model is utilized, the Kalai and Smordinsky [13] model, to determine the profit division between the manufacturer and the retailer. The Kalai and Smordinsky model suggests that both parties equally share the system additional profits in order to achieve channel coordination.

A numerical example is provided to illustrate our findings in section 6 and concluding remarks are in section 7.

2. Model development

We consider a system consisting of one manufacturer and one retailer, or a group of homogeneous retailers. The retailer purchases an item from the manufacturer and resells it at the retail level with a self-determined price. The operating cost facing the retailer includes the purchasing cost, the inventory holding cost, and the ordering cost associated with each order. The operating cost facing the manufacturer includes only the setup cost associated with each production lot. The manufacturer's purchasing cost of materials is assumed to be zero because it can be included in its wholesale price to the retailer. We assume that the manufacturer has an infinite production rate and adopts a lot-for-lot policy. This means that the manufacturer's inventory is immediately transferred to the retailer. Therefore, the inventory holding cost is not considered for the manufacturer.

Similar to Jeuland and Shugan [12], Lal and Staelin [15], Kohli and Park [14], and Weng [27,28], we assume that the retailer's inventory policies can be described by the widely used Economic Order Quantity (EOQ) model. Next, it is assumed that the manufacturer has complete knowledge of the retailer's demands, inventory holding costs, and ordering costs. The retailer is assumed to be in a monopolistic position for the product, i.e., the annual demand at the retail level is a downward sloping function of the retail price.

The manufacturer's wholesale price is ω and the retailer's retail price is p. It is reasonable to assume $p \geq \omega$. In many industries, the retail price does not exceed a certain percentage of the wholesale price. Therefore, we assume $p \leq k\omega$, where k is a constant with $k > 1$. We also assume that there exists a cap, g, for the manufacturer's wholesale price, i.e., $\omega \leq g$. The downward sloping demand function at the retail level is assumed to be $D(p) = \tilde{\alpha} p^{-\beta}$, with $\tilde{\alpha} > 0$ and $\beta > 0$, where $\tilde{\alpha}$ and β are constants and β is the price elasticity of demand ($0 < \beta < 1$). Let S_r and S_m be the retailer's ordering cost per order and the manufacturer's setup cost per setup, respectively. The retailer's annual inventory holding cost is H_r. The retailer's order size is Q.

The manufacturer's annual profit is equal to gross revenue minus the production setup cost. Therefore, the manufacturer's annual profit function is given by

$$\pi_m(\omega, p, Q) = \omega D(p) - S_m D(p)/Q. \tag{1}$$

The manufacturer's decision variable is the wholesale price, ω.

Similarly, the retailer's average annual profit is equal to gross revenue minus the ordering cost and inventory holding cost. Then its functional form is given by

$$\pi_r(\omega, p, Q) = (p - \omega)D(p) - S_r D(p)/Q - QH_r/2. \tag{2}$$

The retailer's decision variables are the retail price, p, and the size of order quantity, Q.

3. Two-stage non-coordination game model

In this section, we model the manufacturer-retailer interaction as a two-stage non-coordinate game with the manufacturer as the leader and the retailer as the follower. The leader, who has the ability to enforce its strategies on the other player, announces its strategies first and imposes them on the follower. The follower then reacts to the leader's action and decides on its strategies.

As we assumed in section 2, the retailer adopts an EOQ order quantity model. Therefore, for any given retail price, p, the retailer's order quantity is $Q_r = [2S_r D(p)/H_r]^{1/2}$ and the associated ordering plus holding cost is $[2S_r H_r D(p)]^{1/2}$. Substituting the EOQ order quantity Q_r into equations (1) and (2) defines the manufacturer's and retailer's annual profit functions specified by equations (3) and (4) as follows:

$$\pi_m(\omega, p \mid Q_r) = \omega D(p) - S_m[H_r D(p)/(2S_r)]^{1/2}, \tag{3}$$

$$\pi_r(\omega, p \mid Q_r) = (p - \omega)D(p) - [2S_r H_r D(p)]^{1/2}. \tag{4}$$

The manufacturer, as the leader, first declares the wholesale price, ω. The retailer then decides on the retail price, p. To determine the equilibrium of the two-stage game, we first solve for the reaction function in the second stage of the game.

For any given wholesale price, ω, the retailer's objective is to choose the retail price, p, that maximizes his/her annual profit in (4) under the constraint that $k\omega \geq p \geq \omega$. Since $\partial\pi_r(\omega, p \mid Q_r)/\partial p > 0$, $\pi_r(\omega, p \mid Q_r)$ is a strictly increasing function, $p = k\omega$ is the optimal retail price for the retailer.

It can be seen that both the annual demand rate and the retailer's order quantity depend on the manufacturer's wholesale price, ω. The optimal wholesale price, ω, is determined at the first stage by maximizing the manufacturer's annual profit. Substituting $p = k\omega$ into $D(p)$ and Q_r, the manufacturer's annual profit can be rewritten as

$$\pi_m(\omega \mid Q_r) = \alpha\omega^{1-\beta} - S_m[\alpha H_r/(2S_r)]^{1/2}\omega^{-\beta/2}, \tag{5}$$

where $\alpha = \tilde{\alpha}k^{-\beta}$. Therefore, the manufacturer's problem is to maximize $\pi_m(\omega \mid Q_r)$ in (5) subject to the wholesale price cap constraint, i.e., $\omega \leq g$. Since $d\pi_m(\omega \mid Q_r)/d\omega > 0$, $\pi_m(\omega \mid Q_r)$ is a strictly increasing function of ω. Therefore, the manufacturer's optimal wholesale price is $\omega^* = g$, the retailer's optimal retail price is $p^* = kg$ and the retailer's optimal order quantity is $Q_r^* = [2\alpha S_r g^{-\beta}/H_r]^{1/2}$. Furthermore, optimal annual profits for the manufacturer and the retailer are

$$\pi_m^* = \alpha g^{1-\beta} - S_m[\alpha S_r/(2H_r)]^{1/2} g^{-\beta/2} \tag{6}$$

and

$$\pi_r^* = \alpha(k-1)g^{1-\beta} - [2\alpha S_r H_r)]^{1/2} g^{-\beta/2}, \tag{7}$$

respectively.

4. Coordination game model

In this section, we consider the situation in which both the manufacturer and the retailer are willing to coordinate to maximize their system profit. The system profit function is defined as the sum of the manufacturer's and the retailer's profits:

$$\pi_s(\omega, p, Q) = \pi_m(\omega, p, Q) + \pi_r(\omega, p, Q), \tag{8}$$

i.e.,

$$\pi_s(p, Q) = pD(p) - (S_r + S_m)D(p)/Q - QH_r/2. \tag{9}$$

For any given retail price, p, it is easy to show that the system ordering, setup and inventory holding cost is minimized by the following system EOQ formula:

$$Q_s = [2(S_r + S_m)D(p)/H_r]^{1/2}. \tag{10}$$

Substituting the system EOQ formula (10) into (1), (2) and (9), the manufacturer's, retailer's and system's profit functions can be rewritten as

$$\pi_m(\omega, p \mid Q_s) = \omega D(p) - S_m[H_r D(p)/(2(S_r + S_m))]^{1/2}, \tag{11}$$

$$\pi_r(\omega, p \mid Q_s) = (p - \omega)D(p) - (S_r/(S_r + S_m) + 1)[H_r D(p)(S_r + S_m)/2]^{1/2}, \tag{12}$$

$$\pi_s(p \mid Q_s) = pD(p) - [2(S_r + S_m)H_r D(p)]^{1/2}, \tag{13}$$

respectively. The following results show that if both the manufacturer and the retailer employ only the system EOQ order quantity in their coordination, the manufacturer's and the system's annual profits are higher but the retailer's annual profit is lower than those at non-coordination.

Theorem 1 For the given retailer's non-coordinated retail price, $p^* = kg$, and manufacturer's non-coordinated wholesale price, $\omega^* = g$, the relationships between $Q_s(p^*)$ and $Q_r(p^*)$, between $\pi_m(\omega^*, p^* \mid Q_s(p^*))$ and $\pi_m(\omega^*, p^* \mid Q_r(p^*))$, between $\pi_r(\omega^*, p^* \mid Q_s(p^*))$ and $\pi_r(\omega^*, p^* \mid Q_r(p^*))$, and between $\pi_s(p^* \mid Q_s(p^*))$ and $\pi_s(p^* \mid Q_r(p^*))$ are as follows:

$$Q_s(p^*) > Q_r(p^*), \tag{14}$$

$$\pi_m(\omega^*, p^* \mid Q_s(p^*)) > \pi_m(\omega^*, p^* \mid Q_r(p^*)), \tag{15}$$

$$\pi_r(\omega^*, p^* \mid Q_s(p^*)) < \pi_r(\omega^*, p^* \mid Q_r(p^*)), \tag{16}$$

$$\pi_s(p^* \mid Q_s(p^*)) > \pi_s(p^* \mid Q_r(p^*)). \tag{17}$$

Proof Since $S_r + S_m > S_r$, $Q_s(p^*) > Q_r(p^*)$. Since $2\alpha(S_r + S_m)H_r < 2\alpha(S_r + S_m)H_r + \alpha H_r(S_m)^2/(2S_r)$, $[2\alpha(S_r + S_m)H_r]^{1/2} < [2\alpha S_r H_r]^{1/2} + S_m[\alpha H_r/(2S_r)]^{1/2}$. Utilizing (6) and (7), we have

$$\begin{aligned}\pi_s(p^* \mid Q_r(p^*)) &= \pi_m^* + \pi_r^* \\ &= \alpha k g^{1-\beta} - \{[2\alpha S_r H_r]^{1/2} + S_m[\alpha H_r/(2S_r)]^{1/2}\}g^{-\beta/2}.\end{aligned} \tag{18}$$

Therefore,

$$\pi_s(p^* \mid Q_s(p^*)) > \pi_s(p^* \mid Q_r(p^*)). \tag{19}$$

Comparing (3) and (11), since $S_r + S_m > S_r$, we have

$$\pi_m(\omega^*, p^* \mid Q_s(p^*)) > \pi_m(\omega^*, p^* \mid Q_r(p^*)). \tag{20}$$

Since $[2S_rH_r]^{1/2} < [S_r/(S_r+S_m)+1][(S_r+S_m)H_r/2]^{1/2}$, by comparing (4) and (12), we have

$$\pi_r(\omega^*, p^* | Q_s(p^*)) < \pi_r(\omega^*, p^* | Q_r(p^*)). \tag{21}$$

□

To improve the manufacturer and the retailer outcomes under coordination, let us discuss full coordination. Since both the manufacturer and the retailer would be willing to accept a coordinated policy only if their profits were not less than non-coordinated policy, we should consider π_m^* as the manufacturer's lower bound and π_r^* as the retailer's lower bound on their respective annual profits $\pi_m(\omega, p | Q_s(p))$ and $\pi_r(\omega, p | Q_s(p))$ for considering fully coordinated policy. Therefore, the system optimal problem, under the system EOQ order quantity, is now formulated as follows:

$$\begin{aligned} \underset{\omega,p}{\text{Maximize}} \quad & \pi_s(p | Q_s(p)) = pD(p) - [2(S_r + S_m)H_rD(p)]^{1/2} \qquad (22) \\ \text{subject to} \quad & \pi_r(\omega, p | Q_s(p)) \geq \pi_r^*, \\ & \pi_m(\omega, p | Q_s(p)) \geq \pi_m^*, \\ & 0 \leq \omega \leq g, \ \omega \leq p \leq k\omega. \end{aligned}$$

Our system coordinate game model belongs to a class of "Cross-Constrained" games defined in Charnes et al. [5, 6], where the players' strategies are interactive through the manufacturer's and the retailer's minimum acceptable profit constraints. In classical game theory, the strategy set of a game is assumed to be a topological product of individual players' strategy sets, i.e., strategies are not interactive between players.

Since $\pi_s(p | Q_s(p))$ does not include ω as its variable, the optimal system retail price, p, can be obtained by solving $\text{Max}_p \pi_s(p | Q_s(p))$ subject to $\omega \leq p \leq k\omega$. Since $d\pi_s(p | Q_s(p))/dp > 0, \pi_s(p | Q_s(p))$ is a strictly increasing function, $p = k\omega$ is the optimal system retail price.

Now, let us discuss the determination of the system optimal wholesale price, ω. Since $\pi_r(\omega, p | Q_s(p)) = (p - \omega)D(p) - S_rD(p)/Q(p) - Q(p)H_r/2$, we can let $\omega_{\max}$ be the retailer's largest acceptable wholesale price from the manufacturer that satisfies $\pi_r(\omega, p | Q_s(p)) \geq \pi_r^*$. If $p = k\omega$ is substituted into the first constraint in (22), $\omega_{\max}$ should satisfy the following condition:

$$\alpha(k-1)\omega_{\max}^{1-\beta} - [S_r/(S_r + S_m) + 1][\alpha H_r(S_r + S_m)/2]^{1/2} \omega_{\max}^{-\beta/2} = \pi_r^*. \tag{23}$$

Similarly, since $\pi_m(\omega, p | Q_s(p)) = \omega D(p) - S_m[H_rD(p)/(2(S_r + S_m))]^{1/2}$, we can let $\omega_{\min}$ be the manufacturer's smallest wholesale price that satisfies $\pi_m(\omega, p | Q_s(p)) \geq \pi_m^*$. Substituting $p = k\omega$ into the second constraint in (22), $\omega_{\min}$ should satisfy the following condition:

$$\alpha k\omega_{\min}^{1-\beta} - S_m[\alpha H_r/(2(S_r + S_m))]^{1/2}\omega_{\min}^{-\beta/2} = \pi_m^*. \tag{24}$$

Since $\pi_r(g, kg \mid Q_s(kg)) < \pi_r^*$, $(\omega, p) = (g, kg)$ is not a feasible solution of (22). Therefore, $\omega_m < g$. Furthermore, we have the following result:

Theorem 2 For any ω satisfying $\omega_{\max} \geq \omega \geq \omega_{\min}$ and $p = k\omega$, (ω, p) is a feasible solution and

$$\pi_s(p \mid Q_s(p)) > \pi_m^* + \pi_r^*. \tag{25}$$

In order to determine the optimal system wholesale price, ω, after substituting $p = k\omega$ into the objective function of (22), problem (22) can be rewritten as

$$\begin{aligned} \underset{\omega}{\text{Maximize}} \quad & \pi_s(\omega) = \alpha k\omega^{1-\beta} - [2\alpha H_r(S_r + S_m)]^{1/2}\omega^{-\beta/2} \\ \text{subject to} \quad & \omega_{\max} \geq \omega \geq \omega_{\min}. \end{aligned} \tag{26}$$

Since $d\pi_s(\omega)/d\omega > 0$, $\pi_s(\omega)$ is a strictly increasing function. Therefore, the optimal solution of (26) is $\omega^{**} = \omega_{\max}$. Furthermore, $(\omega^{**}, p^{**}) = (\omega_{\max}, k\,\omega_{\max})$ is the optimal solution of (22) and $Q_s^{**} = [2\alpha(S_r + S_m)\omega_{\max}^{-\beta}/H_r]^{1/2}$ is the system optimal order quantity. Let $\pi_m^{**} = \pi_m(\omega^{**}, p^{**}, Q_s^{**})$, $\pi_r^{**} = \pi_r(\omega^{**}, p^{**}, Q_s^{**})$ and $\pi_s^{**} = \pi_m^{**} + \pi_r^{**}$. Then we have

$$\pi_s^{**} > \pi_s^*, \tag{27}$$

$$\pi_m^{**} > \pi_m^*, \tag{28}$$

$$\pi_r^{**} = \pi_r^*. \tag{29}$$

5. Division of profits

The previous section revealed that coordination increases system profits. We now consider how to achieve system coordination and to divide these additional profits. There are several alternatives for achieving system coordination. For example, simple contracts, vertical integration, profit sharing, and quantity discounts (for a detailed discussion, see Jeuland and Shugan [12], Li and Huang [17], Li et al. [18,19], and Weng [27,28]. A profit sharing mechanism that results from bargaining between the manufacturer and the retailer is the alternative choice to examine next.

Suppose the manufacturer receives a fraction λ $(0 \leq \lambda \leq 1)$ of the system additional profits and while the retailer receives the remainder of the additional profits. Consequently, the manufacturer and the retailer annual profits are given by

$$\pi_m(\lambda) = \pi_m^* + \lambda\Delta\pi, \tag{30}$$

$$\pi_r(\lambda) = \pi_r^* + (1-\lambda)\Delta\pi, \tag{31}$$

where $\Delta\pi = \pi_s^{**} - \pi_s^*$. There are many negotiation models that can be used to determine the division of system additional profits between the manufacturer and the retailer. Here we only utilize one well-known bargaining model in the literature, i.e., the Kalai and Smordinsky [13] model (see Kohli and Park [14] and Huang [11]). This approach requires no knowledge about the bargaining parties' negotiation power. In our coordination model, the basic assumption is that no degree of bargaining power is assigned to each member. Therefore, the above-mentioned bargaining model is appropriate in the determination of the division of system additional profits.

According to the Kalai and Smordinsky model, the bargaining division fraction λ is the point at which the (π_m, π_r)-curve in the profit space for $\lambda \in [0, 1]$ intersects the line connecting the disagreement point (π_m^*, π_r^*) with the ideal point $(\pi_m(1), \pi_r(0))$ (i.e., the point corresponding to the infeasible outcome at which both the manufacturer and the retailer obtain the entire system additional profits).

It is easy to show that the (π_m, π_r)-curve in the profit space can be expressed by

$$\pi_r = \pi_s^{**} - \pi_m \tag{32}$$

and the line connecting (π_m^*, π_r^*) with $(\pi_m(1), \pi_r(0))$ can be expressed by

$$\pi_r = (\pi_r^* - \pi_m^*) + \pi_m. \tag{33}$$

The intersection point of (32) and (33) is

$$\bar{\pi}_m = \pi_m^* + \Delta\pi/2, \tag{34}$$

$$\bar{\pi}_r = \pi_r^* + \Delta\pi/2. \tag{35}$$

Therefore, the Kalai and Smordinsky model predicts that both the manufacturer and the retailer should equally share the system additional profits, i.e., $\lambda^* = 1/2$.

6. An example

To illustrate the theoretic findings, we utilize a small business example. Suppose a large manufacturer of jeans sells its product to a retailer in a small town. This clothing store is the only store that retails jeans in the town and consequently occupies a type of monopolistic position for its product. We are interested in examining the game theoretic issues and advantages of cooperation between the manufacturer and the small town retailer.

We assume that the demand of jeans at the retail level has the functional form as follows:

$$D = 3407\, p^{-\beta}. \tag{36}$$

Additional assumptions are:

H_r = retailer's annual inventory holding cost per unit is \$1;
S_r = retailer's ordering cost per order is \$50;
S_m = manufacturer's setup cost per setup is \$120;
g = cap of manufacturer's wholesale price is \$35;
k = maximum percentage that the retail price is allowed to exceed the wholesale price is 120%.

In order to see how the price elasticity parameter, β, affects system members' decision variables and their profits, we consider different values of β as 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. Following the sequence of topics developed in the previous sections, we examine the non-coordination structure first. Table 1 summarizes the results. For example, suppose the price elasticity is $\beta = 0.3$, the retailer's retail price for jeans is $p^* = \$42.00$, the manufacturer's wholesale price for jeans is $\omega^* = \$35.00$, the consumer demand for jeans at the retail level is $D^* = 1110$ units, the retailer's

Table 1

Numerical results of non-coordination game model.

β	p^*	ω^*	D^*	Q_r^*	π_m^*	π_r^*	π_s^*
0.1	42.00	35.00	2344.48	484.20	81475.89	15927.18	97403.07
0.2	42.00	35.00	1613.33	401.66	55984.43	10891.62	66876.05
0.3	42.00	35.00	1110.19	333.20	38456.82	7438.13	45894.96
0.4	42.00	35.00	763.96	276.40	25407.03	5071.34	31478.37
0.5	42.00	35.00	525.71	229.28	18124.76	3450.70	21575.46
0.6	42.00	35.00	361.76	190.20	12433.42	2342.13	14775.55
0.7	42.00	35.00	248.94	157.78	8523.63	1584.81	10108.44
0.8	42.00	35.00	171.31	130.88	5838.65	1068.26	6906.91
0.9	42.00	35.00	117.88	108.57	3995.58	716.60	4712.19

order quantity from the manufacturer is $Q_r^* = 333$ units of jeans, the manufacturer's, the retailer's and the system's annual profits are $\pi_m^* = \$38456.82$, $\pi_r^* = \$7438.13$, and $\pi_s^* = \$45894.96$, respectively. Two implications can be drawn from table 1. First, the retail price and wholesale price are not sensitive at all to the price elasticity parameter, β. For any value of β, the retail price is always equal to \$42.00 and the wholesale price is always equal to \$35.00. Second, as the price elasticity, β, increases, all other parameters decrease. For example, when β increases from 0.3 to 0.4, the consumer demand decreases from 1110 units to 764 units.

Now, we study the structure of coordination between the manufacturer and the retailer. There are two cases to discuss for this situation. The first case involves the situation where the manufacturer and the retailer employ only the system EOQ order

Table 2

Numerical results of coordination game model with only system order quantity.

β	$Q_s(p^*)$	$\pi_m(\omega^*, p^* \mid Q_s(p^*))$	$\pi_r(\omega^*, p^* \mid Q_s(p^*))$	$\pi_s(\omega^*, p^* \mid Q_s(p^*))$
0.1	892.82	81741.81	15833.67	97575.49
0.2	740.61	56205.03	10814.05	67019.08
0.3	614.38	38639.82	7373.79	46013.60
0.4	509.65	26558.83	5017.96	31576.80
0.5	422.78	18250.69	3406.42	21657.10
0.6	350.71	12537.88	2305.40	14843.28
0.7	290.93	8610.28	1554.34	10164.62
0.8	241.34	5910.53	1042.98	6953.52
0.9	200.20	4055.21	695.63	4750.85

quantity in their coordination. Numerical results are summarized in table 2. It is apparent that our numerical results are consistent with theorem 1, i.e., for a given value of β, the system order quantity, the manufacturer's annual profit, and the system's annual profit are higher while the retailer's annual profit is lower than those at non-coordination. For example, let $\beta = 0.3$, the system order quantity is $Q_s(p^*) = 614$ units, which is higher than $Q_r^* = 333$ units (an 84% increase), the manufacturer's annual profit is $\pi_m(\omega^*, p^* \mid Q_s(p^*)) = \$38639.82$ which is higher than $\pi_m^* = \$38456.82$, the system's annual profit is $\pi_s(\omega^*, p^* \mid Q_s(p^*)) = \$46013.60$ which is higher than $\pi_s^* = \$45894.96$, the retailer's annual profit is $\pi_r(\omega^*, p^* \mid Q_s(p^*)) = \$7373.79$ which is lower than $\pi_r^* = \$7438.13$. Additionally, it is noted that as β increases, the system order quantity, the manufacturer's, the retailer's and the system's annual profits decrease.

The second case involves the situation where all decision variables are coordinated. Numerical results of this case are illustrated in table 3. It is apparent that for a given value of β, the coordinated retail price and wholesale price are lower, the coordinated order quantity, the manufacturer's and the system's annual profits are higher, and the retailer's annual profit is the same compared with those for the non-coordination situation. For example, let $\beta = 0.3$, the coordinated retail price is $p^{**} = \$29.06$ which is lower than $p^* = \$42.00$, the coordinated wholesale price is $\omega^{**} = \$24.22$ which is lower than $\omega^* = \$35.00$, the manufacturer's annual profit is $\pi_m^{**} = \$38958.45$ which is higher than $\pi_m^* = \$38456.82$, the system's annual profit is $\pi_s^{**} = \$46396.59$ which is higher than $\pi_s^* = \$45894.96$, the retailer's annual profit is $\pi_r^{**} = \$7438.13$ which is the same with $\pi_r^* = \$7438.13$.

Two implications can be drawn from table 3. First, as β increases, the retail and wholesale prices increase. For example, as β increases from 0.3 to 0.4, the retail price increases from \$29.16 to \$31.07 and the wholesale price increases from \$24.22 to \$25.89. Second, as β increases, the consumer demand, system order quantity,

Table 3

Numerical results of coordination game model.

β	p^{**}	ω^{**}	D^{**}	Q_s^{**}	π_m^{**}	π_r^{**}	π_s^{**}	π_m^{KS}	π_r^{KS}
0.1	25.30	21.08	2343.01	892.54	82208.54	15927.18	98135.72	81842.22	16293.51
0.2	27.56	22.97	1610.58	740.00	56591.05	10891.62	67482.67	56287.74	11194.93
0.3	29.06	24.22	1106.32	613.31	38958.45	7438.13	46396.59	38707.64	7688.95
0.4	31.07	25.89	759.02	508.00	26820.96	5071.34	31892.30	26614.00	5278.31
0.5	32.58	27.15	519.68	420.35	18465.07	3450.70	21915.77	18294.92	3620.86
0.6	34.12	28.43	354.50	347.18	12711.34	2342.13	15053.48	12572.39	2481.10
0.7	36.70	30.58	240.13	285.74	8747.66	1584.81	10332.47	8635.65	1696.83
0.8	38.53	32.11	160.26	233.43	6014.11	1068.26	7082.36	5926.38	1155.99
0.9	39.74	33.12	102.97	187.11	4122.31	716.60	4838.92	4058.95	779.97

manufacturer's annual profit, retailer's annual profit, and the system's annual profit decrease. For example, as β increases from 0.3 to 0.4, the consumer demand decreases from 1106 units to 759 units, the system order quantity decreases from 613 units to 508 units, the manufacturer's annual profit decreases from \$38958.45 to \$26820.96, the retailer's annual profit decreases from \$7438.13 to \$5071.34, and the system's annual profit decreases from \$46396.59 to \$31892.30.

Regarding how to share system additional profits, the Kalai and Smordinsky bargaining model suggests that the manufacturer and the retailer should share equally the system additional profits. The results of the share between system members are listed in table 3. For example, for $\beta = 0.3$, the system additional profit is \$501.63 ($\pi_s^{**} - \pi_s^* = \$46396.59 - \$45894.96$). Therefore, the manufacturer's profit is $\pi_m^{KS} = \$38707.64$ and the retailer's profit is $\pi_r^{KS} = \$7688.95$.

7. Concluding remarks

This research attempts to investigate the impact of order quantity, wholesale price and retail price on channel coordination of the manufacturer-retailer system through cross-constrained game theory and bargaining theory. We show that channel coordination yields higher system as well as individual profits.

There are two limitations in this paper. First, in deriving the results we assumed that there is only one retailer or a group of homogeneous retailers selling the manufacturer's product. Future research may relax this assumption and consider multiple non-homogeneous retailers. Second, our concern in this paper was with the issues of the manaufacturer's wholesale price as well as the retailer's retail price and order quantity. It would be fruitful to include and examine analytical issues related to the incorporation of other variables, such as the manufacturer's product quality and the service provided by the retailer to consumers.

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