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Short communication

Duality of Turbulence and Wave in Wind Waves*

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Abstract: The three-seconds power law for wind waves of simple spectra, already derived by TOBA (1972 and 1973), may also be derived by introducing surface-wave properties into the form of the rate of energy dissipation in the theory of turbulence. The universal constant B, which was formerly determined empirically as 0.062 is here obtained as $B=(2\pi)^{-3/2}=0.0635$. Thus wind waves have the duality of turbulence and wave.

Quite different from surface waves produced by a mechanical wave generator on a still water surface, the wind waves are special phenomena in which the skin flow by the shearing stress of wind, the turbulence and the wave motion may not be separated from one another. In the present short note, it is shown that the three-seconds power law, which was derived for wind waves of simple spectra (TOBA, 1972, 1973), may be derived also by the use of a well known form of the rate of energy dissipation in the theory of turbulence. It is interpreted as an evidence that wind waves have the duality of turbulence and wave.

The three-seconds power law is expressed by

$$H^*=BT^{*3/2} \qquad (1)$$

where $H^*=gH/u_*^2$, $T^*=gT/u_*$, and where H is the significant wave height, T the significant wave period, g the acceleration of gravity, u_* the friction velocity of air, and B a universal constant derived empirically as

$$B=0.062 \qquad (2)$$

and the law seems to hold universally.

In the theory of turbulence, it is postulated that energy flux from the largest turbulent eddies to the smallest ones, ε, is equal to the mean energy dissipation by viscosity in the smallest eddies, per unit time per unit mass of fluid, and it is expressed by Kolmogoroff and Obukhov's law as

$$\varepsilon \propto \frac{V^3}{\Lambda} \qquad (3)$$

where Λ represents the size of an eddy and V the variation of velocity over the distance of the order of magnitude of Λ (*e.g.* LANDAU and LIFSHITZ, 1954).

The absolute velocity of surface water particles in surface waves of deep water, u, is given, as well known, by the amplitude a and the angular frequency σ, by

$$u=\sigma a \qquad (4)$$

Now we regard wind waves as turbulence, and take $2a=H$ as Λ, then V or the variation in the velocity of the individual particles at the surface over the distance Λ becomes $2u$, thus V becomes $H\sigma$. This coincides with the form of $\sigma=V/\Lambda$ in the treatment of turbulence. Substituting these for (3), we obtain

$$\varepsilon \propto H^2\sigma^3 \qquad (5)$$

The value of $H^2\sigma^3$ should be determined by physical properties related with wind waves, that is, the rate of momentum transfer from the air to the water represented by u_*, and g since the gravitational force is the restoring force for surface waves. If one includes the capillary-gravity wave range, g may be replaced by $g_*=g(1+S\kappa^2/\rho_w g)$, where S is the surface tension, κ the wave number, and ρ_w the density

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of water. As the simplest form, we assume dimensionally

$$H^2\sigma^3 = u_* g \tag{6}$$

Transforming this to a dimensionless form, we obtain

$$H^{*2}\sigma^{*3} = 1 \tag{7}$$

where $\sigma^* = u_*\sigma/g$. Substituting $\sigma = 2\pi/T$ for (7), immediately follows

$$H^* = \left(\frac{T^*}{2\pi}\right)^{3/2} \tag{8}$$

Comparing (8) with (1),

$$B = (2\pi)^{-3/2} = 0.0635 \tag{9}$$

and we see that it is substantially equivalent to the value given by (2) which was determined empirically by TOBA (1972).

The H^* in (7) represents the "dimensionless diameter of revolution", and σ^{*-1} the "dimensionless time of revolution", of surface water particles normalized by u_* and g. It is noteworthy that the right hand side of (7) is substantially equal to unity, or that the squared dimensionless diameter of revolution is equal to the cubed dimensionless time of revolution, which has been substantiated by coincidence of the B 's given by (2) and (9).

Using the form (8), Lemmas I through III given by TOBA (1972, 1973) may be expressed as follows:

Lemma I. $$u_0 = (1/8)u_* \tag{10}$$

Here u_0 is the mass transport velocity, at the sea surface, of the significant waves substituted by Stokes waves.

Lemma II. $$\delta = \frac{H}{L} = (2\pi T^*)^{-1/2} \tag{11}$$

Here δ is the steepness, L the significant wave length, and a condition for surface waves of deep water $\sigma^2 = g\kappa$ has been used in the derivation.

Lemma III. $$\frac{u_* L}{\nu} = (2\pi T^*)^{1/2}\frac{u_* H}{\nu} \tag{12}$$

TOBA (1973a) showed, by the combination of the three-seconds power law and the similarity in the spectral form of wind waves, that wind waves grow in such a way that the peak of the spectra moves along the σ^{-4}-line, and suggested the existence of a kind of general similarity in the wind wave field itself, by the fact that the gross form of the spectra is also along the σ^{-4}-line. The present note is to give support to this suggestion of similarity. If (6) holds universally for any value of σ in the gravity wave range, the ideal form of the energy spectrum density ϕ is expressed by

$$c\int_{\sigma}^{\infty}\phi d\sigma = \frac{a^2}{2} = \frac{H^2}{8} = \frac{u_* g}{8\sigma^3} \tag{13}$$

so that

$$\phi = \alpha u_* g\sigma^{-4},\quad \alpha = 3/8c = 0.375/c \tag{14}$$

where c is a correction factor, arising from the part of the spectrum lower than σ, and for some other reasons. The values of the coefficient α for the gross form of the spectra, and for the peak point, proposed empirically by TOBA (1973a), were 0.020 and 0.13, respectively.

At any rate, in the "face of turbulence", the energy flux ε is constant for a given u_*. In the course of the growth of wind waves, however, an excess energy should be supplied in order to newly organize larger maximum eddies, and its mechanism should be sought for in the other "face of waves".

References

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