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LETTER TO THE EDITOR

Perspectives on the Energy of the Universe

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Recent progress in computing the energy of the universe including the gravitational contribution is discussed. Various issues are raised including symmetries, energy localization and observational verification.

For many years, when one discussed the energy density or the energy of the universe, the focus was on the "matter" (T_{00}) part. This was understandable because the primary issue regarding the closure of the universe rested upon the value of the matter density. However, we know that gravitation plays a role in the energy of a physical system. This is most clearly revealed in the mass defect when one calculates the energy of an isolated spherically symmetric body. Recently [1,2], we showed that the consideration of the gravitational contribution resolves the problems connected with the equality of inertial and gravitational mass. Thus, the question arises of what role gravitational energy plays in the total energy density and the total energy of the universe.

An immediate motivation for determining the gravitational contribution to the energy of the universe arose from the Albrow [3] and Tryon

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[4] proposal that the universe may have arisen as a quantum fluctuation of the vacuum. Noting that such a universe must have a zero net value for all conserved quantities, Tryon indicated how this might arise using a Newtonian order-of-magnitude estimate. He also referred to a topological argument attributed to Peter Bergmann that a closed universe must have zero total energy. Since the advent of inflationary cosmological theories, some authors, particularly Vilenkin [5], have developed the vacuum fluctuation idea further. Thus, it is now appropriate that the issue of the total energy of the universe, within the context of general relativity, be examined carefully.

To this end, we recently considered [6] the energy-momentum conservation laws as applied to an FRW universe. We found that the covariant conservation laws

$$T^{ik}{}_{;k} = 0 \tag{1}$$

($i, k = 0, 1, 2, 3$ and a semi-colon denotes a covariant derivative) can be expressed in the form of an ordinary divergence

$$\left[\sqrt{-g}\left(T^0_0 - \frac{3}{8\pi}\left[\frac{\dot{a}^2}{a^4} + \frac{k}{a^2}\right]\right)\right]_{,0} = 0 \tag{2}$$

(a comma denotes a partial derivative) using the conformal FRW metric

$$ds^2 = a(t)^2\left[dt^2 - \frac{dr^2}{1 - kr^2} - r^2 d\Omega^2\right]. \tag{3}$$

The traditional manner in which physicists have identified a total density including the contribution from gravity has been that of re-casting (1) into the form of an ordinary vanishing divergence such as we found in (2). However, this generally entailed the introduction of a pseudotensor t_{ik}. For example, in the Einstein form, (1) is changed to

$$[\sqrt{-g}\,[T^k_i + t^k_i]]_{,k} = 0 \tag{4}$$

and from this, one deduces both a density and a Poynting vector as in electromagnetism. However, there is an important distinction between (4) and the procedure in electromagnetism because in the latter, there is no pseudotensor and all the elements are tensorial. In general relativity, the pseudotensor can be changed at will and hence there is an important ambiguity introduced regarding the value of the density. In fact, the question of the meaningfulness of energy localization in general relativity is raised. These issues were first discussed during the early years after the development of general relativity and the debate continued for decades. While local meaning to pseudotensorial quantities are clearly questionable, global measures in the case of isolated systems admitting asymptotically Minkowskian metrics have been shown to yield consistent results under transformations which retain the required conditions. The calculations in [2] were of such form.

The significant aspect of (2) was that it did not conform to any pseudotensorial form. In fact, from (3) and the expression for the Einstein tensor, we found that (2) could be expressed as

$$\left[\sqrt{-g}\left(T_0^0 - \frac{1}{k}\,G_0^0\right)\right]_{,0} = 0 \tag{5}$$

and hence in the absence of a cosmological constant, we [6] identified the total density as zero. Apart from the benefit of determining the actual value, of added interest is that since the elements going into the make-up of the total density are components of tensors, the problems associated with pseudotensors are absent.

At first glance, this result might appear trivial. The vanishing of this density could have been seen from the outset by starting with the Einstein field equations and taking the terms to one side. A divergence of the resulting vanishing quantity remains zero and one might even claim that a conservation law has thus emerged. Indeed, Lorentz [7] and Levi-Civita [8] followed such a procedure, but this was justifiably criticized by Einstein (see Pauli, Ref. 9). However, this was not the procedure which we followed. We used the actual conservation laws and found that when applied to FRW spacetimes, the combined density which could be identified was

$$T_{00} - \frac{1}{k}\,G_{00}.$$

This appears to be a special feature of such spacetimes.

Following our work, Rosen [10] used the Einstein pseudotensor with cartesian coordinates to calculate the total energy of a closed FRW universe. Interestingly, he found that the total energy was again zero. However, a cautionary note must be added here. While there are arguments available to support global pseudotensorial calculations for isolated systems with asymptotically Minkowskian metrics, in the present case we are dealing with FRW cosmologies for which there are no boundaries at all. Moreover, while (5) yields both a local density and a global energy which respect the symmetries of the FRW spacetimes, the pseudotensor calculation not only fails to provide a density consistent with the symmetries but also yields non-vanishing pseudo-energy flux densities.

Similar problems arise in the case of cylindrical gravitational waves [11]. While calculations in cartesian coordinates might appear to be most reliable, the system is not isolated and the metric in [11] is singular along the symmetry axis. Thus, a global asymptotically Minkowskian metric cannot be found for such a system.

The conventional wisdom has been that gravitational energy cannot be localized in principle. However, through the years the conventional wisdom has been challenged in various papers in various ways. Recently [12,13], we took the following approach to the problem: The deduction of total energy including the gravitational contribution stems from the conservation laws (1). These laws have content only in the presence of matter, T_{ik}. In vacuum, (1) degenerates to the empty identity $0 = 0$. However, what researchers have done with (1) through the decades has been to recast (1) into (4) by introducing a pseudotensor and then extracting content from the subsequent integral form of (4) in vacuum. For example, energy supposedly lost by an otherwise insular system through gravitational waves are calculated from the integral form of (4) in the asymptotic vacuum. However, the origin of this purported information stems from (1) which is in fact devoid of any content in vacuum.

As a result, we have introduced the hypothesis that energy, including the gravitational contribution, is most logically localized in regions of non-vanishing T_{ik} and the proper determination of the energy localization is deduced in systems of reference in which the pseudotensor vanishes. The hypothesis builds upon the fact that the pseudotensor is such a problematic entity, which suggests that preferred physical systems for energy localization are those in which it is eradicated. It is eradicated in all Kerr–Schild metrics, for example. If the hypothesis is correct, energy including the gravitational contribution is localized in the structural form $(-g)T_{00}$, and so acquires a tensorial aspect. An interesting question to be answered is the extent and nature of spacetimes in which the pseudotensor can be eliminated. In the present case, we have found that the conservation laws appear to point to an unambiguous vanishing of FRW total energy. This raises the possibility that the FRW cosmologies are degenerate cases of the localization hypothesis, as a density which must both vanish and also be of the form $(-g)T_{00}$ can only be realized in a singular form with $g = 0$ within the context of the hypothesis.

These are issues which must be explored further. Also to be considered is the concurrence of the Rosen energy calculation result with that of our own. Perhaps reasons can be produced which support such pseudotensor calculations in spite of the objections which have been raised against them.

Finally, we consider the issue of observational verification of the calculated energy. For isolated systems, the matter is straightforward: From the asymptotic metric, one can read the value of the total mass (and hence energy) from the coefficient of $1/r$ in g_{00}. The tracking of test bodies moving according to the geodesics of this metric provide the observational link. However, for the cosmological problem, while the motion of bodies within the system are easily deduced, there is no asymptotic metric and hence no $1/r$ coefficient to examine and relate to the observed motions. It would be very useful if a link between observations and the total energy density of an FRW cosmology could be devised. At this point, we have the deduced value of the density as zero from the conservation laws but a confirmation is called for. While it might require considerable effort and/or ingenuity to discover the means by which such a confirmation may be achieved, it is a worthy and meaningful goal. It is important as a concept in its own right and because of the implications for cosmological models. That it is meaningful is inherent in the theory itself: a conserved energy-momentum tensor drives the field equations of cosmology and hence a role for energy in the universe is presumed from the outset.

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