

## FOUNDATION ENGINEERING

### INVESTIGATION OF THE EFFECT OF A VERTICAL SURCHARGE ON HORIZONTAL DISPLACEMENTS AND RESISTANCE OF PILE COLUMNS TO HORIZONTAL LOADS

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In the USSR's construction specifications and regulations SNiP II-B.5-67\* and in the design standards of foreign countries, it is allowed to calculate the horizontal load on piles without consideration of the effect of a vertical surcharge. The principle of the independence of the action of forces is used in the majority of known methods of calculating piles subjected to vertical and horizontal forces and the calculation is made separately for each type of load.

In recent years, suggestions on calculating piles have appeared which substantiate the need to take into account the mutual effect of the loads when calculating piles. On the basis of theoretical investigations, Snitko and Snitko [1, 2] and Shakhirev and Yanyshv [3] showed that the resistance of piles to horizontal loads in the presence of a vertical surcharge is lower than without this surcharge. A vertical surcharge reduces especially considerably (by twofold and more) the bearing capacity of flexible piles with respect to horizontal loads.

Tests of piles by the joint effect of horizontal and vertical loads are described in the literature. Thus, tests of single-column transmission-line towers were conducted by Kananyan [4, 5], pile columns of pipeline trestles by Golubkov, Tugaenko, and Lakizo [6], etc. However, in these works, the effect of a vertical surcharge on the bearing capacity and horizontal displacements of the piles was not examined.

Tests of situ-cast piles, which showed a favorable effect of a vertical surcharge, are presented in [7].

Taking into account the great theoretical and practical importance of the problem, TsNIEPsel'stroi jointly with the Scientific-Research Institute of Industrial Construction (NIIpromstroi) in 1967-1968 and with the Kazakh State Institute of Agriculture (KazGIIZ) in 1969 and 1974 conducted full-scale tests of pile columns of agricultural buildings by horizontal static loads with a different vertical surcharge on two test plots. On the first, the foundation bed was composed of homogeneous saturated clay soils and on the second, of slump-prone soils of very stiff consistency. The physical and mechanical properties of the soils of the plots are given in Table 1.

On plot No. 1, we tested 14 solid prismatic pile columns and 8 hollow prismatic pile columns with a 16-cm-diameter cylindrical cavity. The cross section of the pile columns was  $30 \times 30$  cm, the longitudinal working reinforcement consisted of four 12-mm-diameter bars of class A-I, the concrete was M200. The driving depth of the pile columns was 2 and 3 m, the height above the ground surface was 2.4 m.

On plot No. 2, we tested 21 solid prismatic pile columns and those with bilateral cantilevers designed by TsNIEPsel'stroi [8]. Cross section of the pile columns,  $30 \times 30$  cm; longitudinal working reinforcement, four 12-14-mm-diameter bars of class A-III; concrete, M300. The driving depth was 2 and 4 m, the height above the ground surface was 2.5 m.

On both plots, the pile columns were driven into the ground by a Diesel hammer. The vertical position during driving was checked by two transits installed in mutually perpendicular planes. The driven pile columns had deviations from the vertical of not more than  $1^\circ$ , and the mutual displacement of two adjacent tested pile columns in plan did not exceed 3 cm. The cantilever pile columns were driven into the ground so that the distance from the top of the cantilevers to the ground surface was 20 cm.

On plot No. 1, the horizontal load was created by winches pulling together two adjacent pile columns at the height of their heads, and on plot No. 2 by means of a special device developed by TsNIEPsel'stroi [9].

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TABLE 1

Sampling depth, m	natural structure	Density, g/cm <sup>3</sup>	Porosity, %	Water content, %		Plasticity index	Degree of saturation, fractions of unit	Consistency index, fractions of unit	Rel. slump-proneness at p = 0.2 MPa, fractions of unit	Initial slumping pressure, MPa	Cohesion, MPa	Angle of internal friction, deg	
				natural at liquid limit	at plastic limit								
Plot No. 1													
1	1,88	1,45	47,3	29,8	45,0	23,6	0,21	0,92	0,28	—	0,040	17	
2	1,86	1,41	48,7	31,6	46,0	24,9	0,21	0,82	0,32	—	0,030	20	
3	1,92	1,45	47,2	31,6	46,1	24,4	0,28	0,97	0,34	—	0,043	12	
4	1,96	1,50	45,4	30,1	48,2	24,7	0,24	0,99	0,23	—	0,075	3	
5	1,91	1,46	46,9	31,2	44,4	24,6	0,20	0,97	0,33	—	0,040	17	
Plot No. 2													
0,5	1,29	1,23	54,6	5,1	32,0	21,0	0,11	0,14	-1,45	0,109	0,017	0,050 0,008	24 24
1	1,40	1,32	51,4	6,0	29,0	20,5	0,09	0,15	-1,71	0,108	0,027	0,074 0,011	35 27
2	1,44	1,35	49,9	6,2	28,2	19,5	0,08	0,16	-1,57	0,104	0,038	0,096 0,014	32 26
3	1,40	1,29	52,4	9,0	31,7	22,0	0,10	0,22	-1,27	0,073	0,041	0,100 0,010	37 27
4	1,51	1,38	48,0	7,2	26,3	19,3	0,09	0,23	-1,26	0,052	0,085	0,119 0,017	37 26
5	1,62	1,49	45,0	9,4	29,3	19,7	0,10	0,31	-0,94	0,046	0,086	0,093 0,024	33 25

Note. In the numerator are the indices for soil of natural water content; in the denominator, for saturated soil.

TABLE 2

Type of pile column	Driving depth, m	State of soil	P <sub>V</sub> , kN	P <sub>h</sub> kN at y <sub>0</sub> 10 mm	Appearance of cracks		Opening of cracks to 0,3 mm		Fracture	
					P <sub>h</sub> , kN	y <sub>0</sub> , mm	P <sub>h</sub> , kN	y <sub>0</sub> , mm	P <sub>h</sub> , kN	y <sub>0</sub> , mm
19-P	2	Natural water content	0	13,2	8,8	1,8	—	—	13,3	10,6
20-P	2	»	0	12,9	10	5,2	—	—	14,2	22,6
25-P	2	»	320	19,7	20,4	10,7	—	—	24	20
26-P	2	»	320	21,5	13,8	4,8	—	—	21,6	10,8
3-P	3	»	0	18	7,8	1,4	—	—	18	10
4-C	3	»	0	15,5	9	1,9	—	—	16,2	12,9
27-C	3	»	320	fract.	25,8	3,8	—	—	31,8	9
28-C	3	»	280	32,5	26,7	4,1	—	—	33,5	45
51-C	4	»	0	18,8	12	2,1	16	3,2	24	57,3
53-C	4	»	0	20,5	14	3,1	18	5,9	26	59,3
30-P	2	Saturated	0	10,7	14,5	25,5	14,5	25,5	17	74,3
31-P	2	»	0	10,1	12,5	22	—	—	16	190,2
23-P	2	»	67	9,3	6	1,3	6	1,3	12	381,3
24-P	2	»	67	10,1	10,5	11,1	10,5	11,1	12	110
42-P	4	»	184	9,7	11	16,3	—	—	13	160
5-C	2	»	0	14,5	13	4,4	15	14	20	210,3
6-C	2	»	0	15	6	0,4	9,5	2,4	20	172,8
15-C	2	»	71	13,8	13	5,1	—	—	14	11,3
16-C	2	»	71	18,7	15	5,2	19	11,2	23	244,8
41-C	4	»	188,2	17	13	4,4	—	—	20	134,7
43-C	4	»	183,2	17,6	12	2,4	—	—	20	204,6

Note. The types of pile columns with index P are prismatic and with C cantilever, designed by TsNIIEPsel'stroi.

The vertical surcharge P<sub>V</sub> on both plots was created by means of a platform with weights hinge-suspended from the head of the pile columns [9].

Horizontal load testing of the pile columns was carried out by the method of TsNIIEPsel'stroi [9]. Taking into account that the horizontal load transmitted to pile columns of agricultural buildings is brief (mainly the wind load), as the conditional criterion of stabilization of horizontal displacements at the level of the ground surface from each load increment we took a rate of displacements of not more than 0.1 mm during 10-min observation. In the experiments with a vertical surcharge horizontal loading of the pile columns began after stabilization of settlements from the given vertical surcharge. The conditional criterion of stabilization of settlements was taken according to standard GOST 5686-69 [10] — no more than 0.1 mm during 2-h observation.

The maximum vertical surcharge on plot No. 1 was 142 kN (about 0.8% of the limiting) and on plot No. 2 in soil of natural water content 320 kN and in saturated soil 188.2 kN (respectively, about 0.4 and 0.7% of the limiting). The tests on the plots were preceded by tests of analogous pile columns by vertical static loads.

The horizontal displacements of the pile columns at the level of the ground surface  $y_0$  were measured by Maksimov deflectometers secured on anchor pile columns at the same level.

On plot No. 2, the tests were conducted both in soil of natural water content and in soil soaked to full saturation (at a degree of saturation  $G \geq 0.8$ ). To accelerate soaking, six 19-cm-diameter drain wells were drilled to a depth 1 m below the driving depth of the piles around each pile column at a radius of 0.8-0.9 m. By banking the earth around the pile columns at a distance of 1-1.1 m, pits about 30 cm deep were created for soaking. The bottom of the pits and wells were filled with rubble to eliminate warping. The achievement of full saturation was checked by means of an NIV-1 neutron moisture meter.

Observation of the appearance and development of cracks under the effect of the horizontal load was carried out by means of a MPB-2 microscope. The pile columns were dug up to detect cracks located below the ground surface. In some experiments, observations of the formation of cracks were carried out during loading of the pile columns by way of narrow pits 0.2 m wide and up to 1.5 m deep located on the side of the pile column opposite to the direction of the horizontal force. Digging of such a pit does not introduce substantial errors into the test results, since in this zone the pile comes loose from the soil under a horizontal load. Some experiments on plots Nos. 1 and 2 were conducted by a cyclic horizontal load with its gradual increase up to 0.8 of the breaking load. On plot No. 2, the change in the dry density of the slump-prone soil in the near-pile zone caused by driving the prismatic and cantilever piles was investigated by the method described in [11]. On plots Nos. 1 and 2, the gaps between the soil and faces of the pile columns caused by transverse vibrations of the piles during driving were measured by a thin metal probe.

The results of the horizontal load tests of the pile columns under various vertical surcharges are given in Table 2 and in Figs. 1-4 in the form of graphs  $y_0 = f(P_h)$ .

As we see from Figs. 1 and 2 (plot No. 1, driving depth 3 m), for displacements  $y_0 = 8-10$  mm a vertical surcharge up to 54 kN on hollow prismatic pile columns and up to 142 kN on solid prismatic pile columns has practically no effect on displacements from a horizontal load. Under a vertical surcharge of 112-142 kN, the bearing capacity of hollow prismatic pile columns was 1.24 times greater than when  $P_v = 0$ . On plot No. 2, it was established during testing of pile columns with a 2-m driving depth in saturated slump-prone soil ( $G \geq 0.8$ ) that a vertical surcharge insignificantly affects the resistance of the pile columns to horizontal loads. We see from a comparison of graphs 30-P, 31-P, 23-P, and 24-P (Fig. 4) that for prismatic pile columns a surcharge has no effect within the limits of accuracy of the experiment. From a comparison of graphs 5-C, 6-C, and 16-C (Fig. 4) we can conclude that for displacement  $y_0 = 10$  mm a vertical surcharge increased the resistance of the cantilever pile columns by 1.1 times.

A different picture was observed in the case of testing pile columns in slump-prone soils of very stiff consistency. In this case, the vertical surcharge led to a substantial increase of the bearing capacity of the pile columns with respect to horizontal loads. For displacement  $y_0 = 10$  mm the vertical surcharge increased the resistance of prismatic pile columns to a horizontal load by 1.57 times (graphs 19-P, 20-P, 25-P, 26-P in Fig. 3) and of cantilever pile columns by 1.93 times (graphs 3-C, 4-C, 27-C, 28-C in Fig. 3).

The tests of pile columns of different design with a small relative embedment  $h/d < 15$  (where  $h$  is the depth of embedment,  $d$  is the side dimension of the cross section of the pile column) made it possible to establish certain characteristics of their behavior in the ground. Solid prismatic pile columns with a 3-m driving depth on plot No. 1 with displacement  $y_0 = 8$  mm had a greater resistance than the hollow ones (graphs 95, 86, 107, 108 in Fig. 2 and 99, 104 in Fig. 1). On plot No. 2, the cantilever pile columns with a 2-m driving depth and displacement  $y_0 = 10$  mm carried a horizontal load in slump-prone soil of very stiff consistency that was 1.28 times and in saturated slump-prone soil 1.42 times greater than the prismatic (graphs 3-C, 4-C, 19-P, 20-P in Fig. 3 and 5-C, 6-C, 30-P, 31-P in Fig. 4). Saturation of the slump-prone soil led to a decrease of the resistance of the pile columns to a horizontal load  $P_h$  for the same displacement  $y_0 = 10$  mm. For a driving depth of 2 m, the value of  $P_h$  decreased on the average by 1.3 times for the unsurcharged prismatic pile columns, by 2.13 times for the surcharged, by 1.14 times for the unsurcharged cantilever, and by 1.98 times for the surcharged (see Table 2). We see from Table 2 and a comparison of the graphs in Fig. 4 that in saturated slump-prone soils, the value of  $P_h$  for  $y_0 = 10$  mm is about the same for prismatic pile columns with a 4-m driving depth as for the cantilever pile column with a 2-m driving depth.

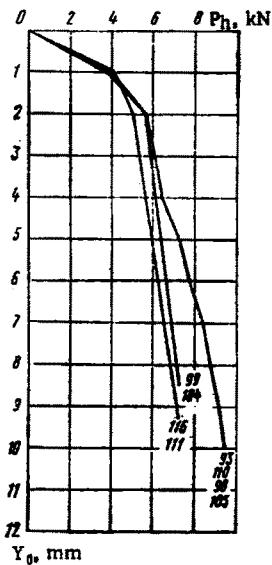


Fig. 1

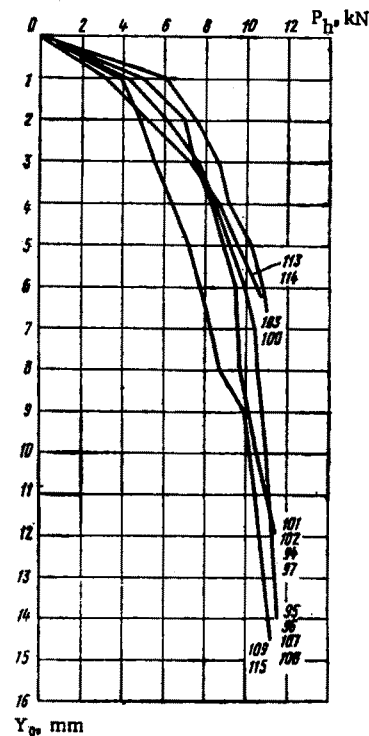


Fig. 2

Fig. 1. Averaged graphs of horizontal load tests of hollow prismatic pile columns on plot No. 1. Nos. 116, 111 ( $P_V = 54$  kN); 99, 104 ( $P_V = 0$ ); 93, 98 ( $P_V = 142$  kN); 110, 105 ( $P_V = 112$  kN); driving depth 3 m.

Fig. 2. Averaged graphs of horizontal load tests of solid prismatic pile columns on plot No. 1. Driving depth 3 m: No. 109 ( $P_V = 142$  kN); 115 ( $P_V = 112$  kN); 95, 96, 107, 108 ( $P_V = 0$ ); 113, 114 ( $P_V = 54$  kN); driving depth 2 m: Nos. 101, 102 ( $P_V = 112$  kN); 94, 97 ( $P_V = 142$  kN); 103, 100 ( $P_V = 54$  kN).

Observations of the formation of cracks during a gradual increase of the horizontal load showed that in the case of a small percentage of reinforcement of the pile columns by deformed bars of class A-III, the first microcracks appear at a load of about 0.4 of the breaking. The width of opening of the cracks subsequently increases to 0.3-0.5 mm and more. However, experiments with multiple cyclic loading made it possible to establish that after removing the brief horizontal load at its total magnitude up to 0.7-0.8 of the breaking, the cracks close so much that their residual width does not exceed 0.05-0.2 mm. Cyclic loading showed that the portion of recoverable deformation reaches 70% for slump-prone soils of very stiff consistency at  $P_h = 0.7-0.8$  of the breaking load and drops to about 30% for soaked slump-prone and saturated clay soils. Fracture of the cantilever pile columns occurred in the slump-prone soils of very stiff consistency at the level of the top of the cantilevers. For the prismatic pile columns, the fracture zone was located at a depth to 1.5d in the very stiff slump-prone soils and at depth 3-4d in the saturated slump-prone and ordinary clay soils on plot No. 1. The depth of the gap between the soil and faces of the prismatic pile columns in the investigated soils did not exceed 0.9 m. For the cantilever pile columns, gaps with a depth to 0.3 m were observed on the two faces where cantilevers were absent. An investigation of the dry density of the soil in the near-pile zone showed the presence of compacted zones under the cantilevers of the pile columns. In the investigated soils, zones of upward yielding of the soil were not observed at any stages of horizontal loading up to fracture of the pile columns.

Analyzing the results obtained, we need note that a vertical surcharge can both somewhat reduce and increase the resistance of pile columns (and piles) to horizontal loads. In weak soils under the effect of a horizontal load, a pile with a small relative embedment ( $< 15$ ) turns in the soil relative to the point of zero deflections "0." In this case, a vertical surcharge will create an additional bending moment  $M_1 = P_V e$ , where  $e$  is the eccentricity of force  $P_V$  relative to the center of gravity of the cross section at point 0. The moment  $M_1$  is greater, the greater the amount of rotation of the pile and, consequently, eccentricity  $e$ . At the same time, upon turning of the pile a moment  $M_2$  occurs under the foot from the nonuniform distribution of pressures,

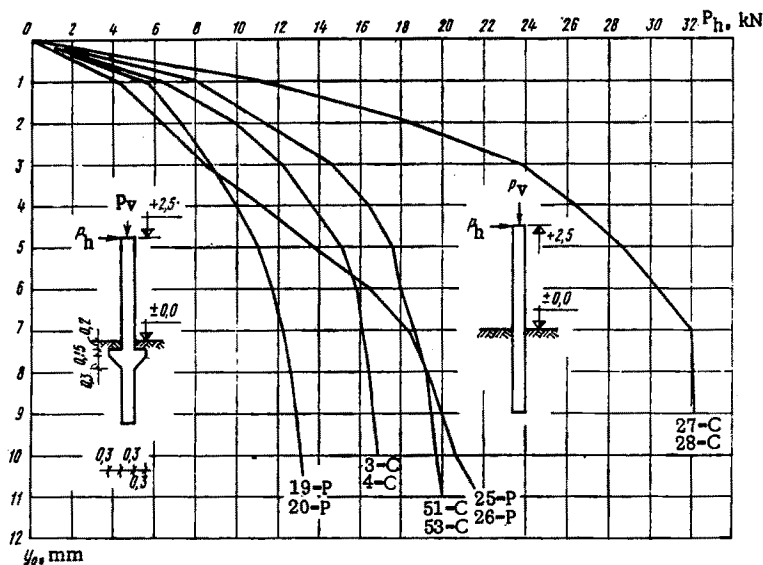


Fig. 3

Fig. 3. Averaged graphs of horizontal load tests of solid prismatic and cantilever pile columns in slump-prone soil of natural water content on plot No. 2.

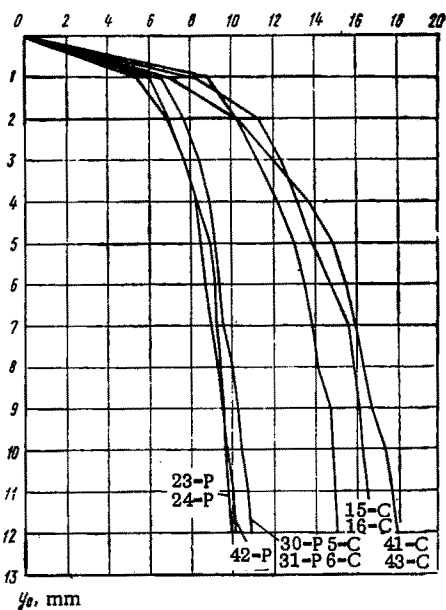


Fig. 4

Fig. 4. Averaged graphs of horizontal load tests of solid prismatic and cantilever pile columns in saturated slump-prone soil on plot No. 2.

whose direction is opposite to moment  $M_1$ . Rotation of the pile column is prevented also by moment  $M_3$ , which is equal to the product of the frictional force on the end of the pile T and its arm relative to point 0. The frictional force T and moments  $M_2$ ,  $M_3$  are greater, the greater the pressure under the foot of the pile. Moment  $M_2$  increases also with increase of the area of the base of the pile. A vertical surcharge promotes an increase of pressure under the toe. In weak soils, the pressure is small, whereas the angles of rotation are considerable. The resulting moment proves to be of the same sign as  $M_1$ . This reduces the bearing capacity of the pile with respect to horizontal loads.

In strong soils, the opposite picture is observed and a vertical surcharge has a favorable effect. In the case of small horizontal displacements of the pile columns, the vertical surcharge reduces the calculated eccentricity, which increases its eccentric compressive strength as a reinforced-concrete element. In very stiff slump-prone soils, the favorable effect of the surcharge proved to be considerable: for prismatic pile columns the resistance increased by 1.57 times and for cantilever by 1.93 times. The effect is greater for the cantilever pile columns, since to moments  $M_2$  and  $M_3$  is added moment  $M_4$  of the same sign from the pressure occurring under one of the cantilevers.

The increase of resistance of pile columns under the effect of a vertical surcharge in strong soils that was established by the authors agrees with the results of testing rigid (situ-cast) piles in analogous soils [7]. A comparison of the values of  $P_h$  for displacement  $y_0 = 10$  mm and the breaking values for prismatic and cantilever pile columns shows that the cantilever pile columns are distinguished by a higher resistance to horizontal loads. In very stiff slump-prone soils in the case of a 2-m driving depth and  $30 \times 30$  cm section, this increase amounts to 1.3-1.6 and in soaked slump-prone soils to 1.4-1.6, increasing in both cases in the presence of a vertical surcharge (see Table 2). In the investigated soils the involvement of the cantilevers proved to be equivalent to a 1.5-2 m increase of embedment of the prismatic pile columns. This increase of bearing capacity can be explained by the different character of the work of prismatic and cantilever pile columns in the upper zone of the soil. For the cantilever pile columns, driving of the cantilevers into the ground causes the formation of compacted zones ( $\gamma_d$  to  $1.65$  g/cm<sup>3</sup> vs  $\gamma_d$  of natural soil  $1.23$ - $1.32$  g/cm<sup>3</sup>). For prismatic pile columns, conversely, after driving, gaps form in the upper zone between the faces of the pile columns and the soil with a depth to 0.9 m. The ineffective work of prismatic piles in the upper zone was noted earlier by G. S. Kolesnik, A. A. Grigoryan, and others. This circumstance was taken into account when developing the method of calculating cantilever and prismatic pile columns in the Instructions [12].

Since the total length of pile columns usually cannot exceed 8 m with the pile-driving units presently used in rural construction, the use of cantilevers substantially expands their area of use.

The results of the tests show that the use of Table 12 of SNiP II-B.5-67\* for designing pile columns can lead to substantial errors. According to the experimental data, the depth of conditional embedment is closer to the ground surface than according to the aforementioned table. For clays of very stiff consistency, it should be no more than 1.5-2d and of medium consistency no more than 4-5d. Similar results were obtained earlier during tests of pile columns on other plots [1]. On the whole, the depth of conditional embedment should be determined by calculation in conformity with the Instructions [12].

Since the brief effect of a horizontal load of the wind type on pile columns (for  $P_h$  up to  $P_h^{frac}$ ) does not cause the appearance of residual cracks (the latter amount to 0.05-0.2 mm), when calculating pile columns as elements of the frame of a one-story agricultural building, the permissible width of cracks should be increased to 0.3-0.4 mm instead of 0.2 mm according to SNiP II-B.5-67\*. This suggestion is consistent with the new approach to the calculation of reinforced-concrete structures according to SNiP II-21-75 [13].

### CONCLUSIONS

1. The principle of independence of the action of forces in the general case is not applicable to calculation of pile columns with relative embedment  $< 15$ . A vertical surcharge can somewhat decrease (in weak saturated soils) or increase (in clays of very stiff consistency) the resistance of pile columns to horizontal loads. In the investigated saturated soils, the bearing capacity decreased by no more than 15%, and in very stiff slump-prone soils it increased by a factor of 1.9.

2. Under equal conditions, pile columns with cantilevers absorb a 1.3-1.6 times greater horizontal load than prismatic.

3. When designing pile columns, the depth of their conditional embedment in the soil should be determined in conformity with the requirements of Instructions [12] and not by Table 12 in SNiP II-B.5-67\*, since this leads to uneconomical reinforcement of the pile columns.

4. For pile columns of agricultural buildings a permissible crack width of 0.3 mm should be adopted instead of the 0.2 mm according to SNiP II-B.5-67\*.

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CALCULATION OF CIRCULAR FOUNDATION SLABS  
WITH CONSIDERATION OF NONLINEAR  
DEFORMATIONS OF THE REINFORCED CONCRETE

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At present there exist several methods of calculating circular reinforced-concrete slabs (in a linear formulation) lying on a base taken as a semi-infinite elastic solid, including Gorbunov-Posadov's method [1, 2] of expansion of the soil reaction pressure  $p(\rho)$  in a series

$$p(\rho) = \sum_{n=0}^{\infty} a_{2n} \rho^{2n}, \quad (1)$$

where the  $a_{2n}$  are coefficients subject to determination.

The author compiled detailed tables of dimensionless quantities for some main cases of loads and thereby maximally facilitated practical calculations.

It is expedient to extend this method to problems in which the reinforced-concrete slab works beyond the elastic limit and the elastic base remains linearly deformable. In such a simplified formulation, as was shown by Korenev [3], we can study rather accurately the behavior of flexible and extended slabs, though in the general case, the plastic deformations, of course, must be taken into account both in the slab and in the soil. The possibility of such a simplified approach was confirmed clearly on a practical example of calculation in [4].

At the same time, consideration of plastic deformations in a reinforced-concrete structure lying on an elastic base reduces the rigidity of the structure and, in turn, the forces in it. A reduction of forces is related to economy of materials.

A calculation method is given in [3] which permits taking into account the formation of plastic hinges in structures. This method is based on loading the structure in the section where the plastic hinge was formed by an angular deformation.\* In this case, the problem reduces to summation of the results of two calculations of an elastic structure — with respect to the given load and with respect to the load in the form of a certain angular deformation. The magnitude of the deformation is selected so that it together with the given load causes in the section under consideration a bending moment equal to the limiting. This method can be used also in Gorbunov-Posadov's method, for which the tables of this method should be supplemented by data on a calculation of the loads in the form of deformations. Such loads permit solving also certain other problems, for example, calculation of a nonisolated slab on an elastic base.

Let us proceed to obtain these data. Let a circular slab of radius  $R$  and constant thickness lying freely on a semi-infinite elastic solid be loaded by an angular deformation  $\varphi$  distributed about a circle of radius  $a$  ( $0 < a < R$ ) [3].

For such a load, the differential equation of the unknown function of bending of the slab  $w$  of the dimensionless variable  $\rho = r/R$ , according to [2] has the form

$$\frac{d^4 w}{d\rho^4} + \frac{2}{\rho} \cdot \frac{d^3 w}{d\rho^3} - \frac{1}{\rho^2} \cdot \frac{d^2 w}{d\rho^2} + \frac{1}{\rho^2} \cdot \frac{dw}{d\rho} = -\frac{R^4}{D} p(\rho),$$

\*Loads in the form of deformations (linear and angular) were introduced by Umanskii [5] and Korenev [3].