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MEASUREMENT ERROR AND THE ALBERT-LOEWER PROBLEM

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Modal interpretations of QM have the welcome consequence that unitarily evolved post-measurement states which superpose eigenstates of the anticipated pointer observable can represent devices registering determinate measurement outcomes. Albert and Loewer have claimed that modal interpretations cannot account for the outcomes of "error-prone" measurements. But Albert, Loewer, and their commentators have not always appreciated the relation of measurement error to the Albert-Loewer problem. I argue that measurement error is neither necessary nor sufficient to generate the Albert-Loewer problem, and use the Araki-Yanase theorem to show that measurements of a large class of observables, if they are error-free, are beset by the Albert-Loewer problem.

Key words: modal interpretations, measurement problem, Albert, imperfect measurement.

1. THE MEASUREMENT PROBLEM

According to a standard quantum mechanics (QM) text, a physical system possesses a determinate value for a magnitude A "if, and only if, the [quantum] state of the physical system is represented by an eigenfunction $|a_i>$ of the Hermitean operator A associated with A" [27; 19, 26, 29]. Call this principle of determinate value assignment *the eigenvector/eigenvalue link.*

quantum measurements don't have outcomes. The problem persists when the simplifying assumptions constraining the measurement model just discussed are relaxed [9].

Recognizing this problem, von Neumann [32] responded by invoking the *deus ex machina* of measurement collapse, a sudden, irreversible, discontinuous change of the state of the measured system to an eigenstate of the observable measured. So, for example, a collapse episode would instantly change the superposition $\Sigma_i c_i |o_i\rangle|p_i\rangle$ to an $\mathbf{O} \otimes \mathbf{P}$ eigenstate; collapse to the eigenstate $|o_n\rangle|p_n\rangle$ occurs with probability $|c_n|^2$. Many texts accord this "Collapse Postulate" axiomatic status, but they do not thereby repair the conceptual damage it does. Perhaps the most unsettling implication of the Collapse Postulate is that laboratory tests of the empirical adequacy of QM succeed only if the fundamental dynamical law of the theory breaks down.

2. MODAL INTERPRETATIONS

All that is familiar. Presently growing in notoriety, an elegant family of interpretations of QM which I'll call *modal interpretations* purport to evade the measurement problem by maintaining the universality of Schrödinger evolution, and revising the eigenvector/eigenvalue link [10,14,15,20,24,25]. A stock example of a modal interpretation exploits a theorem von Neumann attributes to Schmidt. According to this theorem, any vector $|\Psi^{sr}\rangle$ in the tensor product space $H^s \otimes H^r$ admits a decomposition of the form

$$|\Psi^{sr}\rangle = \Sigma_i c_i|a_i\rangle|b_i\rangle,$$

where $\{c_i\}$ are complex coefficients, $\{|a_i\rangle\}$ and $\{|b_i\rangle\}$ are sets of orthogonal vectors on H^s and H^r respectively, and the summation index i does not exceed the dimensionality of the smaller factor space. If the set $\{|c_i|^2\}$ is non-degenerate, then this *polar decomposition* of $|\Psi^{sr}\rangle$ is unique.[1]

Modal interpretations[2] replace the orthodox eigenvector/eigenvalue link with the following semantic rule:

> MSR: If $|\Psi^{sr}\rangle = \Sigma_i c_i|a_i\rangle|b_i\rangle$ is the unique polar decomposition of the state of a composite S + R system, then subsystem S has a determinate value for each H^s observable with eigenbasis $\{|a_i\rangle\}$, and subsystem R has a determinate value for each H^r

QM texts standardly assert that quantum states, governed by the time dependent Schrödinger Equation, evolve unitarily. Unfortunately, the conjunction of unitary dynamics and the eigenvector/eigenvalue link fails to account for measurement phenomena. In an ideal von Neumann measurement [32,12] an object system S is coupled to a recording apparatus R. Associated with the systems are Hilbert spaces H^s and H^r. Consider non-degenerate object observable **O**, with eigenbasis $\{|o_i>\}$ and eigenvalues $\{o_i\}$, on H^s and pointer observable **P**, with eigenbasis $\{|p_i>\}$ and eigenvalues $\{p_i\}$, on H^r. Suppose that S's pre-measurement state is an eigenstate $|o_n>$ of **O**, and that R's initial state is some apparatus "ready" state $|p0>$. In the tensor product Hilbert space $H^s \otimes H^r$, an ideal **O** by **P** measurement unfolds as follows:

$$|o_n>|p0> \dashrightarrow |o_n>|p_n>. \quad (1.1)$$

Let $\mathbf{U}_{\mathbf{IvN}}$ represent an $H^s \otimes H^r$ operator which is a unitary extension of the map (1.1). In (1.1) the pre-measurement composite state is an $\mathbf{O} \otimes \mathbf{I}^r$ ($\mathbf{I}^r$ the identity operator for H^r) eigenstate associated with the eigenvalue o_n; the post-measurement composite state is an $\mathbf{I}^s \otimes \mathbf{P}$ eigenstate associated with the eigenvalue p_n. The evolution operator $\mathbf{U}_{\mathbf{IvN}}$ replicates the object observable's pre-measurement value in the pointer observable's post-measurement value, and so conforms to intuitive measurement desiderata.

$\mathbf{U}_{\mathbf{IvN}}$ falters when asked to work on an object system whose premeasurement state is the superposition $|\psi^s> = \Sigma_i c_i |o_i>$. Because unitary operators are linear,

$$\mathbf{U}_{\mathbf{IvN}}(|\psi^s>|p0>) = \Sigma_i c_i \mathbf{U}_{\mathbf{IvN}}(|o_i>|p0>). \quad (1.2)$$

Specifying $\mathbf{U}_{\mathbf{IvN}}$'s action on $|o_i>|p0>$, (1.1) generates the post-measurement composite state

$$|\Psi^{sr}> = \Sigma_i c_i |o_i>|p_i>. \quad (1.3)$$

$|\Psi^{sr}>$ is not an eigenstate of the pointer observable $\mathbf{I}^s \otimes \mathbf{P}$. By the eigenvector/eigenvalue link, the pointer observable has no determinate value. Crudely put, the pointer doesn't point. According to one version of the *measurement problem*, if the Schrödinger Equation governs measurement evolutions, and if the eigenvector/eigenvalue link governs the determinate observable values characterizing quantum systems, then most observable with eigenbasis $\{|b_i>\}$. $|c_n|^2$ gives the probability that these observables' actual values are the eigenvalues associated with $|a_n>|b_n>$.

Expressing modal semantics in terms of the density matrix formalism makes their family resemblance to orthodox eigenvector/eigenvalue semantics more perspicuous [31]. The density matrix $\mathbf{W}^s$ representing the reduced state of subsystem S of a composite S + R system may be obtained from the composite state $|\Psi^{sr}>$ by "tracing out" over the degree(s) of freedom pertaining to the "remainder" system R. Where $\{|x_i>\}$ is a complete orthonormal basis spanning H^r,

$$\mathbf{W}^s \equiv \Sigma_i <x_i| \Psi^{sr}><\Psi^{sr}| x_i>. \tag{2.1}$$

The reduced state $\mathbf{W}^s$ encodes $|\Psi^{sr}>$'s statistical implications for observables pertaining to S alone: for each observable $\mathbf{Q}$ on H^s, $Tr(\mathbf{Q}\,\mathbf{W}^s) = <\Psi^{sr}|\mathbf{Q} \otimes \mathbf{I}^r|\Psi^{sr}>$. While the composite state determines a unique reduced state for each of its component systems, the converse does not hold. Now, $\mathbf{W}^s$ is an operator whose eigenbasis is the complete orthonormal basis furnishing the polar decomposition of $|\Psi^{sr}>$ and whose eigenvalues are the squared norms of the expansion coefficients of the polar decomposition. If this set is non-degenerate, the operator $\mathbf{W}^s$ is non-degenerate as well. We may express the consequences modal semantics hold for subsystem S as follows:

(MSR^S) If $\mathbf{W}^s$ is the non-degenerate reduced state of a quantum system S, and $\mathbf{W}^s$ has eigenvectors $\{|a_i>\}$ with eigenvalues $\{|c_i|^2\}$, then S has a determinate value for each H^s observable with eigenbasis $\{|a_i>\}$. The probability an observable determinate on S has as its actual value the eigenvalue associated with $|a_n>$ is $|c_n|^2$.

Orthodox semantics assign S a determinate value for an observable if and only if S's reduced state is a pure state, represented by a projection operator π^s, an extremal member of the set of density operators. Observables containing π^s in their spectral resolutions are determinate, according to the eigenvector/eigenvalue link. The modal semantics extend orthodox semantics to systems whose states $\mathbf{W}^s$ aren't pure. An impure $\mathbf{W}^s$ cannot be identified with a projection operator, but it may be associated with a set of projection operators $\{\pi_i\}$ which provide its spectral resolution. Observables containing $\{\pi_i\}$ their spectral resolutions are determinate, according to (MSR^S). Thus we may express both orthodox and modal semantics in terms of the spectral resolutions of the density operators representing the states of quantum systems. On both interpretations, it is necessary that an observable determinate on a system with reduced state $\mathbf{W}^s$ have in its spectral resolution the projectors in $\mathbf{W}^s$'s spectral resolution. But eigenvector/eigenvalue semantics additionally require $\mathbf{W}^s$ to be a pure state, so that it coincides with its spectral resolution.

Not only do modal semantics seem a natural generalization of orthodox semantics, modal semantics also deal elegantly with the unitarily evolved post-measurement states which confounded the eigenvector/eigenvalue link. Recall the post-unitary-measurement composite state $|\Psi^{sr}\rangle = \Sigma_i \, c_i \, |o_i\rangle|p_i\rangle$, which induces the reduced apparatus state $\mathbf{W}^r = \Sigma_i \, |c_i|^2 \, |p_i\rangle\langle p_i|$. According to (MSR), the observables determinate on the apparatus system are H^r observables conspiring in $|\Psi^{sr}\rangle$'s polar decompositon, observables whose eigenbasis diagonalizes $\mathbf{W}^r$. By the modal semantics, then, the pointer observable **P** is determinate on the apparatus system after measurement, and the Born Rule furnishes a probability distribution over its possible values. The ignorance interpretation of mixtures also essays this pattern of value assignment to subsystems of the composite system. But so doing, the ignorance interpretation implies that the composite state is the mixture

$$\mathbf{W}^{sr} = \Sigma_i \, |c_i|^2 \, |o_i\rangle\langle o_i| \otimes |p_i\rangle\langle p_i|,$$

a mixture empirically distinguishable from the system's true pure state $|\Psi^{sr}\rangle$. Modal interpretations avoid this debacle because their value assignments are not only *compatible* with the attribution of the pure state $|\Psi^{sr}\rangle$ to the composite system, they're also *derived from* that pure state attribution! Thus would the Modal Interpretation explain what textbook interpretations can not: how measurement interactions obedient to the laws of quantum dynamics issue determinate outcomes corraborating the predictions of the quantum statistical algorithm.

3. THE ALBERT-LOEWER PROBLEM

Albert and Loewer contend that modal interpretations, however marvellously they account for model post-measurement states such as $|\Psi^{sr}\rangle$, are stymied by the post-measurement states of actual laboratory devices [1-3]. The coupling U_{IvN} establishes a *perfect* correlation between pointer eigenstates and orthogonal states of the object system. This, Albert and Loewer contend, requires error-free measuring devices, and even "the more expensive sorts of ...measuring devices" [1] will introduce some noise into the measurement process. A utopian device for measuring $\sigma(\mathbf{z})$, the z-component of spin on a spin 1/2 system, will have two indicator states, $|"+"\rangle$ (signalling the +1/2 value of $\sigma(\mathbf{z})$), and $|"-"\rangle$ (signalling the -1/2 value). $|"+"\rangle$ and $|"-"\rangle$ comprise the eigenbasis of the pointer observable **P**. The utopian measurement evolution will correlate these eigenstates perfectly with $\sigma(z)$ eigenstates $|+\rangle$ and $|-\rangle$. When the initial state of the object system is $|\phi\rangle = a|+\rangle + b|-\rangle$, an ideal spin measurement develops as follows:

$$(a|+\rangle + b|-\rangle)|p0\rangle \longrightarrow a|+\rangle|"+"\rangle + b|-\rangle|"-"\rangle = |\Phi\rangle. \quad (3.1)$$

But in "in a real measurement, there is always some probability of the measuring device making an error." So after an error-prone (and ergo realistic) spin measurement, the state of the composite system will be not $|\Phi\rangle$ but

$$|\Phi'\rangle = a'|+\rangle|"+"\rangle + b'|-\rangle|"-"\rangle + c|+\rangle|"-"\rangle + d|-\rangle|"+"\rangle. \quad (3.1')$$

where "the components $|-\rangle|"+"\rangle$ and $|+\rangle|"-"\rangle$ represent errors"[2].

Now the rub for modal interpretations is that in virtue of Schmidt's theorem, $|\Phi'\rangle$ will admit of a polar decomposition in terms of some bases $\{|\uparrow\rangle, |\downarrow\rangle\}$ on H^s and $\{|U\rangle,|D\rangle\}$ on H^r. $\{|\uparrow\rangle,|\downarrow\rangle\}$ will be the basis of a component of spin $\sigma(\mathbf{z'})$ different from $\sigma(\mathbf{z})$, and $\{|U\rangle,|D\rangle\}$ the basis of some observable **P'** on the apparatus system which is different from **P**, and whose physical interpretation is unclear. (MSR) implies that *these* observables--and not the $\sigma(\mathbf{z})$ observable we take ourselves to be measuring, and not the pointer observable we look to in measuring it--are the observables which have determinate values when $|\Phi'\rangle$ represents the composite system. The Albert-Loewer criticism is that, interpreting actual postmeasurement states like $|\Phi'\rangle$, modal semantics "fail to assign a definite position to the pointer which is supposed to register the measurement outcome" [1] and so offer us "no guarantee that measurements will have definite outcomes" [3]. Slavish adherence to modal semantic rules in the face of realistically error-prone devices allots the "wrong" observables determinate values.[3]

4. MODELLING DEVICE ERROR: ROGUE HAMILTONIANS

Albert and Loewer submit the generic possibility that real world measurement evolutions leave apparatus systems in reduced states which aren't diagonalized by the eigenbasis of the anticipated pointer observable. Call interactions issuing such states *AL-prone*. Albert and Loewer motivate the possibility of AL-prone measurements by an intuitive appeal to device error. Modal proponents were quick to point out that measurements can be error-prone in Albert and Loewer's intuitive sense without being AL-prone, that is, without visiting indeterminate pointer observables on measurements modally described. To model situations in which "the pointer registers ["+"] when the value of [spin] is actuallly [-]" [11] Bub summons rogue measurement Hamiltonians which, governing errant measurement interactions, leave devices in misleading, but nevertheless determinate, indicator states. For instance, a rogue evolution operator U_e might *anti*-correlate spin and pointer eigenstates:

$$U_e(|+\rangle|p0\rangle) = |+\rangle|\text{"-"}\rangle, \tag{4.1}$$

$$U_e(|-\rangle|p0\rangle) = |-\rangle|\text{"+"}\rangle. \tag{4.2}$$

Performed on a spin system with initial state $a|+\rangle + b|-\rangle$, the misfiring measurement driven by this rogue Hamiltonian issues the post-measurement composite state

$$|\Phi\rangle = a|+\rangle|\text{"-"}\rangle + b|-\rangle|\text{"+"}\rangle. \tag{4.3}$$

The eigenbasis $\{|\text{"+"}\rangle, |\text{"-"}\rangle\}$ of the anticipated pointer observable **P** furnishes a polar decomposition of $|\Phi\rangle$. (MSR) therefore allots **P** a determinate value. Its value is "+" when the spin of the object system is -, but still the apparatus registers an outcome. "How else could the instrument err," Bub asks Albert and Loewer, "*if it registers at all?*"

In a similar vein, Healey [21] offers "Almost-**O**" measurements, unitary extensions of the map

$$|n_i\rangle|p0\rangle \longrightarrow |n_i\rangle|p_i\rangle, \tag{4.4}$$

where $\{|n_i\rangle\}$ are eigenstates of an observable "close to" the anticipated object observable **O** in that $\langle n_i|o_j\rangle \approx \delta_{ij}$. Acting on the initial object state

$$|\psi^{s}\rangle = \Sigma_i\, c_i\, |o_i\rangle = \Sigma_{ij}\, c_i\, \langle n_j|o_i\rangle|n_j\rangle, \qquad (4.5)$$

(4.4) issues the post measurement composite state

$$|\underline{\Psi}^{sr}\rangle = \Sigma_{ij}\, c_i\, \langle n_j|o_i\rangle\, |n_j\rangle|p_j\rangle. \qquad (4.6)$$

In (4.6), the pointer observable **P** gives the polar decomposition of $|\underline{\Psi}^{sr}\rangle$. Thus (MSR) attributes **P** a determinate value, even though the interaction fails to correlate **P**'s eigenstates perfectly with **O**'s. Almost-**O** measurements comprise a particularly well-behaved class of rogue measurements. Because $\langle n_i|o_j\rangle \approx \delta_{ij}$, the r.h.s. of (4.6) indicates that **P** is not only determinate but also manifests statistics which very nearly reproduce the **O** measurement statistics implicit in the initial object state.

Albert and Loewer worry that modal interpretations cannot expect Nature to leave laboratory devices in states diagonalized by just the right apparatus observable. Bub's and Healey's error-prone devices misfire, but they misfire so providentially that they *are* left in reduced states diagonalized by just the right apparatus observable. Albert and Loewer may well ask: Why should we expect *all* imperfect devices to misfire in exactly this way?! Offering these cooperatively uncooperatively devices in response to the Albert-Loewer argument, Bub and Healey seem to miss its point. But identifying the uncooperative situations as situations in which the device errs by *recording the wrong value*, Albert and Loewer *invite* Bub and Healey to miss their point. Identifying error-prone measurements with respect to *what the device registers* enables a modal proponent to incorporate her own criterion of observable determinateness in an account of device error. And an account of device error predicated on modal semantics—this is essentially what Bub and Healey offer—is unlikely to threaten modal interpretations.

5. ERROR: THE MISTRANSCRIPTION OF PROBABILITIES

I propose to cull from the quantum theory of measurement a notion of measurement error which does not presuppose any particular account of observable determinateness. Suspending the intuitive desideratum that measurements should communicate pre-interaction object observable values to post-measurement pointer observable values, what do we expect of measurement? Van Fraassen suggests that "the most general notion of measurement requires . . . only that we be able to infer from information of outcomes to information of the measured system's initial state" [30]. A sort of transcription criterion serves to capture this intuition. Let $|\psi>$ be an arbitrary premeasurement object state. Then what Busch, Lahti and Mittlestaedt [12] call *the probability reproducibility condition* (PR) requires the coupling **U** between object system S and recording device R (initially in ready state $|p0>$) to transcribe the probability distribution $|\psi>$ defines for the object observable **O** to the probability distribution the post-measurement composite state $\mathbf{U}(|\psi>|p0>)$ defines for the pointer observable $\mathbf{I^s} \otimes \mathbf{P}$. Where $\mathbf{W^r}$ is the reduced apparatus state induced by $\mathbf{U}(|\psi>|p0>)$, we can express (PR) as follows:

> The quadruple $<H^r, |p0>, \mathbf{P}, \mathbf{U}>$ satisfyies PR for **O** if and only if, for any initial object state $|\psi>$,
> $$\mathrm{Tr}(|\psi><\psi|o_i><o_i|) = \mathrm{Tr}\,(\mathbf{W^r}|p_i><p_i|) \qquad \text{(PR)}$$

In an interaction satisfying (PR), if the initial object state assigns object eigenvalue o_n probability $|c_n|^2$, then the final apparatus state will assign pointer value p_n the same probability. We can infer $|\psi>$ from pointer statistics, and so satisfy van Fraassen's intuitive desideratum.

Busch, Lahti and Mittelstaedt offer (PR) as a necessary, but not sufficient, condition for measurement. They would supplement (PR) by an "objectification" requirement, a criterion of observable determinateness by whose lights the pointer observable designated in a PR-satisfying interaction is determinate. But in the last section, we saw how, in the context of the Albert-Loewer problem, relying on an objectification requirement to characterize measurement error runs us in dialectical circles. It may be more strategic in that context to characterize measurement error with respect to (PR) unsupplemented. Thus I suggest we take quadruples $<H^r, |p0>, \mathbf{P}, \mathbf{U}>$ fulfilling (PR) for **O** to be *error-free O-measurements*, and take quadruples failing (PR) to be *error-prone O-measurements*. (Fine [18] develops a formal account of quantum measurement which encompasses both error-free and error-prone measurements.) The approach I suggest not only liberates our notion of measurement error from preconceptions about observable determinateness, but also answers to the intuitively constituted examples of measurement error canvassed so far: Albert and Loewer's error-prone spin measurement (3.1') fails to transcribe probabilities unless it so happens both that $|a|^2 = |a'|^2 + |d|^2$ and that $|b|^2 = |b'|^2 + |c|^2$. Bub's rogue evolution (4.1-4.3) fails to transcribe probabilities unless $|a|^2 = |b|^2$. And Healey's almost-**O** measurements (4.4-4.6) fail to transcribe probabilities unless $|c_i|^2 = \Sigma_j |c_j\langle n_i|o_j\rangle|^2$. For arbitrary initial object states, these intuitively error-prone interactions fail to transcribe probabilities, and so they fail (PR).

AL-prone measurements leave apparatus systems in states not diagonalized by the eigenbasis of the anticipated pointer observable, which are states from which (MSR^S) withholds determinate pointer values. Bub and Healey offer examples of measurements which are error-prone, insofar as they violate (PR), but not AL-prone. In the next section, I'll offer examples of measurements which are AL-prone but not error-prone.

6. GUM DISEASE

A theorem due to Beltrametti, Cassinelli, and Lahti [8] characterizes the unitary evolutions satisfying (PR). Where **O** is discrete and **P** is discrete and non-degenerate, (PR) is fulfilled if and only if **U** is a unitary extension of the mapping

$$|o_{ij}\rangle|p0\rangle \dashrightarrow |q_{ij}\rangle|p_i\rangle, \qquad (6.1)$$

where j is a degeneracy index and $\{|q_{ij}\rangle\}$ are a set of unit vectors orthogonal in that index. Call such evolutions *Normal Unitary Measurements* or NUMs. Some special cases of NUMs merit our attention. In one special case, the $\{|q_{ij}\rangle\}$ are orthogonal in the first index as well; they comprise an orthonormal set of vectors, eigenvectors of some observable **Q** on H^S. Such a NUM will leave the composite system in a state which perfectly correlates **Q** eigenstates with eigenstates of the pointer observable **P**. These *strong state correlating* measurements leave the apparatus system in a reduced state $\mathbf{W}^r$ diagonalized by **P**, and leave the object system in a reduced state $\mathbf{W}^s$ diagonalized by **Q**; (MSR) deems **Q** and **P** determinate on S and R respectively. Another special kind of NUM occurs when the $\{|q_{ij}\rangle\}$ are identical with eigenvectors $\{|o_{ij}\rangle\}$ of the object observable **O**. The ideal von Neumann measurement (1.1) is an example of such a NUM. But the $\{|q_{ij}\rangle\}$ in (6.1) aren't required to be orthogonal in i. For a non-degenerate object observable **O**, the most general sort of unitary measurement takes the form

$$|o_i\rangle|p0\rangle \dashrightarrow |q_i\rangle|p_i\rangle \qquad (6.2)$$

$\{|q_i\rangle\}$ a non-orthogonal set of unit vectors.

Call this most general class of NUM interactions, interactions conforming to the template (6.2), *general unitary measurements*, or GUMs.[4]

Subject S in initial state $\Sigma_i\, c_i|o_i\rangle$ to a GUM:

$$\Sigma_i\, c_i|o_i\rangle|p0\rangle \dashrightarrow \Sigma_i\, c_i|q_i\rangle|p_i\rangle = |\Psi'\rangle. \qquad (6.3)$$

Because the $\{q_i\rangle\}$ are non-orthogonal, $\{|p_i\rangle\}$ does not furnish a polar decomposition of $|\Psi'\rangle$. (MSR) does not attribute the apparatus a determinate value for the pointer observable **P**, and the measurement has no outcome—the very difficulty emphasized by Albert and Loewer. We can conclude that GUMs are AL-prone. Satisfying (PR), GUMs are not error-prone. Thus it is not necessary for an interaction to be AL-prone that it be error-prone.

7. THE ARAKI-YANASE THEOREM

I have argued that the class of measurements to which Albert, Loewer, and some of their commentators direct their remarks—the class of error-prone measurements—neither contains nor is contained in the class of measurements to which Albert, Loewer, and their commentators should direct their remarks—the class of AL-prone measurement interactions. I will close by discussing an ironic implication of the Araki-Yanase theorem: there exist a large class of observables whose error-free measurements are necessarily AL-prone.

When von Neumann showed how to construct an ideal unitary measurement $\langle H^r, |p0\rangle, \mathbf{P}, \mathbf{U}\rangle$ for any discrete object observable, he did not take constraints on allowable interactions, such as conservation laws, into account. Wigner [33] argued by means of an example that conservation laws make the ideal measurement of certain object observables impossible; Araki and Yanase [4] generalized this result. In what follows, I sketch a simple version of their theorem, and extract its consequences for the Albert-Loewer problem.[5]

Let object observable **M** be discrete and non-degenerate, and let the composite quantity $\mathbf{L} = \mathbf{L1} \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{L2}$ be conserved. (Because **L** for the composite system is the sum of **L** for each subystem, Araki and Yanase call it an "additively conserved quantity".) As a constant of the motion, **L** commutes with composite system evolution operator **U**. An ideal measurement of **M** by **P** will unfold as follows:

$$U(|m_i\rangle|p0\rangle) = |m_i\rangle|p_i\rangle \qquad (7.1)$$

Claim: (7.1) is possible only if $[\mathbf{L1}, \mathbf{M}] = 0$.

Proof:

$$\begin{aligned} &\langle m_i|\langle p0|\ \mathbf{L}\ |m_j\rangle|p0\rangle \\ &\quad = \langle m_i|\langle p0|\ \mathbf{U}^{-1}\mathbf{UL}\ |m_j\rangle|p0\rangle && (\mathbf{U}\ \text{unitary}) \\ &\quad = \langle m_i|\langle p0|\ \mathbf{U}^{-1}\mathbf{LU}\ |m_j\rangle|p0\rangle && ([\mathbf{U},\mathbf{L}] = 0) \\ &\quad = \langle m_i|\langle p_i|\ \mathbf{L}\ |m_j\rangle|p_j\rangle. && (\text{by } 7.1) \end{aligned}$$

The calculation above implies that, if **L** is conserved, (7.1) is possible only if

$$\langle m_i|\langle p0|\ \mathbf{L}\ |m_j\rangle|p0\rangle = \langle m_i|\langle p_i|\ \mathbf{L}\ |m_j\rangle|p_j\rangle. \qquad (7.2)$$

Rewrite **L** as $\mathbf{L1} \otimes \mathbf{I} + \mathbf{I} \otimes \mathbf{L2}$. The l.h.s. of (7.2) becomes

$$\langle m_i|\mathbf{L1}|m_j\rangle\langle p0|p0\rangle + \langle m_i|m_j\rangle\langle p0|\mathbf{L2}|p0\rangle, \qquad (7.3)$$

and the r.h.s. becomes

$$\langle m_i|\mathbf{L1}|m_j\rangle\langle p_i|p_j\rangle + \langle m_i|m_j\rangle\langle p_i|\mathbf{L2}|p_j\rangle. \qquad (7.4)$$

Use (7.3), (7.4), and the orthogonality of $\{|p_i\rangle\}$ and $\{|m_i\rangle\}$ to rewrite (7.2) as

$$\langle m_i|\mathbf{L1}|m_j\rangle + \delta_{ij}\langle p0|\mathbf{L2}|p0\rangle = \delta_{ij}\langle m_i|\mathbf{L1}|m_j\rangle + \delta_{ij}\langle p_i|\mathbf{L2}|p_j\rangle. \quad (7.5)$$

For $i \neq j$, (7.5) holds only if:

$$\langle m_i|\mathbf{L1}|m_j\rangle = 0. \qquad (7.6)$$

That is, only if $[\mathbf{M},\mathbf{L1}] = 0$. Therefore, if **L** is conserved, (7.1) is possible only if **M** commutes with **L1**. In other words, an ideal measurement of **M** is possible only if $[\mathbf{M},\mathbf{L1}] = 0$. This is what was claimed.

The argument that a measurement of **M** is possible only if **M** commutes with all additively conserved quantities applies as well to strong state correlating measurements. But it is blocked for (GUM)s. For these, not all of the orthogonality relations invoked to write the δ_{ij}'s in (7.5) obtain. It follows from the Araki-Yanase theorem, then, that if an object observable **M** fails to commute with an additively conserved quantity of the composite measurement system, the only PR-satisfying measurements of **M** available are GUMs. (Using a result of Fine [18], Hellman [23] makes a terminological variant on this point. For further discussion, see ref [28].) But these measurements are AL-prone. In the context of measurements of quantities subject to the Araki-Yanase theorem, the demand that measurements be *free from error* generates the Albert-Loewer problem.

8. CONCLUSION

I mean these remarks to disentangle the dialectical situation surrounding the Albert-Loewer problem. But I direct them also against a misconception which seems to arise concerning that problem. According to the misconception, the Albert-Loewer problem looms when something goes wrong: when an instrument misfires, or when the apparatus system becomes entangled with its environment in such a way that the anticipated pointer observable fails to diagonalize $\mathbf{W}^r$. This misconception tempts us to dismiss AL-prone interactions as conceptually uninteresting contingent complications. Others have addressed this misconception. Elby [16] conjures "the problem of tails" for measurements whose outcomes are registered either directly or indirectly in the positions of systems. A standard example is a Stern-Gerlach measurement in which the position of a spin-1/2 particle shot through an inhomogeneous magnetic field functions as a record of its spin in the direction of the field's inhomogeneity. Let the field be inhomogeneous in the z direction, and let $|\xi 0(z)\rangle$, the wave packet describing the premeasurement position of the localized particle, be centered at z=0. Interaction with the magnetic field shifts the wave packet of a spin up particle so that it's centered at z=L and shifts the wave packet of a spin down particle so that it's centered at z = -L. Call the shifted wave packets $|\xi +(z)\rangle$ and $|\xi -(z)\rangle$ respectively. If $|\xi 0(z)\rangle$ has significant tails (that is, if $\xi 0(z)$ is non-zero for $|z| > L$), $|\xi +(z)\rangle$ and $|\xi -(z)\rangle$ will not, strictly speaking, be orthogonal, and so cannot by modal lights be eigenstates of a determinate pointer observable. Elby points out that the logical structure of QM gives us every reason to suppose that $|\xi 0(z)\rangle$ will have tails. Even if at some instant the particle's wave packet is confined to a finite region, the Schrödinger equation implies that an instant later, that wave packet will have *infinite* tails.[6] So QM itself implies that the archetypical Stern-Gerlach measurements discussed in every introductory quantum course are subject to the Albert-Loewer problem.

Dickson [16] casts this argument in a more general form to conclude that QM itself implies that a class of "typical" measurements, a class including the Stern-Gerlach measurement, are subject to the Albert-Loewer problem.

The unitary operator driving the Stern-Gerlach measurement sketched above acts as follows:

$$U_{SG}(|+\rangle|\xi 0(z)\rangle) = |+\rangle|\xi+(z)\rangle \quad (8.1)$$

$$U_{SG}(|-\rangle|\xi 0(z)\rangle) = |-\rangle|\xi-(z)\rangle \quad (8.2)$$

If $\langle\xi-(z)|\xi+(z)\rangle \neq 0$, the measurement is AL-prone. According to the theorem of Busch, Casinelli and Lahti [8], if $\langle\xi-(z)|\xi+(z)\rangle \neq 0$, the measurement fails (PR) as well. Typical Stern-Gerlach measurements are not only AL-prone, but also error-prone. I've argued here that there exist a class of measurements which are error-free but AL-prone. Even in a benign universe of perfectly isolated, flawlessly accurate devices, error-free measurements of certain observables are available only if they're AL-prone. The presence of AL-prone interactions is secured not by the familiar and uninteresting phenomenon of bad luck, but by physical law.

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NOTES

1. States with non-unique polar decompositions present modal interpretations with a special set of difficulties [5,15,20]. As these are not the difficulties with which I am here concerned, I will confine my attention in what follows to states whose polar decompositions are unique.
2. Neither Healey nor van Fraassen (among prominent modal proponents) offer interpretations justly encapsulated by MSR. The following discussion is nevertheless pertinent to their interpretations because those interpretations generally agree with the valuations offered by MSR at the close of measurement-type interactions, and so confront some variant of the Albert-Loewer problem.
3. There's a consensus emerging that the strongest modal response to Albert and Loewer takes to heart their insistence on describing the measurement interaction realistically. Dropping the idealization that S and R are hermetically isolated from their environment, modal

proponents cite decoherence accounts of quantum measurement [34] to contend that environmental interactions leave the apparatus in a reduced state nearly diagonalized by the eigenbasis of the anticipated pointer observable. Such an apparatus, modal defenders maintain, will have a determinate value for an observable "close enough" to the anticipated pointer observable for experimental practice to flourish. Bacciagaluppi and Hemmo [7] provide a splendid review of this literature; see also [6,13,15,22]. I intend my remarks here as a prelude to these decoherence discussions: modal defenders should, after all, be clear on what problem their appeal to the environment is intended to solve.

4. It must be acknowledged that calling these interactions "measurements" is contentious. Those who subject measurements to an objectification requirement which GUMs fail would object to the characterization. A defense of GUMS might appeal to a long and apparently respectable history of treating them as measurements. For example, the opening chapter of Landau and Lifshitz's text [26] remarks—almost in passing—that most actual measurement interactions take the form (6.1) but fail to correlate orthogonal object states with distinct "reading" states of the instrument—in other words, that most actual measurements are GUMs. Given the weight of tradition and practice, I'm inclined to believe that the onus is on those who would append an objectification criterion to their notion of measurement to explain why it's permissable to thereby disqualify a large class of interactions hitherto recognized as measurements.

5. In a discussion which is not an explicit reaction to the Albert-Loewer problem, Healey [20] allows that the Araki-Yanase theorem limits the class of observables for which what he calls M-suitable interactions exist. (In my vocabulary, M-suitable interactions are neither error- nor AL-prone—they're the "good" measurements of which modal interpretations make good sense.) Healey's semantics prevent him from appealing to GUMs for error-free measurements of observables subject to Araki-Yanase limitations. Rather, he would minimize these limitations by recognizing that no apparatus is ever truly isolated.

6. It is probably worth mentioning that, introducing his modal interpretation, Kochen [24] calculates in detail how it would describe the unfolding of a realistic Stern-Gerlach measurement. He concludes that, except in cases of near-exact degeneracy, the bases he privileges on the apparatus

system approach the anticipated pointer basis very rapidly. (Refs. [6] and [16] reach similar conclusions.)