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CHOICE BEHAVIOR IN INTERACTIVE MULTIPLE-CRITERIA DECISION MAKING

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Abstract

Choice behavior in an interactive multiple-criteria decision making environment is examined experimentally. A "free search" discrete visual interactive reference direction approach was used on a microcomputer by management students to solve two realistic and relevant multiple-criteria decision problems. The results revealed persistent patterns of intransitive choice behavior, and an unexpectedly rapid degree of convergence of the reference direction approach on a preferred solution. The results can be explained using Tversky's [20] additive utility difference model and Kahneman–Tversky's [5] prospect theory. The implications of the results for the design of interactive multiple-criteria decision procedures are discussed.

1. Introduction

Interactive multiple-criteria decision making has now been a popular research topic for more than a decade. Several dozen procedures have been developed for solving both continuous and discrete optimization problems having multiple criteria. The specifics of these procedures vary, but they have several common characteristics. For example, at each iteration a solution, or set of solutions, is generated for a decision maker's (DM's) examination. As a result of the examination, the DM inputs information in the form of tradeoffs, pairwise comparisons, aspiration levels, etc. His/her responses are used to generate a, presumably, improved solution. The procedures terminate in a satisfactory or a satisfactorily near-optimal solution or, simply, when the DM so chooses. For a review of several interactive multiple-criteria procedures, see Steuer [19] or Lotfi and Teich [12].

The nature and type of information requested from a DM differs from one procedure to another. Also, the mathematical assumptions upon which the procedures are based vary. For example, researchers continue to postulate different assumptions about the form and stability of a DM's composite value (utility) function. However, only a few are concerned about the behavioral realism of such assumptions. A major focus of this paper is to attempt to determine whether such various assumptions are plausible or reasonable from a behavioral perspective.

In this paper, we discuss the results and implications of a laboratory experiment, whose purpose was to examine actual choice behavior in interactive multiple-criteria decision-making environments. As a research instrument, we use a visual interactive method called VIMDA (Korhonen [7]) for solving discrete multiple-criteria problems. The method was implemented on an IBM/PC1 microcomputer with color graphics to solve two realistic and relevant decision problems. This particular method was chosen as our instrument for several reasons. The method is a "free search" type of approach that makes no assumptions, except monotonicity, about the properties of the DM's value function – thus allowing us to observe choice behavior in an unrestricted manner. Furthermore, VIMDA allows us to control some of the framing parameters of the problem, such as color. We emphasize that our purpose is not to compare or contrast VIMDA against any other decision procedure (structured or unstructured). We simply use it as an instrument to observe and explain choice behavior in an interactive setting. Why are we interested in this problem? Simply, because this issue has significant implications for the design and development of interactive multiple-criteria decision procedures. One of the purposes of this paper is to convince the multiple-criteria optimization researchers that they must integrate the knowledge of normative and behavorial decision theorists and, conversely, to spark synergistic and functional advances in both fields.

This paper consists of five sections. In the first section, we have described the purpose of the study. The second section provides the details of the experiments, and the third section the results. In the fourth section, the observed choice behavior is discussed in the context of Tversky's [20] additive difference model and Kahneman–Tversky's [5] prospect theory. In the fifth section, the implications of the results for the design and development of multiple-criteria interactive methods are enumerated.

2. Experiments

2.1. PRELIMINARIES

We consider two discrete multiple-criteria decision problems. In general, we assume that there is a single DM, a set of n deterministic decision alternatives, and p criteria ($p > 1$), which define an $n \times p$ decision matrix X whose elements are denoted by x_{ij}, $i \in I = \{1, 2, \ldots, n\}$ and $j \in J = \{1, 2, \ldots, p\}$. We use χ_i or i to refer to the decision alternative in row i. Thus, each decision alternative is a point in the criterion space $\mathbb{R}^p$.

Assuming that a DM wishes to maximize each of the p criteria, the problem is to

$$\text{"max"}_{i \in I}\ \chi_i. \tag{1}$$

Since the above problem rarely has a unique solution, any efficient solution $\chi_k \in E$, $k \in I$, is a reasonable and possible compromise solution. E is the set of efficient (nondominated) alternatives.

Classically, it is assumed that a DM makes choices using an increasing (explicit or implicit) value function υ: $\mathbb{R}^p \rightarrow \mathbb{R}$. Thus, the problem is considered in the following form:

$$\max_{i \in I} \upsilon(\chi_i). \tag{2}$$

In multiattribute value theory, the value function is represented explicitly by using a priori preference information obtained from a DM [6]. Interactive multiple-criteria optimization procedures do not assume the existence of an explicit value function. However, it is fairly common to assume that it belongs to a specific class of functions, e.g. concave, quasi-concave, linear [19]. If such assumptions are made about the form and existence of the value function, then choice behavior is limited by the method. For instance, the use of a linear composite function permits the DM to consider only convex nondominated solutions [26]. However, convex dominated (but efficient) solutions may very well be of interest to the DM and should not be excluded. We provide a definition of a convex nondominated vector for the convenience of the reader.

DEFINITION

A vector χ_i, $i \in I$, is convex nondominated if and only if (iff) there exists no convex combination of other distinct vectors χ_k, $\chi_i \neq \chi_k$, $k \in I$, $k \neq i$, which dominates χ_i; that is, the following set of constraints has no solution:

$$\begin{aligned} \sum_{k \neq i} \mu_k \chi_k &\geq \chi_i , \\ \sum_{k \neq i} \mu_k &= 1, \\ \mu_k &\geq 0. \end{aligned} \tag{3}$$

(Isermann [4] uses the terms A- and B-efficiency to refer to the concepts of nondominance and convex nondominance, respectively.)

On the other hand, if nothing is assumed about the value function, all efficient choices are acceptable and reasonable.

2.2. DESCRIPTION OF THE DECISION PROBLEMS

2.2.1. *Choosing a washing machine (I)*

This problem was extracted from Zeleny ([25], pp. 210–211). The original decision problem consisted of thirty-three washing machines that were evaluated using four criteria: price, total washing time, electricity consumption (directly proportional to the cost of electricity per washing cycle), and water consumption. In our experiment, we used the first three criteria, which all were to be minimized (table 1). The reason for omitting one of the criteria was simply that we wanted to have a larger variation between the number of criteria in the two problems, respectively.

Table 1

Washing machines

Machine number	Type*	Price ($)	Total washing time (min)	Electricity consumption (kWh)
1	D	509	74	1.5
2	D	425	80	1.5
3	C	446	72	1.6
4	D	564	65	1.6
5	C	547	53	1.8
6	C	450	68	1.6
7	C	473	65	1.6
8	N	484	56	1.7
9	C	456	68	1.6
10	D	488	72	1.6
11	C	530	55	1.7
12	D	477	76	1.5
13	N	589	53	1.6
14	N	534	61	1.4
15	D	536	57	1.7
16	C	494	71	1.5
17	N	425	65	1.8
18	N	555	53	1.7
19	D	543	57	1.6
20	C	515	68	1.5
21	D	452	76	1.5
22	D	547	68	1.5
23	N	421	76	1.4
24	D	498	68	1.6
25	C	467	65	1.7
26	N	595	50	1.8
27	N	414	68	1.7
28	C	431	66	1.7
29	C	452	72	1.5
30	D	408	77	1.6
31	C	478	59	1.8
32	N	395	76	1.5
33	N	543	57	1.5

* D = dominated (total number = 11), C = convex dominated (total number = 12), N = convex nondominated (total number = 10).

2.2.2. *Buying a home (II)*

The second problem consisted of choosing one out of forty-three actual homes in the Helsinki metropolitan area. The data were collected from the main daily newspaper (Helsingin Sanomat), published in Helsinki. Five different criteria were used to evaluate the alternatives: price, location (measured on a 1–10 scale), area in square meters, number of rooms, and the condition of the unit (measured on a 1–10 scale) (table 2).

Table 2

Homes in Helsinki

Home number	Type*	Price (FMK)	Location*	Area (m^2)	Number of rooms	Condition*
1	N	250,000	3	75	3	6
2	N	252,900	2	46	1	10
3	N	255,000	7	55.5	2	8
4	D	265,000	3	50	1	7
5	D	275,000	6	44	1	6
6	C	275,000	6	60.5	1	6
7	N	283,785	5	47	2	10
8	D	284,750	1	67	3	5
9	C	285,000	2	47.5	2	10
10	C	295,000	6	60	2	5
11	N	295,102	5	61	2	10
12	N	298,500	10	38	1	8
13	N	308,992	8	68	2	8
14	D	310,000	5	68	3	5
15	D	310,000	7	72	3	5
16	N	310,000	7	81	3	6
17	N	313,065	7	81	3	9
18	D	314,275	2	48.5	2	10
19	N	316.200	3	93	3	5
20	D	320,000	3	51	2	7
21	D	330,000	3	76	3	7
22	D	333,410	4	77	1	6
23	C	335,000	7	82.5	3	8
24	D	335,830	2	48.5	2	10
25	D	334,330	5	54	2	10
26	D	338,000	6	63,5	3	4
27	D	339,739	2	60	2	10
28	C	350,000	9	54	1	6
29	N	351,000	7	88.5	3	8
30	N	354,739	2	61	3	10
31	N	355,400	2	65.5	3	10
32	D	356,360	2	55	2	10
33	D	360,325	2	55	2	10
34	N	361,000	2	66.5	3	10
35	N	385,000	10	47.5	1	5
36	D	351,500	5	55.5	2	10
37	D	350,000	1	60.5	2	10
38	N	255,300	4	44.5	1	10
40	N	384,407	5	67	3	10
41	D	392,600	2	60.4	2	9
42	N	400,000	9	66	3	9
43	D	391,050	5	66	3	10

*D = dominated (total number = 19), C = convex dominated (total number = 6), N = convex nondominated (total number = 18).

*On the 1–10 scales, 10 is best for location and condition.

Note that regarding the criterion "location", no specific location was associated with different values. For example, a value of 10 (best location) might imply different residential areas for different individuals. The context and the alternatives were defined so that all criteria except price were to be maximized.

2.3. SUBJECTS

A group of seventy-two management students at the Helsinki School of Economics and Business Administration made decisions on both problems individually. Most of the students were upper division undergraduates majoring in management science or accounting. They had prior experience in using microcomputers, interactive multiple-criteria decision methods and, specifically, the visual reference direction approach. The subjects' motivation was enhanced by providing extra credit for students participating in the experiments.

2.4. THE VISUAL REFERENCE DIRECTION APPROACH

The reference direction approach of Korhonen [7], which was used as the research instrument, is a visual, interactive procedure for solving discrete multiple-criteria decision problems. It has been implemented on an IBM/PC1 microcomputer (under the name VIMDA = Visual Interactive Method for Discrete Alternatives).

The main steps of the method are as follows:

Step 0: Choose an arbitrary efficient solution as a starting point. (The currently implemented version simply chooses as the starting point the alternative which has the best value on the last criterion, if it is nondominated.)

Step 1: Ask the DM to specify aspiration levels for the criteria (fig. 1). Use the aspiration levels to define a reference direction, namely, a direction that emanates from the current alternative and passes through the point defined by

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                            V I M D A
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The Specification of the Aspiration Levels

The Name of the Current Solution: 14

                    Lower   Upper    Current   Aspiration
Criteria            Bounds  Bounds   Values    Levels
========            ======  =======  =======   ==========
Price                395     595      534       395
Wash.time            50      80       61        50
El.consumption       1.4     1.8      1.4       1.4


                       Press Esc to Exit or Correct Values
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Fig. 1. Specification of aspiration levels.

the aspiration levels. If at later iterations the DM does not wish to change the aspiration levels, stop. Otherwise, proceed to step 2.

Step 2: Generate a subset of efficient solutions by projecting the reference direction on the set of efficient solutions. (Mathematically, this is accomplished by minimizing an achievement (scalarizing) function, as suggested by Wierzbicki [23] in his reference point approach. When the achievement function is applied to the reference direction vector, a set of efficient solutions is generated.)

Step 3: Present the subset, generated in the previous step, to the DM graphically and numerically (fig. 2) and ask him/her to choose the most preferred solution from this set; return to step 1. (The criterion values in fig. 2 are shown on the ordinate.

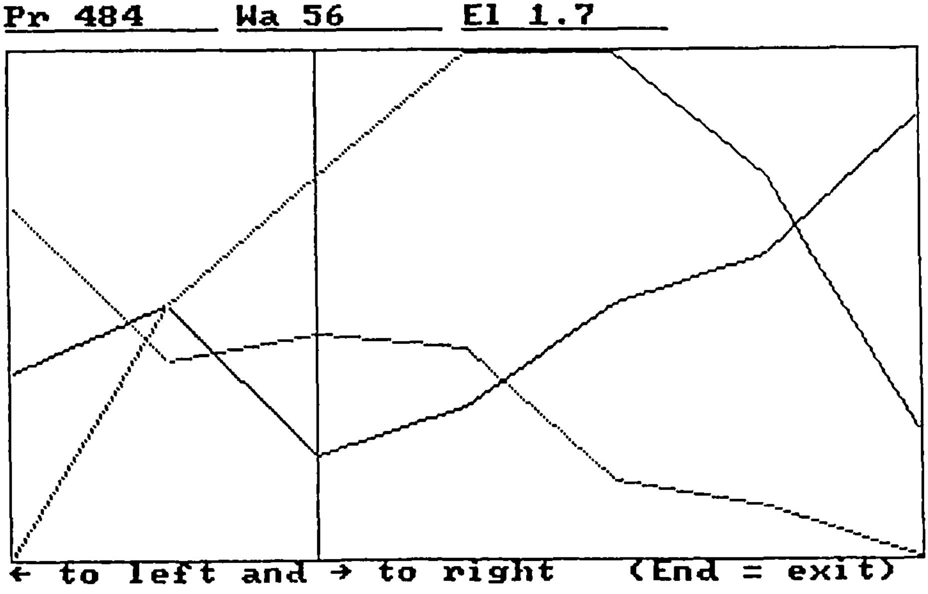

Fig. 2. Sample display of step 3.

The current alternative is shown in the left-hand margin. The criterion values of consecutive alternatives have been connected with lines using different colors and patterns. The cursor characterizes that alternative whose criterion values are printed numerically at the top of the screen. The cursor moves to the right and to the left, and each time the criterion values are updated. The DM is asked to choose his/her most preferred alternative from the screen by moving the cursor to point to such a solution.)

The reference direction approach is not based on any assumptions, except for monotonicity, regarding the properties of the value function. Using the procedure, a DM is free to examine any efficient solution. Furthermore, this freedom is not restricted by previous choices. The currently implemented version of VIMDA does not include a mathematical optimality test. The process is terminated when the DM is satisfied with the currently best solution. If a more formal termination criterion is used, it would be necessary to make assumptions about the value function at the time of termination.

2.5. DESIGN

Each subject, starting at the same point, solved both decision problems, first problem I, then problem II. The problems were perceived as being very different and independent. Also, the subjects were beforehand familiar with the research instrument and the setting. Hence, dependence effects, such as learning, were minimal. At the beginning of each session, the subjects were provided with one-page problem descriptions. The subjects then made choices in each problem and were allowed to iterate as long as they desired. Their choices during the solution process were documented for subsequent analysis. After making decisions on the problems, they were subjected to a post experimental interview, reflecting on their choices, choice process, the procedure, etc.

3. Experimental results

The results of both experiments were interesting and useful. Subjects, however, considered problem II more relevant personally than problem I.

In the washing machine purchase problem (I), the average number of interactive iterations was 1.9, while in the home buying problem (II), it was 2.3. The difference in rate of convergence is presumably due to the greater number of choice criteria and, apparently, the greater perceived relevance and interest of problem II. The subjects were aware that problem II was based on real data. In addition, their motivation was further enhanced by informing them in advance that (after the experiment) they would be provided with more detailed information regarding their final choice (home). As many students were in the process of apartment hunting, this served as a strong inducement for them to make choices consistent with their true preferences. The observed rapid degree of convergence of the reference direction approach on a preferred solution in the context of multiple objective linear programming, in general, has also been previously experimentally exhibited [8].

Similar types of choice behavior were exhibited in both problems. Typical profiles are shown in table 3 for the washing machine problem. All subjects started the VIMDA program from the same solution (number 14). Also, the ideal solution values served as default values for the first set of aspiration levels. Accordingly, many subjects chose the ideal solution as their first aspiration levels. Whether or not this has biased the final outcome is an interesting question (that we cannot answer based on our data). This all ties together with prospect theory, as explained later in this paper. (A careful reader will notice that alternative number 19 is weakly nondominated. See the second

Table 3

Typical profiles of choice behavior (washing machine purchase problem)

Subject number	1*	8*	42	53*
Initial solution:	14	14	14	14
Initial aspiration level:	395, 50, 1.4	395, 50, 1.4	430, 55, 1.5	400, 55, 1.5
Choices: iteration 1:	14, 7, 8, 31, 17, 27, 32	14, 7, 8, 31, 17, 27, 32	14, 7, 8, 31, 17, 27, 32	14, 7, 8, 25, 17, 27, 32
Preferred choice:	7	7	32	7
Aspiration level:	395, 65, 1.6	400, 55, 1.4	435, 65, 1.5	473, 50, 1.5
Choices: iteration 2:	7, 6, 27, 32	7, 8, 31, 11, 5, 18, 26	32, 3, 6, 7, 8, 19, 11, 18, 26	7, 8, 11, 18, 26
Preferred choice:	32	11	6	8
Aspiration level:		400, 55, 1.7		480, 56, 1.5
Choices: iteration 3:		11, 8, 31, 7, 17, 27, 32		8, 14, 23
Preferred choice:		8		8
Aspiration level:		400, 55, 1.7		400, 55, 1.5
Choices: iteration 4:		8, 31, 7, 17, 27, 32		8, 7, 25, 28, 27, 32
Preferred choice:		8		7

*Subjects 1, 8, and 53 made cycles. (Note: Ideal aspiration level: (395, 50, 1.4).)

iteration for person 42. The early version of the VIMDA program produced weakly nondominated alternatives. This feature has subsequently been corrected.)

3.1. INTRANSITIVITIES (CYCLES)

Thirty-two percent and eighteen percent of the subjects in problems I and II, respectively, exhibited inconsistent or intransitive preferences at least once: namely, at some point they would prefer choice A to B, even though they earlier preferred B to A (table 3). These frequencies are clearly high compared to the small number of average iterations (roughly two). The transitivity axiom was violated in two different ways: (a) *explicitly*, if the subject chose the same alternative as the best at least twice but not at subsequent iterations (e.g. subject no. 53); (b) *implicitly*, if an alternative was chosen as the best subsequently but not when it was available for the first time (e.g. subject no. 1). The first type of violation results in what we call a cycle of type A, and the second is a cycle of type B. The observed frequencies are depicted in table 4.

Table 4

Frequencies of cycles

Type of cycle	Problem I	Problem II
A	2	1
B	21	12

Only six of those individuals who made a cycle in problem I did so in problem II. Thus, we found no evidence of a typical "cycle-maker", that is, a person who would systematically make cycles. The subjects were not informed about their cycles, and we never tested whether they would be willing to change their responses. We simply wanted to understand subjects' choice behavior.

Two of the authors of this paper initially thought that perhaps the subjects who made such cycles did not take the task seriously enough, and used the reference direction approach themselves. To their surprise, they also exhibited cyclic behavior!

When subjects "choose" an alternative at an earlier stage, no real commitment is necessarily implied since they can continue for as long as they like. Thus, one might speculate that these "intransitivities" may be as indicative of exploratory behavior as any violation of fundamental consistency axioms. This could conceivably happen. However, we do not think that such behavior was exhibited. On the average, only about two iterations were performed. This does not seem to imply exploratory behavior. At any rate, we do not have enough data to compare the average number of iterations of subjects who made cycles with that of subjects who did not make cycles. Furthermore, even if we had access to such data, we would not know which would be the cause and which would be the effect: cycles increasing the number of iterations or the number of iterations increasing the likelihood of cycles! However, the above is an interesting question. It would also be interesting to learn whether the number of cycles per iteration increases as a function of the number of iterations.

There is one additional issue that needs to be discussed. What if a subject (say no. 1) was truly indifferent between alternatives 7 and 32? Is it possible that the apparent preference reversal is simply due to the "forced choice paradigm" being used? If this were true, type B cycles would seem less consequential. True or not, this question deserves further analysis. At this point, we can only reference some of the behavioral decision theory literature, such as Tversky [20], which has shown that consistency violations are predictable and persistent. Also, if true indifference is frequent, our ability to make inferences about the DM's behavior is limited.

3.2. FORM OF THE VALUE FUNCTION AT TERMINATION

Originally, we also wished to examine if the subjects' choices were consistent with a linear or another specific value function [9]. Because of the cycles observed in the experiment and the limited data available, however, we only investigated the form of the function at termination.

We tested the linearity of the value function by using a surrogate measure, namely the relative frequencies of convex dominated solutions preferred by subjects. The test of quasi-concavity would be meaningless at termination, since the function would always be quasi-concave. The absolute frequencies of convex dominated (but efficient) and nondominated alternatives for both problems (column T), as well as the subjects' choices (column E) are shown in table 5.

The following hypotheses were formulated and tested, where acceptance of the null hypothesis implies a nonlinear value function:

Table 5

The classification of efficient solutions

Type	Problem I*		Problem II*	
	T	E	T	E
Convex dominated	12	42	6	4
Convex nondominated	10	30	18	67
Total	22	72	24	71

*T = theoretical distribution, E = empirical distribution.

H_0: $p = p_0$.

There is no difference between the theoretical and observed proportion of convex nondominated solutions ($p_0 = 10/22$ and $18/24$ for problem I and II, respectively).

H_1: $p > p_0$.

The observed proportion of convex nondominated solutions is higher than the theoretical proportion.

The null hypothesis was tested using a normal density approximation. The z values were -0.65 and 3.77, respectively, for both problems. This leads to accepting the null hypothesis for problem I and rejecting the null hypothesis for problem II, at a level of significance less than 1%. (The result is roughly the same when a correction factor is used.) Our conclusion is that, overall, it is important to realize that subjects do choose convex dominated solutions. Also, although in the aggregate we rejected the null hypothesis for problem II, on an individual level we need to go beyond a linear model. See the next section for additional details.

3.3. ASPIRATION LEVELS

The concept of an aspiration level is a well-established idea in decision making (see, for example, ref. [10]). Since the specification of one's aspiration levels is of considerable importance in the Korhonen [7] approach, we present some *tentative* results regarding how subjects seemed to establish and modify their aspiration levels. The results are based on observations regarding subjects' choices of aspiration levels and on interviews with several subjects. Often, the aspiration levels may be regarded as the perceived best solution of what a DM believes is achievable. A reference direction specified by a DM leads directly from the current solution to this "optimum" [8]. It was interesting to observe that most subjects realized the unachievability of the ideal solution. They either chose less than ideal aspiration levels on all criteria, or chose an ideal aspiration level on one criterion only (probably their perceived most important criterion). Typical profiles of aspiration levels chosen are shown in table 3 for problem I. Aspiration levels chosen for problem II were similar. Simply stated, as subjects believed that the ideal solution would be unachievable, they chose values for the initial aspiration levels they perceived to be feasible and satisfactory, either on all criteria, or on all but the most salient criterion. In the latter case, they could then search for an "optimum" on the most salient criterion by adjusting its aspiration level based on prior results, holding the other aspiration levels fixed. Subsequent aspiration levels, in general, were adjusted upward or downward depending on their degree of achievement at each iteration.

3.4. EFFECT OF COLORS AND SCREEN POSITIONING OF ALTERNATIVES

In the experiments, the impact of two kinds of display effects was also tentatively investigated, namely, the use of different colors and the position of the alternatives on the screen. The use of different type colors to represent choice profiles did not significantly affect subjects' behavior. However, one should be careful in generalizing the results. A significant impact of colors has been previously observed, e.g. by Benbasat and Dexter [1], although it appears that the benefits of color are more specific than some of the general claims made in the literature would suggest. We were also interested in observing whether the subjects' choices were biased by their position on the screen (left, middle, or right). However, a rapid termination of the choice process precluded us from observing any significant screen position effect. All this relates to what Payne [15] calls contingent decision behavior (viz., behavior is contingent on the task environment). This needs to be examined more carefully, both theoretically and empirically.

4. Discussion

The persistence of the intransitivities observed in our experiments is similar to those originally observed by Tversky [20], and later by Lindman and Lyons [11] and Ranyard [17]. Moreover, Tversky [20] has provided a choice theory that predicts and explains intransitive preferences between multidimensional alternatives. In the case where alternatives are evaluated based on comparisons of criterion-wise differences between alternatives, Tversky's additive difference model (lexicographic semiorder) is applicable. If the difference functions (which determine the contribution of the particular subjective difference to the overall evaluation of the alternatives) are nonlinear, intransitivities may systematically occur. In interactive procedures (as in the reference direction approach), comparisons are made with respect to a so-called reference outcome (current solution). This clearly favors the use of an additive difference model, and hence accounts for the persistence of the cycles exhibited. Mathematically, Tversky's additive difference model can be interpreted as follows:

$$\Phi(x_i - x_r) = \sum_{j \in J} \Phi_j(x_{ij} - x_{rj}), \quad i, r \in I \tag{4}$$

and

$$\Phi_j(-\delta) = -\Phi_j(\delta) \qquad \text{for all } j \in J \text{ and } \delta \in \mathbb{R}, \tag{5}$$

where $\Phi_j : \mathbb{R} \to \mathbb{R}$ are the marginal difference functions, and $\delta \in \mathbb{R}$ is the component-wise difference between two alternatives.

Later, Kahneman and Tversky [9] developed prospect theory. In prospect theory, outcomes are expressed as positive or negative deviations (gains or losses) from a reference outcome. Although value functions differ among individuals (and criteria), Kahneman and Tversky proposed that they are commonly S-shaped: concave above the reference outcome and convex below it. Furthermore, according to prospect theory, value functions are commonly steeper for losses than for gains. Although prospect theory should be viewed as an approximate and simplified description of choice behavior, many empirical studies support it (see, for example, refs. [14] and [16]). Prospect theory was originally developed for single-criterion problems, but the ideas are relevant to multiple-criteria decision problems as well. In fact, the additive difference model may be regarded as a generalization of prospect theory to the multiple-criteria context if the symmetry assumption in (5) is modified as follows:

$$\Phi_j(\delta) \le -\Phi_j(-\delta), \quad \text{iff} \quad \delta \ge 0, \quad \text{for } j \in J. \tag{6}$$

Above, we have assumed that x_r is a reference outcome.

Prospect theory provides a reasonable explanation for the rapid degree of convergence of the Korhonen [7] as well as many other interactive procedures. Many interactive algorithms have been notoriously rapid in convergence, overwhelmingly more than (single objective) mathematical optimization routines. Subjects becoming fatigued, in general, does not explain the difference, but prospect theory does. In fact, we may end up with a situation where a DM prefers A to B (if A is the reference outcome) and B to A (if B is the reference outcome), since the losses would carry more weight than the gains. Simply, try A = (1,2) and B = (2,1), where the numbers indicate criterion values.

Next consider a choice problem having four alternatives A, B, C, and D, evaluated using two criteria (table 6). Let us define the marginal difference functions $\Phi_i : \mathbb{R} \to \mathbb{R}$, $i = 1, 2$, used in the additive difference model as follows:

$$\Phi_1(\delta) = \begin{cases} \delta, & \text{if } \delta \ge 0, \\ \alpha\delta, & \text{if } \delta < 0, \end{cases} \tag{7}$$

$$\Phi_2(\delta) = \begin{cases} \sqrt{\delta}, & \text{if } \delta \ge 0, \\ -\alpha\sqrt{-\delta}, & \text{if } \delta < 0, \end{cases} \tag{8}$$

where $\alpha > 1$ is a multiplier that is used to control the steepness of the function for losses in relation to gains (fig. 3). The aggregate function is simply

$$\Phi(x_i - x_r) = \Phi_1(x_{i1} - x_{r1}) + \Phi_2(x_{i2} - x_{r2}). \tag{9}$$

Table 6

Decision matrix of the example

Alternatives	Criterion	
	1	2
A	0.0	1.5
B	1.1	0.5
C	0.6	1.0
D	1.4	0.4

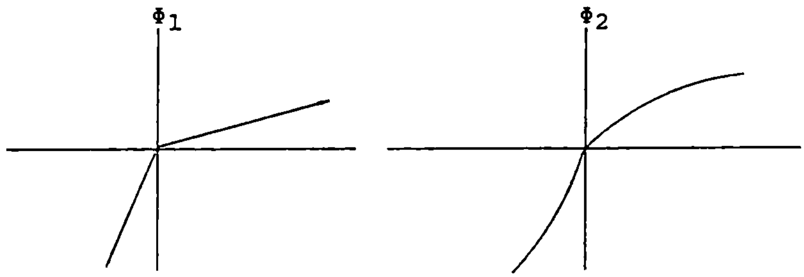

Fig. 3. Examples of the Kahneman–Tversky marginal difference functions.

Table 7

Strength of preference matrix

To:		A	B	C	D
From:	A		0.03	−0.16	0.27
	B	−0.18		0.17	−0.04
	C	0.06	−0.26		−0.03
	D	−0.46	−0.01	−0.09	

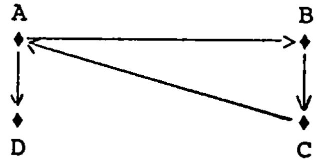

Fig. 4. A preference graph.

An $\alpha = 1.075$ will produce the following strength of preference matrix (table 7), which is illustrated with the preference graph in fig. 4. The row indicates the reference outcome and the column the alternative against which the reference outcome is compared (in a binary comparison). For example, the value 0.03 in the first row and second column implies that the DM's value increases by this amount if he/she moves from A to B (A being the reference outcome). The aggregate value function produces a cycle A $\rightarrow$ B $\rightarrow$ C $\rightarrow$ A, where the arrow points to the preferred alternative. In other words, if the DM is at point A (the reference outcome), he/she wants to proceed to B; also, if the DM is at B (the reference outcome), he/she wants to proceed to C, and so forth. Furthermore, D is an "absorbing" state (using the terminology of Markov chains); it can be reached only from A. If the process starts from D, it stops immediately. In other words, the DM would prefer D to all other alternatives at D. For $\alpha = 1$, the (additional) arrows go from D $\rightarrow$ B and C $\rightarrow$ D. This will produce another cycle: B $\rightarrow$ C $\rightarrow$ D $\rightarrow$ B. Moreover, for $\alpha \geq 1.5$, all alternatives are absorbing, namely, wherever the process starts, it stops immediately.

We have also linked together the experimental results of section 3 with the Tversky model discussed in this section. Accordingly, we generated for each individual (for problem II because of its perceived higher relevance) the pairwise preference information that could be derived from his/her responses. Then, we ran a test based on a linear programming formulation for each subject, as explained below, to verify whether the subjects' choices were consistent with prospect theory, assuming piecewise linear marginal value functions (fig. 3).

For each choice χ_i, $i \in I$, preferred to the (current) reference outcome χ_r, we generated an inequality as follows:

$$\mu^{+\prime} z_i^+ + \mu^{-\prime} z_i^- \geq \varepsilon, \tag{10}$$

and for each reference outcome χ_r preferred to the available choices χ_i, we generated a set of inequalities of the following type:

$$\mu^{+\prime} z_i^+ + \mu^{-\prime} z_i^- \leq -\varepsilon, \tag{11}$$

where ε is a scalar variable, vectors z_i^+ and z_i^- are

$$z_{ij}^+ = \begin{cases} x_{ij} - x_{rj}, & \text{if } x_{ij} - x_{rj} \geq 0, \\ 0, & \text{if } x_{ij} - x_{rj} < 0, \end{cases}$$

$$z_{ij}^- = \begin{cases} x_{ij} - x_{rj}, & \text{if } x_{ij} - x_{rj} < 0, \\ 0, & \text{if } x_{ij} - x_{rj} \geq 0. \end{cases}$$

μ^+ is a vector of weights corresponding to the gains, and μ^- is a vector of weights corresponding to the losses (with respect to the reference outcome χ_r). To estimate ε, we solve the following problem:

$$\begin{array}{ll} \text{maximize} & \varepsilon \\ \text{subject to} & (10), (11), \text{and} \\ & \sum_{j \in J} \mu_j^+ + \sum_{j \in J} \mu_j^- = 1, \\ & \mu_j^+ \leq \mu_j^-, \quad j = 1, \ldots, p, \end{array}$$

where the last set of inequalities forces the marginal value functions to be steeper for losses than for gains (or at least equally steep). If max ε is less than or equal to 0, the model is said to be "Tversky-inconsistent". Otherwise, it is not.

The results were as follows: there were six Tversky-inconsistent individuals and thirty-nine Tversky-consistent individuals, of whom nineteen individuals were consistent with a purely linear model (in terms of all criteria), out of a total of forty-five subjects. Only forty-five subjects were tested for Tversky-consistency, since the others did not make enough iterations. A much larger data bank would be needed to perform a more extensive analysis. However, the results seem to indicate that prospect theory is a reasonable model of choice for many individuals, although we used a very simple type of function. Of course, we are using a sufficiency type of argument: assuming certain value functions and certain parameter values for these functions is *sufficient* to produce the behavior observed in the experiemnt.

In our approach, when χ_i, $i \in I$, is preferred to the reference outcome χ_r, we generated an inequality (10). When the DM chooses χ_i, this implies that χ_i is preferred to χ_k for all $k \in I$, χ_k not equal to χ_i. We have not added constraints corresponding to these preferences into our formulation. The reason for not doing this was that we wanted to test the original Tversky–Kahneman idea, where subjects are assumed to compare a reference outcome against other alternatives. Conceivably, one could consider the best choice (χ_i) as the reference outcome (also for this iteration) and add constraints corresponding to these preferences to our formulation.

5. Conclusions and implications

This paper makes several contributions to research on the relation between multiple-criteria decision methods and behavioral decision research. Briefly, the paper provides a plausible behavioral explanation (based on Kahneman–Tversky's prospect theory [5] and Tversky's [20] difference model) of why preferences converge so rapidly despite the presence of inconsistent (intransitive) preferences. Obviously, there may exist other explanations (such as the "shifting attention" or "switching dimensions" paradigms [20]), and as Fischer et al. [2] have pointed out, choice behavior is not always stereotype (that is, it cannot always be explained). However, we feel that our explanation is plausible and that human subjects have conditional value functions that depend on the reference outcome. Actually, the notion that decision behavior is contingently rational goes back to at least Luce and Raiffa [13]. Additional, carefully designed experiments with interactive methods are needed to further substantiate our arguments.

The possible intransitivity in the DM's choices is well known, per se, in the decision theory literature. In some sense, our research seems to replicate many of the observations and recommendations made by Tversky and Kahneman in a (descriptive) paper-and-pencil context. However, we focus on a specific software system that makes extensive use of computer graphics.

What are the implications of the results of this investigation – and prospect theory – for the design and development of multiple-criteria interactive decision methods?

(1) A direct implication of prospect theory is that, whether we like it or not, the "path" or sequence in which alternatives are presented may affect the final choice. It would therefore seem important to look at the problem from different perspectives, use multiple representations, multiple starting points, and so forth. This helps the DM reconcile between different solutions and finally make up one's mind.

(2) Behavioral convergence of interactive procedures is important. At least in our experiments, decision makers were not willing to wait and see if they would converge upon a good solution in, say, fifty iterations. Therefore, interactive procedures should be designed to make "good progress" in a few initial iterations; the later iterations seem less important.

(3) At the risk of being simplistic, we think that interactive procedures should have built-in mechanisms (e.g. intelligence in the form of expert systems) to deal with inconsistencies, since such inconsistencies are not uncommon. What types of mechanisms might be used deserves further analysis. Obviously, specific mechanisms to handle inconsistencies could lead to an increase in the cognitive as well as the computational load.

(4) Ceteris paribus, the less restrictive "ad hoc" behavioral assumptions are made, the better. Restrictive behavioral assumptions lead to excluding potentially viable solutions from further consideration, and do not allow preference reversals, and so forth.

Based on (other) existing behavioral studies, we also believe that it is important that attention be paid to "framing" a problem properly, such that the choice process of a DM is consistent with the model being used and its assumptions. Otherwise, severe discrepancies and biases may exist between the model results and the DM's solution. Also, as the form of presentation can affect a DM's processing strategy, careful consideration must be given to how and what information is displayed in an interactive algorithm [3,21].

Rational behavior has been and still is one of the cornerstones of contemporary (normative) decision analysis. It has its virtues. However, in this paper we are not advocating "irrationality" but "conditional rationality". "Conditional rationality" means that a DM's preferences are a function of the reference outcome. From this perspective, the Kahneman–Tversky models are extensions of classical rationality. They can be used to predict and explain both transitive and intransitive behavior. An obvious question is, however, what is the reference outcome. In this study, we have assumed that the currently best available choice is such a reference outcome. This may or may not be true.

Additional behavioral experimentation with interactive methods is needed to clarify several of the open-ended research issues raised in this paper, and to further investigate the impact of framing and graphics on interactive choice behavior in multiple-criteria decision problems. An interesting future study would be to compare choices and choice patterns using the VIMDA approach to those made with another type of aid and/or without any aid at all. An open question is whether the rate or pattern of observed intransitivities would vary. More generally, the issue is to determine whether (and how) the interactive approach changes the process and outcome of the DM's deliberations.

What we have implicitly proposed is an evolution of the field of interactive multiple-criteria decision making toward paying added respect and attention to the behavioral realities of decision making, and integrating the results of behavioral decision theory into the design and development of interactive multiple-criteria methods (see, also, Larichev [10]). This is certainly an area that has been overlooked in the operations research literature.

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