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Physics with Nonperturbative Quantum Gravity: Radiation from a Quantum Black Hole†

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We study quantum gravitational effects on black hole radiation, using loop quantum gravity. Bekenstein and Mukhanov have recently considered the modifications caused by quantum gravity on Hawking's thermal black-hole radiation. Using a simple ansatz for the eigenstates of the area, they have obtained the intriguing result that the quantum properties of geometry affect the radiation considerably, yielding a discrete spectrum, definitely non-thermal. Here, we replace the simple ansatz employed by Bekenstein and Mukhanov with the actual eigenstates of the area computed using loop quantum gravity. We derive the emission spectra, using a classic result in number theory by Hardy and Ramanujan. Disappointingly, we do not recover the Bekenstein–Mukhanov discrete spectrum, but — effectively — a continuum spectrum, consistent with Hawking's result. The Bekenstein–Mukhanov argument for the discreteness of the specrum is therefore likely to be an artifact of the ansatz, rather than a robust result (at least in its present kinematical version). The result is an example of concrete (although somewhat disappointing) application of nonperturbative quantum gravity.

KEY WORDS : Area eigenstates

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Quantum gravity research has traditionally suffered from a great scarcity of physical applications where theories and ideas could be tested, at least in principle [1]. One of the few areas in which ideas on quantum gravity may be tested is black hole physics [2]. The loop approach to quantum gravity [3] is now sufficiently developed that we may begin to probe it within "physical" applications. It is thus natural to investigate what loop quantum gravity asserts about black hole physics.

Recently, Bekenstein and Mukhanov [5] have suggested that the thermal nature of Hawking's radiation may be affected by quantum properties of gravity (For a review of earlier suggestions in this direction, see Ref. 6). As is well known, Hawking derived the black hole thermal emission spectrum from quantum field theory in curved spacetime, therefore within the approximation in which the quantum properties of gravity are neglected. Attempts have been made to relate Hawking's temperature to gravitational dynamics, but the problem of how quantum gravity affects black hole emission can be convincingly addressed only within a full theory of the quantum gravitational field. Bekenstein and Mukhanov observe that in most approaches to quantum gravity the area can take only quantized values [7]. Since the area of the black hole surface is connected to the black hole mass, black hole mass is likely to be quantized as well. The mass of the black hole decreases when radiation is emitted. Therefore emission happens when the black hole makes a quantum leap from one quantized value of the mass (energy) to a lower quantized value, very much as atoms do. A consequence of this picture is that radiation is emitted at quantized frequencies, corresponding to the differences between energy levels. Thus, quantum gravity implies a discretized emission spectrum for the black hole radiation.

By itself, this result is not physically in contradiction with Hawking's prediction of a continuous thermal spectrum. To understand this, consider the black body radiation of a gas in a cavity, at high temperature. This radiation has a thermal Planckian emission spectrum, essentially continuous. However, radiation is emitted by elementary quantum emission processes yielding a discrete spectrum. The solution of the apparent contradiction is that the spectral lines are so dense in the range of frequencies of interest, that they give rise — effectively — to a continuous spectrum. Does the same happen for a black hole?

In order to answer this question, we need to know the energy spectrum of the black hole, which is to say, the spectrum of the area. Bekenstein and Mukhanov pick up a simple ansatz: they assume that the area is quantized in multiple integers of an elementary area A_0. Namely, that the area can take the values

$$A_n = nA_0\,, \tag{1}$$

where n is a positive integer, and A_0 is an elementary area of the order of the Planck area

$$A_0 = \alpha\hbar G, \tag{2}$$

where α is a number of the order of unity (G is Newton's constant and $c = 1$). Ansatz (1) is reasonable; it agrees, for instance, with the partial results on eigenvalues of the area in the loop representation given in [8], and with the idea of a quantum picture of a geometry made by elementary "quanta of area". Since the black hole mass is related to the area by

$$A = 16\pi G^2 M^2, \tag{3}$$

it follows from this relation and the ansatz (1) that the energy spectrum of the black hole is given by

$$M_n = \sqrt{\frac{n\alpha\hbar}{16\pi G}}\,. \tag{4}$$

Consider an emission process in which the emitted energy is much smaller than the mass M of the black hole. From (4), the spacing between the energy levels is

$$\Delta M = \frac{\alpha\hbar}{32\pi GM}\,. \tag{5}$$

From the quantum mechanical relation $E = \hbar\omega$ we conclude that energy is emitted in frequencies that are integer multiple of the fundamental emission frequency

$$\bar{\omega} = \frac{\alpha}{32\pi GM}\,. \tag{6}$$

This is the fundamental emission frequency of Bekenstein and Mukhanov [5] (they assume $\alpha = 4\ln 2$). Bekenstein and Mukhanov proceed in [5] by showing that the emission amplitude remains the same as the one in Hawking's thermal spectrum, so that the full emission spectrum is given by spectral lines at frequencies multiple of $\bar{\omega}$, whose envelope is Hawking's thermal spectrum.

As emphasized by Smolin in [6], however, the Bekenstein–Mukhanov spectrum is drastically different than the Hawking spectrum. Indeed, Hawking temperature is

$$T_{\mathrm{H}} = \frac{\hbar}{8\pi kGM} \tag{7}$$

(k is the Boltzmann constant); therefore the maximum of the Planckian emission spectrum of Hawking's thermal radiation is at

$$\omega_{\rm H} \sim \frac{2.82kT_{\rm H}}{\hbar} = \frac{2.82}{8\pi GM} = \frac{2.82 \cdot 4}{\alpha}\,\bar{\omega} \approx \bar{\omega}. \tag{8}$$

That is, the fundamental emission frequency $\bar{\omega}$ is of the same order as the maximum of the Planck distribution of the emitted radiation. It follows that there are only a few spectral lines in the regions where emission is appreciable. Therefore the Bekenstein–Mukhanov spectrum is drastically different from the Hawking spectrum: the two have the same envelope, but while Hawking spectrum is continuous, the Bekenstein–Mukhanov spectrum is formed by just a few lines in the interval of frequencies where emission is appreciable. Notice that such a discretization of the emission spectrum is derived by Bekenstein and Mukhanov on purely kinematical grounds, that is using only the (assumed) spectral properties of the area. To emphasize this fact, we will denote it as the kinematical Bekenstein–Mukhanov effect.

This result is of great interest because, in spite of its weakness, black hole radiation is still much closer to the possibility of (indirect) investigation than any quantum gravitational effect of which we can think. Thus, a clear quantum gravitational signature on the Hawking spectrum is a very interesting effect. Is this Bekenstein–Mukhanov effect credible?

One of the most definite results of loop quantum gravity is a calculation of the spectrum of the area from first principles [9]. Thus, following a suggestion in [6], we may use loop quantum gravity to check the Bekenstein–Mukhanov result, by replacing the naive ansatz (1) with the precise spectrum computed in this approach to quantum gravity.

Consider a surface Σ — in the present case, the event horizon of the black hole. According to loop quantum gravity, the area of Σ can take only a set of quantized values. These quantized values are labeled by unordered n-tuplets of positive integers $\vec{p} = (p_1, ..., p_n)$ of arbitrary length n. The spectrum is then given by

$$A_{\vec{p}} = 16\pi\hbar G \sum_{i=1,n} \sqrt{\frac{p_i}{2}\left(\frac{p_i}{2}+1\right)}, \tag{9}$$

For a full derivation of this spectrum, see Ref. 9. The spectrum (9) is not complete. There is an additional sector corresponding to a class of "degenerate" states [10]. These degenerate states play no role in the present discussion, however.

If we disregard for a moment the term +1 under the square root in (9), we obtain immediately the ansatz (1), and thus the Bekenstein–Mukhanov result. However, the +1 is there. Let us study the consequences of its presence. First, let us estimate the number of area eigenvalues between the value $A \ggg l_0$ and the value $A + dA$ of the area, where we take dA much smaller than A but still much larger than l_0. Since the +1 in (9) affects in a considerable way only the terms with low p_i, we can neglect it for a rough estimate. Thus, we must estimate the number of unordered strings of integers $\vec{p} = (p_1, ..., p_n)$ such that

$$\sum_{i=1,n} p_i = \frac{A}{8\pi\hbar G} \gg 1. \tag{10}$$

This is a well known problem in number theory, called the partition problem. It is the problem of computing the number N of ways in which an integer I can be written as a sum of other integers. The solution for large I is a classic result by Hardy and Ramanujan [14]. According to the Hardy–Ramanujan formula, N grows as the exponent of the square root of I. More precisely, we have for large I that

$$N(I) \sim \frac{1}{4\sqrt{3}\,I}\, e^{\pi\sqrt{(2/3)I}}. \tag{11}$$

Applying this result in our case we have that the number of eigenvalues between A and $A + dA$ is

$$\rho(A) \approx e^{\sqrt{\pi A/12\hbar G}}. \tag{12}$$

Now, because of the presence of the +1 term, eigenvalues will overlap only accidentally: generically all eigenvalues will be distinct. Therefore, the average spacing between eigenvalues decreases exponentially with the inverse of the square of the area. This result is to be contrasted with the fact that this spacing is constant and of the order of the Planck area in the case of the naive ansatz (1). This conclusion is devastating for the Bekenstein–Mukhanov argument. Indeed, the density of the energy levels becomes

$$\rho(M) \approx e^{\sqrt{4\pi G/3\hbar}\, M}, \tag{13}$$

and therefore the spacing of the energy levels decreases *exponentially* with M. It follows that for a macroscopical black hole the spacing between energy levels is infinitesimal, and thus the spectral lines are virtually dense in frequency. We effectively recover in this way Hawking's thermal spectrum (except, of course, in the case of a Planck scale black hole). The conclusion is that the Bekenstein–Mukhanov effect disappears if we replace the naive ansatz (1) with the spectrum (9) computed from loop quantum gravity. More generally, we have shown that the kinematical Bekenstein–Mukhanov effect is strongly dependent on the peculiar form of the naive ansatz (1), and it is not robust. In a sense, this is a pity, because we loose a possible window on quantum geometry.

Mukhanov and, independently, Smolin have noticed that the possibility is still open for the existence of a "dynamical" Bekenstein–Mukhanov effect [12]. For instance, transitions in which a single Planck unit of area is lost could be strongly favored by the dynamics. To explore if this is the case, one should make use of the full machinery of quantum gravity, for instance by computing transition probabilities between horizon's area eigenstates induced in a first order perturbation expansion by the coupling between the area of the horizon and a surrounding radiation field. This could perhaps be done following the lines of Ref. 13.

We have argued that the "kinematical" discretization of the black hole emission spectrum suggested by Bekenstein and Mukhanov disappears if we use quantitative result from loop quantum gravity. Our result indicates that loop quantum gravity is sufficiently mature to begin addressing concrete physical problems.

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