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The Meaning of the Quantifiers in the Logic of Leśniewski

Quine has claimed that Leśniewskian quantification is substitutional[1]. But this interpretation is incorrect (cf. Küng and Canty [16]). Actually Leśniewskian quantification constitutes a third possibility that lies between objectual (referential) quantification and substitutional quantification[2], and it overcomes the drawbacks of each of its better known alternatives: while objectual quantification is restricted because some names do not have objets and substitutional quantification is restricted because some objects do not have names[3], Leśniewskian quantification works both for empty names and for nameless objects. This is so because, as we shall see, the range of quantification is neither the set of objects nor the set of names but the set of extensions (i.e. of extensional meanings). And even empty names have an extension, and even nameless objects belong to extensions.

The formulas of substitutional and of Leśniewskian quantification belong to the object language, but their readings are in a certain sense metalinguistic. For instance, according to Ruth Barcan Marcus '$(\exists x)Fx$' is to be read "Some substitution instance of 'Fx' is true" and correspondingly '$(x)Fx$' is to be read "Every substitution instance of 'Fx' is true" (cf. [32] p. 252–253). How is that to be understood? We shall see that in an adequate reading of those formulas names of expressions occur only in an "implicit" and not in an "explicit" way.

In my opinion the question of how to read quantified statements is of some consequence. The habit of giving merely model-theoretic interpretations and no intuitive paraphrases has tended to obscure some subtle, but very important aspects of oblique speech. This can best be made clear by taking as a starting point some recent discussions concerning "saying that".

1. Prologue-functors

It has been customary to think that a sentence such as

(1) *Galileo said that the earth moves.*

is of the form $S(x, p)$ where 'S' is a sentence-forming functor which takes names and sentences as argument-expressions. But actually what does it mean that the second argument-expression is a sentence? Should the argument of a function not be named? Should the second argument-expression of the functor 'S' not be the name of a proposition?

Donald Davidson ([4], cf. also [11] and [23] p. 143) has made an interesting remark that sheds light on this question; he has found that (1) can be turned around in the following way:

(2) *The earth moves. Galileo said that.*

At first sight one might therefore be misled to think that (1) is really of the form $p. R(x, y)$. But obviously this is not correct because in (1) the expression '*the earth moves*' is not a coordinated sentence: from (1) one cannot logically infer that the earth moves.

In a previous article [17] I have therefore insisted on the subordinating connection, spelling out that (1) is short for:

(3) *Galileo said that which follows: the earth moves.*

I have pointed out that this means a return to the form $S(x, p)$, but that we have learnt something important on the way. Davidson's remark has made us aware of the demonstrative nature of the '*that*' and we have noticed its subordinating function. Both these features are made explicit by the expanded expressions '*that which follows:*'.

A functor such as 'S' which contains such a demonstrative and subordinating expression may be dubbed a "prologue-functor", and an expression which is subordinated to such an expression may be called a "play".[4] As a matter of fact the speaker who utters (3) is behaving like an actor and the expression '*the earth moves*' actually "replays" what Galileo is supposed to have said.[5]

This analysis is important because it shows why prologue-functors are not predicate-expressions or relation-expressions whose arguments always name certain objects. In (3) the argument-expression '*the earth moves*' is clearly not the name of some abstract object (the name of a proposition or of a state of affairs), but it is simply what its grammatical form says it is, namely a sentence. Only this sentence is not used in the way in which an isolated sentence is used, but it is used in a special subordinate way, namely "playfully", replaying what Galileo had said in earnest.

As a matter of fact expanding (1) into (3) shows that our example can also be structured according to the form $R(x, y)$[6]. All one has to do is to take '*that which follows: the earth moves*' as one unit, namely as the second argument of the relation-expression 'R'. Now this argument is indeed a name: the name of a saying. And there is also a well-known and less cumbersome way of writing the name of a saying: simply use quotation marks. By replacing '*that which follows:*' by quotation marks, the second argument of '*said*' becomes ''*The earth moves*''.

The difference between interpreting (1) according to the form $S(x, p)$ and interpreting it according to the form $S(x, y)$ is therefore simply a difference concerning the place where we introduce our basic cuts. Taking our cue from Nelson Goodman's article "The way the world is" [9] we can say that not only the world, but also the formulations of ordinary language "are many ways", i.e. can be reconstructed in different ways in our logico-grammatical theories (cf. also Lejewski [23]). There is, however, an asymmetry between the two options: if we cut according to the form $R(x, y)$, then we can still subdivide the second argument into '*that which follows:*' plus '*the earth moves*', that is, into a quotation-functor (which is the most elementary kind of prologue-functor) and its argument. We thus obtain the form $R(x, Q(p))$.[7] If, however, we cut according the form $S(x, p)$, then we have amalgamated '*that which follows:*' into the prologue-functor 'S'. Prologue-functors are by definition functors which contain a hidden quotation-functor.

From the point of view of the long formulation (3) the interpretation according to the form $R(x, Q(p))$ seems preferable, because the parts corresponding to 'R', 'x', 'Q', 'p', are all "bodily" present; but in colloquial speech one can very well say "the same" in less words, with less machinery, by saying simply

(4) *Galileo said the earth moves.*

For this short wording $S(x, p)$ seems quite adequate.[8]

2. Prologue-quantifiers

Having learnt how to treat sentences as "plays", we are now able to give accurate readings of the quantifiers. For instance the following formula of propositional logic

(5) $(p)(p \vee \sim p)$

can be read:

(6) *Whatever-the-inscriptions-equiform-with-the-following-item-are-*
-taken-to-say-: p
the-following-is-herewith-asserted-: $p \vee \sim p$.

In an extensional system this is equivalent with

(7) *Whatever-extension-the-inscriptions-equiform-with-*
-the-following-item-are-taken-to-have-: p
the-following-is-herewith-asserted-: $p \vee \sim p$.

Let us call this the Leśniewskian reading because as a matter of fact it is the one which is best in accordance with Leśniewski's intentions.[9]

But substitutional quantification too can be read according to this prologue method:[10]

(8) *Whatever-propositional-expressions-are-taken-to-be-substituted-for-*
-the-inscriptions-equiform-with-the-following-item-: p
the-following-is-herewith-asserted-: $p \vee \sim p$.

We can now see in what sense the Leśniewskian and the substitutional readings are metalinguistic. They are metalinguistic in the sense that the quantification-prologues "implicitly" contain a quotation-functor. But notice that the quotation-functors do not occur in an "explicit" way: as in the case of the prologue-functor '*S*', the quotation-functors are amalgamated into the quantification-prologues. The formulas of Leśniewskian and substitutional quantification, which belong to the object language are not to be confused with corresponding formulas of the metalanguage where the quotation-expressions occur in an explicit way.[11]

Unlike Leśniewskian and substitutional quantification objectual quantification does not have to be read in an "implicitly" metalinguistic way. This is so because objectual quantification is the special case where the range of quantification and the domain of denotation are identical.[12] As a matter of fact the different kinds of quantification (objectual, substitutional, and Leśniewskian) differ not in their domain of denotation but in what is taken as values of the variables: objects, expressions, or extensions (i.e. extensional meanings). Actually one could adopt uniform readings for all three kinds of quantification. For the universal quantifier this reading would be:

(9) *For-any-value-the-inscriptions-equiform-with-the following-item--may-be-taken-to-have-*: *x*.

But with respect to objectual quantification this reading is unnecessarily complicated, because there (and only there) (9) is equivalent with:

(10) *For any object x*.

In objectual quantification it is not necessary to name (either explicitly or implicitly) the letter which functions as a variable in order to be able to express something concerning its range. In objectual quantification the values in the range of the variable can be talked about by simply using that variable, i.e. by making use of its denoting function.

But it is only in the case of quantification with respect to non-empty individual names that this simplified reading of objectual quantification goes without problems. Notice that an objectual reading of the above mentioned formula of propositional quantification (5) cannot simply say:

(11) *For any proposition p, p or not p.*

If (11) is really objectual, and not merely an elliptical version of (6) which is Leśniewskian, then (11) is not well-formed, because the letter '*p*' is first used as a name-variable (a variable for names of propositions) and afterwards as a sentence-variable.[13] A correct objectual reading of (5) must therefore say:

(12) *For any proposition p, either p or the negation of p is true.*

And this reading is in a certain sense a metalinguistic statement: it contains a semantical predicate such as '*is true*' and its domain of denotation is made up of propositions, i.e. of certain linguistic entities. But it is not metalinguistic in the sense that it would contain names of expressions. As a matter of fact it is rather like a statement of set theory (see below section 4) in that it reifies meanings into abstract objects.

Leśniewskian and substitutional quantification have the further advantage that they make quantification into quotation contexts possible. For instance

(13) (p)(*'p' is true* $\equiv p$)

cannot be given an objectual reading; because in objectual quantification all the bound variables are variables for names, and in (13) "*p*" must be the quotation of a sentence and not of a name. But (13) can be read in Leśniewskian fashion:[14]

(14) *Whatever-propositional-extension-the-inscriptions-equiform-with--the-following-item-are-taken-to-have-: p*
the-following-is-herewith-asserted-: 'p' is true if and only if p.

or substitutionally:[15]

(15) *Whatever-propositional-expressions-are-taken-to-be-substituted-for--the-inscriptions-equiform-with-the-following-item-: p*
the-following-is-herewith-asserted-: 'p' is true if and only if p.

Of course, precautions will have to be taken to prevent the occurrence of semantical paradoxes, but this is nothing unusual (cf. Harman [12], Davis [5], Belnap and Grover [1], Kripke [15] pp. 367–368, 417).

It might be objected that in (14) and (15) it is said that the letter '*p*' is true, and that this is nonsense. But this objection overlooks the fact that Leśniewskian and substitutional quantification let the variables play specific roles. Our readings show that in (14) the letter '*p*' is a sentence (though the quantifier leaves it up to the reader to choose which meaning it should have); and a sentence can meaningfully be said to be true. In (15) the letter '*p*' is only a placeholder and the quantifier leaves it to the reader to substitute a sentence for it; thus in this case the quotes do not operate on the letter '*p*' (to form a name of this letter), but they operate on whatever sentence the reader considers substituting for this letter, i.e. they form a name of this sentence, so that there is no problem.[16]

3. Syntactical, semantical and ontological categories

What variables somebody uses indicates what categories he distinguishes: different styles of variables correspond to different categories. Sofar this correspondence between variables and categories has especially been noticed in its application to the different styles of variables "within" one given kind of quantification. Thus somebody using objectual quantification with respect to individual-variables, set-theoretical variables and propositional variables is said to sort all objects into individuals, sets, and propositions.

It is illuminating, however, to notice that this principle also applies with respect to the differences that exist "between" the three kinds of quantification. Substitutional, Leśniewskian and objectual quantification are then seen to be expressions of different kinds of categorial systems, namely of syntactical, semantical and ontological systems of categories respectively.[17] There is thus a new angle from which the classical questions concerning the "modi significandi" and the "modi essendi" might be studied.

It is well-known that Leśniewski always insisted on the semantical nature of his categories. This distinguishes him from the logical atomists for whom the categories where primarily ontological in nature and from the neopositivists of the 30ies for whom the categories where merely syntactical. That Leśniewski's categories are not syntactical appears clearly in the fact that according to Leśniewski's precise and exhaustive *Terminological Explanations* (cf. [29] and [39]) there are expressions which belong to no category. Such expressions are e.g. the quantifiers.[18] From a syntactical point of view there is no reason why the quantifiers should not belong to a category, for instance to the category of operators (cf. for instance [3] p. 41f), and substitutional quantification with respect to quantifiers would be quite meaningful, though up to now nobody seems to have found it useful to introduce this.

From the semantical point of view of Leśniewski, however, it is primarily the constants in their semantical relation to the objects talked about that are sorted into categories. We thus obtain the semantical categories of names, sentences, and of all kinds of functors (even functors such as *All... are...*). But a Leśniewskian quantifier, especially because it is not an objectual quantifier, is not a constant which has a direct semantical relation to the object talked about. Its function is not to talk about objects: as we have seen, its reading is in a sense metalinguistic. As a matter of fact it mainly serves to signal that in what follows it, certain symbols do not have to be taken as fixed constants but may be taken as variables. Thus the universal quantifier (which is the only quantifier Leśniewski really deemed necessary) signals that the symbols which it binds can be taken to have whatever extensional meaning we like (as long as it is an extensional meaning chosen within the appropriate semantical category).

4. Why Leśniewski's "ontology" is not set theory

Not only sentences can be used "playfully" (i.e. can be used in the way in which the expressions which occur between quotation marks are used), but this usage can apply to any category of expressions whatsoever. Understanding this helps us to understand the essential difference that exists between Leśniewski's "ontology" and set theory. It provides us with a general procedure for reading all the functors of Leśniewski's "ontology" in a way which is specifically Leśniewskian, but which has at the same time the merit that it "implicitly" contains the set-theoretical interpretation of those functors.

For instance

(16) $a \circ b$

can be read

(17) *The-following-two-items-have-identically-the-same--extension-*: *a*, *b*.

In (17) the letters '*a*' and '*b*' are simply general names, they are not individual names of a special kind of abstract objects, such as sets. They are not taken to "name" an extension, they merely are taken to have an extension. Leśniewski's "ontology" is a nominalistic theory in the sense that it avoids reification of extensions into objects (cf. [18]).

The quantified formulas of "ontology", where quantification even with respect to empty names is allowed, present no problem either. For instance

(18) $(\exists a)\big(\sim ex(a)\big)$

can be read

(19) *For-some-extension-which-the-inscriptions-equiform-with--the-following-item-may-be-taken-to-have-*: *a*
the-following-is-herewith-asserted-: $\sim ex(a)$.

(18) is a logical truth, because if '*a*' is taken to have the null extension (something which is logically possible) then '*a does not exist*' (resp. '*a do not exist*') can truthfully be asserted.

5. Understanding "from without" and understanding "from within".

In this paper I have mainly been concerned to give "readings" of logical formulas, that is, I have translated them into a curious kind of "logicians' English"[19]. Some reflections on the role of such readings or paraphrases seem therefore in order.

What have we actually gained with these readings? In dealing with a logical language is it not enough (a) to know the syntactical rules which determine when a formula is well-formed, and (b) to know the truth conditions of all well-formed formulas in terms of a model-theoretical semantics?

I want to maintain that a formulation of the rules and truth conditions is certainly something a logician should aim at, but that beside this understanding of a logical formula "from without" there is also something which might be called the understanding of a logical formula "from within", and that it is with respect to this understanding "from within" that the readings play an important role. As long as this intuitive understanding "from within" is not present, there seems to be something like a gap between our thinking and the formula. Our thoughts "revolve around" the formula, we analyze it, operate with it, but our thoughts are not totally *one* with it. Of course there is nothing morally wrong with such an understanding "from without", and thus making a formula into an *object* of our thoughts has even its own, irreplaceable advantages. It makes us aware that every language is a calculus and it helps us keeping its mechanism in order. But a language is not only a calculus, it is a calculus which expresses a grammar of human thought; and that this is the case we can check only "from within", by experiencing that the formula expresses an unfolding thought.

Knowing the rules of a calculus and the method of its application enables us to calculate meaningfully with it. But a calculus, even a meaningful one, is not yet ipso facto a language. For instance we can all verify that the following addition is correct:

$$\begin{array}{r} 14 \\ 9 \\ \underline{23} \\ 46 \end{array}$$

But what is written down in this case is not a sentence. In dealing with this addition I may have many different propositional thoughts, or even no articulated propositional thought at all. Dealing with this addition is therefore not the same as reading the sentence "Counting 14 units, and then 9 more units, and then 23 more units, gives as a result 46 units." In the latter case I am thinking through one specific propositional thought; if I am reading carefully my thinking espouses exactly the development prescribed by the sentence. There is thus a difference between calculating meaningfully with certain symbols and thinking in the sentences of a language.

Of course, unlike the just mentioned addition, logical formulas are indeed sentences, they are in fact expressions with a full logical articulation. But the question is how we come to internalize this logical articulation, how we learn to think "intuitively" in terms of this articulation.

Thinking in a language is a feat which we have mastered for the first time by learning our mother tongue. Furthermore many of us know from experience how by practicing a foreign language we arrive at the point where we can think in that language. But can we learn a logical language simply by "practicing" it? This question arises because a logician's "practice" with respect to a logical language is not quite the same as the practice in everyday life with respect to an ordinary language.

A logicians practice consists first of all in deriving theorems, and this is rather like the mathematician's practice of calculating with symbols.

But a logician also studies and intuitively understands the semantical rules of the logical language in question (he possesses already an intuitive understanding of a metalanguage), and this may indeed bring about an intuitive understanding of the formulas "from within". I want to insist, however, that the semantical rules do not "express" this intuitive understanding. They merely express the rules and truth conditions of the way of thinking and speaking in question, i.e. they give a description of this language "from without".

The only possible way of "expressing" the intuitive thinking-through of a logical formula, other than by means of the formula itself, is with the help of a paraphrase or translation, with the help of a reading. And therefore such readings play a specific role in the teaching and in the control of the accuracy of our understanding "from within" of a logical formula. As long as we cannot give an exact reading of the formula our understanding is not complete. Under normal circumstances paraphrasing and translating plays an important role even in the learning of ordinary languages, but there it is not absolutely necessary, because there the accuracy of the intuitive understanding is effectively produced by the practice of everyday life.

One might object that translating is a very deficient way of checking accuracy, since no translation into another language can ever be exact. But this objection overlooks the fact that our readings of logical formulas are not translations into ordinary language. As a matter of fact they belong to what I have called "logicians' English", and "logicians' English" deviates from ordinary English precisely because it wants to be a faithful wording of the meaning of the logical formulas in question.

Footnotes

[1] Cf. [37] p. 99, [36] pp. 63, 104, 106. (Earlier, in [35], Quine had thought that Leśniewski's "ontology" was a kind of set theory). In [36] p. 106 Quine claims that Leśniewski mentioned to him in conversation in 1933 that substitutional quantification made good sense no matter what substitution class we take — even that whose sole member is the left-hand parenthesis. It is, however, a fact that in Leśniewski's system parentheses do not belong to a semantical category, and that therefore quantification over left-hand parentheses is not allowed. (See below footnote 17.) Thus the alledged remark proves, if anything, that Leśniewskian quantification is not substitutional. Leśniewski may actually have made this remark in order to give a kind of reductio ad absurdum of substitutional quantification. Or he may have been talking about his metalogical *Terminological Explanations,* where left-hand parentheses belong to the domain of individuals.

[2] Kielkopf [14], who has read [16], agrees that Leśniewskian quantification is neither substitutional nor objectual-referential, but he still calls Leśniewskian quanti-

fication "referential", i.e. referential not with respect to the domain of existing objects, but referential with respect to a "realm" of "mind dependent entities", for instance sets. I think that this terminology is confusing, because, as we shall see, Leśniewskian quantification can only be understood if one realises that these "sets" are *not referred to* by the names of the system, but that they are merely extensions which these names *have*. Kielkopf's terminology is probably due to the fact that the terms 'non-substitutional quantifier', 'domain-and-values-quantifier' and 'referential quantifier' have been used synonymously, see footnote 12 below.

[3] For Quine's objections against substitutional quantification see [38] p. 273, [37] p. 140, [36] pp. 64, 95. For a precise discussion of the case of nameless objects cf. Dunn and Belnap [6] and Weston [43]. Data concerning the history of the discussion of substitutional quantification can be found in Marcus [34] pp. 46–47, 50.

Ruth Barcan Marcus (cf. [33] pp. 244–245) objects against objectual quantification that it mixes logic and ontology, that the logical use of quantification should be ontologically neutral; because in the ordinary and philosophical discourse which we want to paraphrase with the help of our formulas, the ontological status of the objects is often not settled. Lejewski [24] voices a similar opinion when he insists that logic should be ontologically neutral so that the opponents in an ontological discussion can carry on their dispute in a common logical system.

[4] It would be false to say that every argument of a prologue-functor is a "play". In (3), for instance, the name 'Galileo' is an argument of the prologue-functor '*S*', but it is not a "play" because it does not depend on the subordinating expression '*that which follows*:'.

[5] To be fully explicit we would have had to replace the expression '*that*' not merely by the phrase '*that which follows*:' but by the even longer phrase '*something of which that which follows is a samesaying*:'. Wilfrid Sellars (cf. [40], [41]) has strongly insisted on the importance of the notion of "samesaying" or "playing-the-same-linguistic-role", and he has introduced the use of dot-quote-expressions as general names whose extension consists of all utterance-tokens (from all possible languages) which play the same role as that normally played by expression-tokens equiform to the expression-token occurring between the dots. Therefore his explication of our example would presumably read:

Galileo uttered something which is ·a the earth moves·. (For a clear account of Sellars' views on this and related topics cf. Loux [28].)

Compare also the semantical analysis of *oratio obliqua* and *oratio recta* given by W. Marciszewski (cf. [30], [31] chapter 10), especially his remark in [31] p. 153 according to which the Polish word for '*that*' is an "indicator of an accomplished reproduction", i.e. (in my terminology) of a "play". The author reminds us, however, that in oratio oblique the personal pronouns and other indexicals normally do not occur in their original form; e.g. what another person has said in the first person is being reproduced in the third person. But I will not discuss here these special transformations of oratio obliqua.

[6] Prof. Rolf Eberle (Rochester) has drawn my attention to this fact.

[7] Notice that there can be many different kinds of quotation-functors, differing according to the kind of equivalence relation between sayings one prefers, and/or differing according to whether the quotation-functor is forming a general name of concrete objects or an individual name of an abstract object. Cf. [17].

[8] Notice that I do not write '*Galileo said 'The earth moves*'' since in spoken language one does not "hear" quotation-marks. But as a matter of fact the sentence may be said to contain "tacit" quotation-marks, i.e. a "tacit" quotation-functor. However, this tacit quotation-functor does not have to be thougth of as being "incorporated" in the phrase '*the earth moves*', but it can very well be thought of as being incorporated in '*said*'. If the tacit quotation-functor were incorporated in the phrase '*the earth*

moves', then this phrase would no longer be a sentence, but the name of a sentence, and the form of (4) would be $R(x, y)$. – Marciszewski [31] argues that quotation-marks are not always name-forming functors, that they can also be mere punctuation signs which signal that a certain expression-token is a reproduction, but which don't turn that expression-token into a name of the expression which it is reproducing. Marciszewski then goes on to interpret the Polish word '*że*' ('that') in the same way as a mere punctuation sign (the difference being merely that quotation-marks signal oratio recta whereas the word '*że*' signals oratio obliqua). This is all the more convincing as the Polish word '*że*', unlike the English word '*that*', never occurs as demonstrative pronoun. Does the Polish two-word expression '*to że*' (for 'that') prove that the name-forming demonstrative function and the reproduction-signal can actually be dissociated? This would amount to distinguishing in (3) between '*that which follows*' on the one hand and the colon (as a sort of non-name-forming quotation-mark) on the other hand.

In a transformational grammar, should '*S*' be taken to be more basic then '*R*'? As far as definability is concerned, '*S*' and '*R*' seem to be on the same footing; '*S*' can be defined in terms of '*R*' and '*Q*':

$$S(x, p) =_{\mathrm{df}} R(x, Q(p)),$$

and '*R*' can be defined in terms of '*S*' and '*Q*':

$$R(x, y) =_{\mathrm{df}} (\exists p)(S(x, p) . y = Q(p)).$$

[9] Cf. Küng and Canty [16] and Küng [17]; see also section 4 below. – In the early paper [25], where the theorems are not yet given in symbolic notation, Leśniewski expressed the universal quantifier by saying "przy każdem znaczeniu wyrazu '*a*'" ("for every meaning of the expression '*a*'") and the particular quantifier by saying "przy pewnym znaczeniu wyrazu '*a*'" ("for some meaning of the expression '*a*'"); i.e. he explicitly referred to the meaning of the expression. In [26] he referred on p. 187 to this former usage. But at the same time he claimed the this usage was in agreement with the symbolic formulations of Peirce and Whitehead-Russell, apparently overlooking the fact that the quantification in Peirce and *Principia Mathematica* is objectual. And in [27] Leśniewski quoted on p. 12 a passage from Tarski [42], including the footnote 3 from p. 196 where Tarski gives the following explanation of the term 'quantifiers':

> Au sens de Peirce ("On the algebra of logic" *American Journal of Mathematics*, vol. VII, 1885, p. 197) qui appelle ainsi les symboles "Π" (quantificateur général) et "Σ" (quantificateur particulier) représentant les abréviations des expressions: "pour toute signification des termes..." et "pour quelque signification des termes...".

Again the formulation refers to meanings, but again – curiously enough – neither Tarski nor Leśniewski insisted on the difference between Leśniewskian quantification and objectual quantification. The passage in Peirce (*Collected Papers* 3.393) clearly refers to objectual quantification over individuals:

> ... in order to render the notation as iconical as possible we may use Σ for *some*, suggesting a sum, and Π for all, suggesting a product. Thus $\Sigma_i x_i$ means that x is true of some one of the individuals denoted by i or $\Sigma_i x_i = x_i + x_j + x_k +$ etc. In the same way, $\Pi_i x_i$ means that x is true of all these individuals, or $\Pi_i x_i = x_i x_j x_k$, etc.

Lejewski has repeatedly tried to convey to logicians which are not familiar with Leśniewski's system the meaning of Leśniewskian quantification (cf. [20], [21], [22], [24], see also Henry [13] pp. 25–32), but with limited success. The reason for this seems to me due to the fact that although he gave an excellent presentation of the rationale for "unrestricted" (i.e. Leśniewskian) quantification, he did not offer an intuitive reading. Henry [13] p. 28 explains the specificity of Leśniewskian quantification by saying:

> "Somehood" has been concentrated in the quantifier and *only* "somehood"; existence (as distinct from "somehood") can be separately and overtly expressed elsewhere in quantified sentences.

This helps only if this notion of "somehood" is further clarified in terms of extension and meaning.

[10] In my previous publications, [17] and [18], I had not yet realized this, and I had thought that substitutional quantification was "more" metalinguistic than Leśniewskian quantification.

[11] The *formulas* of Leśniewskian and substitutional quantification are therefore not metalinguistic; I say only of their *readings* that they are in a certain sense metalinguistic, namely in this peculiar "implicit" way. Using Wittgensteinian terminology one could say: what our readings "say" in hyphenated form, that the formulas themselves do not "say", they merely "show" it. Dunn and Belnap ([6] p. 184) are therefore right when they deny that the substitution interpretation makes quantification essentially a metalinguistic device. Substitutional quantification is not the same as objectual quantification over expressions. "The utility of the substitutional quantifier lies in the fact that while the referential quantifiers over terms take names of terms as substitutes, the substitutional quantifiers take the terms themselves, which can be denotationless or can denote other things" (Kripke [15] p. 353). Of course one can also have substitutional quantifiers in the metalanguage (cf. [6] p. 184, [1] p. 27, [15] p. 341), but that is another matter. All these affirmations about substitutional quantification hold, mutatis mutandis, also for Leśniewskian quantification.

[12] In Küng and Canty [16] it has been pointed out that Leśniewski's systems are characterized by the fact that the range of quantification is not identical with the universe of discourse, but that for instance the category of names has as its range of quantification the power set of the universe of discourse. Canty [2] has formulated this distinctive feature very neatly in the anti-Quinean slogen "it is false that to be is to be the value of a variable". – Dunn and Belnap [6] p. 184 deny that the variables in substitutional quantification take expressions "as values" because for them substituends are by definition not values. They distinguish therefore between "substitutional quantifiers" and "domain-and-values quantifiers". But this is largely a terminological matter. I prefer my way of speaking because for my purposes it is important to stress not only the differences, but also the common features of the different kinds of quantification. – According to Grover [10] p. 114 the "values" of a "domain-and-values interpretation" do not have to be objects which are named: she speaks of a "domain-and-values interpretation" even in the case of a propositional logic where the variables are explicitly not variables for names, "pro*nouns*", but "pro*sentences*". Such a "domain-and-values interpretation" is exactly a Leśniewskian interpretation.

[13] Dorothy L. Grover [10], who gives the most careful analysis of this problem that I have come across, is aware of the fact that there is something wrong with the word 'proposition' in sentences like (11). She suggests on p. 121 that what we need in this place is not a common noun such as 'proposition' but a "common sentence". Unfortunately there are no "common sentences" in English, and it is hard to see to what category such an expression should belong. Miss Grover has already invented "pro*sentences*" in analogy to pro*nouns*, but there the clues could be taken from the context. With "common sentences" the case is different, because if "common sentences" are sentences and not nouns, then they will no longer fit into the context '*For any...*'.

[14] Leśniewski himself did not consider quantification into quotation contexts. For an extension of Leśniewski's system, but with a special kind of quotation functor,

see Davis [5] and Küng [17]. Of course quotation contexts are very often part of opaque contexts, and it would seem that in order to handle such contexts in Leśniewskian fashion one would need a range of quantification consisting not of extensional but of intensional meanings.

[15] Dunn and Belnap [6] p. 185 and Belnap and Grover [1].

[16] Goddard and Routley [7] p. 35 think that quantification into quotation contexts presupposes a special kind of quotation functor. But I believe that this is a mistake which is due to the fact that they neglect to clarify first the meaning of their quantifiers.

[17] The word '*ontological*' is here used in its customary non-Leśniewskian sense.

[18] J. T. Canty has drawn my attention to this fact. The parentheses and the subquantifiers are other kinds of expressions which belong to no semantical category.

[19] Cf. the very pertinent explanations concerning "philosophers' English" in Grover [10].

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