

## **A NONLINEAR BILEVEL MODEL FOR ANALYSIS OF ELECTRIC UTILITY DEMAND-SIDE PLANNING ISSUES**

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### **Abstract**

An application of bilevel programming in the electric utility industry is presented. The model is nonlinear and is used to analyze various economic issues that affect electric utility planning. The electric utility at the upper level of the model seeks to minimize costs or maximize benefits while controlling electric rates and subsidizing energy conservation programs. Customers at the lower level attempt to maximize their net benefit by consuming electricity and investing in conservation. This model considers factors such as free riders and the rebound effect which affect the net benefits of utility resource plans but are ignored by most planning models. The model's solutions shed light on utility issues including whether there can be a practical difference between various objectives, including minimizing cost ("least cost" planning) and maximizing net social welfare ("value based" planning).

### **1. Introduction**

Many energy planning problems can be stated as single large-scale convex mathematical programs, but for convenience are solved by multilevel decomposition methods. Methods for solving such problems include the well-known Dantzig–Wolfe decomposition technique [1], among other approaches. For instance, Bloom [2] uses generalized Benders' decomposition in a model for planning investments in electric generating capacity while explicitly considering random plant outages. Bienstock and Shapiro [3] present a two-stage stochastic programming with recourse model of a utility resource acquisition problem, and also use Benders' algorithm to solve it. Wollmer [4] adopts a similar approach to solve a coal transportation problem.

However, many hierarchical problems, including some in the energy field, cannot be viewed as a decomposition of a single large-scale convex mathematical problem because the objectives of the upper and lower level are fundamentally in conflict. These are generally known as bi- or multilevel problems. An example is the Stackelberg game in which the follower (lower level) seeks to maximize his individual objective, naively assuming that the leader (upper level) will not change the values of the decision variables he controls. However, the Stackelberg leader makes his decision with full knowledge of how the follower will react [5]. Anandalingam [6],

for instance, analyzes Stackelberg strategies and applies them to a multi-sector linear programming model of the Tunisian economy.

The bilevel programming problem has the following structure:

$$\begin{aligned} & \underset{x}{\text{minimize}} && F(x, y_1, y_2, \dots, y_n) \\ & \text{subject to} && x \in X = \{x : f(x) \geq 0\}, \\ & && \underset{y_i}{\text{minimize}} && G_i(x, y_i), \quad i = 1, 2, \dots, n \\ & && \text{subject to} && y_i \in Y_i = \{y_i : g_i(x, y_i) \geq 0\}. \end{aligned}$$

where  $x$  is the vector of decision variables controlled by the upper level,  $y_j$  is the vector controlled by the subsystem "i" at the lower level, and  $F$ ,  $f$ ,  $G_j$ , and  $g_j$  are all vector valued functions. For most bilevel problems that have been studied in the literature, these functions are linear. However, the problem we examine is nonlinear in both the objectives and constraints.

Unfortunately, such programs generally have a non-convex feasible region which makes solving them difficult. Examples of solution algorithms which have been proposed include those of Bialas and Karwan [7], who use a linear programming based extreme point search technique, and Bard and Falk [8]. The latter approach is based on embedding the Kuhn-Tucker (KT) conditions for the lower-level problem in a single non-convex mathematical program and solves it by a branch and bound method. Although use of linear bilevel algorithms has been proposed for use in transportation network design [9], no application of bilevel methods has been made in electric utility planning. In this paper, we formulate and solve a nonlinear bilevel electric utility planning model in which the lower (follower) level is the power customer and the upper (leader) level is the utility. Our object is to show how bilevel programming can contribute to the understanding of important issues in utility "demand side" planning.

The following section discusses why the bilevel approach is helpful in the analysis of utility planning issues. Then, section 3 summarizes the basic structure of the model. The model formulation and the solution procedure are given in section 4. A few illustrative results of the model are presented in section 5. Section 6 offers some concluding remarks.

## 2. Need for a bilevel utility economic analysis model

The electric utility industry in the United States is very capital intensive, with total revenues over US\$ 150 billion per year. Traditionally, utilities have had a planning process of identifying the least cost supply resources to meet growing demand. Examples of such resources include hydroelectric, nuclear, and fossil-fueled generation. However, after the oil embargo in 1973, there has been an

increasing emphasis on efficient usage of energy. Demand side management programs (DSM programs) have emerged in the late 70's and early 80's as alternative means of supplying services (heat, light, etc.) provided by energy. DSM includes activities and methods which lower overall power demands (energy conservation), modify load shapes of customers (load management), or increase demands during the off-peak periods (load building) [10]. Examples of DSM programs include promotions of efficient appliances and adoption of time-of-use electric rates.

The emergence of DSM led to the concept of "least cost planning" (LCP), which can be described as a process which considers both supply-side and demand-side resources and identifies the combination which minimizes costs to utilities and consumers. During the last few years, "value-based planning" (VBP) has begun to be viewed as an alternative to LCP [11]. The main objective of VBP is to maximize net customer value while meeting the customer's needs, as opposed to LCP, which aims to minimize the cost of providing energy services. Consumer surplus has been proposed as an appropriate measure of net value [12]. The main impetus for VBP is the competitive environment that is gradually transforming the electric utility industry. It has been argued that competition from other fuels, cogeneration, and bypass now makes it unrealistic for utilities to assume that the energy services they provide to customers will be fixed, which is the implicit assumption of LCP [11].

Although LCP and VBP would appear to involve conceptually simple maximization (or minimization) problems, the societal issues involved are complex, affecting all aspects of utility planning. There exist many questions over which debate rages among utilities, regulators and public interest groups. Examples include:

- Should DSM be an administrative or market-driven process?

In an administrative-based process, planners would decide which DSM programs are justified and all ratepayers would subsidize the chosen programs. In a more market-driven approach, the focus would be upon giving appropriate price signals so that both utilities and consumers are motivated to make efficient decisions. With limited exceptions, DSM programs would be paid for by the participants without subsidies from nonparticipants [13]. The market would then determine which programs are economic. The disagreement over the issue is primarily a philosophical one, and is perhaps unresolvable by empirical studies. Yet our bilevel model can still be useful. For instance, it can analyze the possible effect of administrative versus market-driven approaches upon participation rates, an important factor in evaluating programs.

- To what extent do market failures exist which justify DSM programs?

DSM has been justified on the basis that it is a second-best solution to failures in the electricity market. Market failures which could lead to underinvestment in conservation by customers include marginal costs which exceed the price of electricity, environmental externalities, ignorance of the potential energy savings from conservation investments, lack of access to credit, and split incentives, in which consumers of energy services do not control the capital investments affecting

energy use (e.g. apartment tenants) [14]. The extent and even existence of these failures has been debated [13]. Additional empirical studies are needed to settle that debate. Our bilevel model can show the implications of any failure identified in such studies for the evaluation of DSM.

- Does the objective matter?

What should the primary objective of the utility be? Some advocate "least cost" [14], whereas others, in effect, argue that "least rates" to the customers should be the utility's goal [15]. Maximization of consumer surplus or "value" has also been proposed [12, 16, 17]. The bilevel model explicitly analyzes the effects of each of these objectives.

- Do free riders matter in the evaluation of the DSM programs?

Free riders are customers who take advantage of DSM programs to subsidize conservation investment they would have made anyway. There is controversy as to the extent of free riders in DSMs and the implications for their evaluation (e.g. [16, 18]). The bilevel model can estimate the portion of program participants who are free riders, their impact on DSM economics, and how they are affected by distortions in the capital market and electric rate structures.

- Is rebound likely to be significant?

The rebound effect occurs when the customer responds to DSM by increasing his consumption of energy services (e.g. by increasing room temperature, rather than by conserving energy). Although rebound can decrease the effectiveness of conservation programs, it does enhance the welfare of consumers. Some feel that this effect could be significant [19], whereas others disagree [20]. Given assumptions about customer decision-making processes, the bilevel model can show their implications for (1) the amount of rebound and the portion of participants who are free riders, (2) their impact on DSM economics, and (3) how they are affected by distortions in the capital market and electric rate structures.

Both empirical and theoretical research are needed to answer the above questions. Empirical studies could document, for instance, the extent of market failure or rebound in a particular circumstance. Theoretical studies, such as this paper, provide a framework for interpreting the results of empirical investigations and for exploring the consequences of different assumptions. Both types of analyses are desirable. Incorrect conclusions regarding the implications of empirical results for the evaluation of DSM may be drawn unless a sound theoretical basis is created for interpreting those results.

The purpose of our model is to provide a theoretically rigorous framework for "what if" analyses. For instance, if there exist capital market imperfections and customers buy electricity and invest in conservation to maximize their net benefits, would the rebound effect or free riders become important? Or can they be safely ignored in evaluating DSM programs? Would economically efficient levels of participation take place in market-oriented DSM programs? Would choice of planning

objective matter? The answers to such "what if" questions can help utility planners decide which objective is appropriate, which parameters are relevant, and what data are needed in DSM planning.

The reason why this bilevel model could be effective in addressing such "what if" questions is that it makes explicit the assumptions about consumer behavior that most other models leave implicit. This explicitness permits a rigorous analysis of these questions, the conclusions of which contradict those of previous studies whose logical flaws are hidden behind qualitative, descriptive arguments. Furthermore, our model is more comprehensive in several respects since it simultaneously accounts for many factors ignored in other analyses. These include:

- consumer reactions to conservation subsidies including free riders and rebound;
- sub-optimal consumer investment in conservation due to capital market distortions; and
- electric rates which are based on average rather than marginal cost.

A bilevel model is needed here because the instruments available to utilities for motivating consumers to make efficient decisions are imperfect. These instruments include electric rates and DSM programs. Electric rates in the US are generally based on average cost, rather than marginal cost, which distorts incentives and can cause the consumer's objective to be inconsistent with the utility's objective (e.g. maximizing net social benefits). Another distortion results from the "double payments" problem, in which DSM payments by utilities to consumers might provide too strong a motivation for conservation [13]. A "double payment" occurs when the consumer is paid twice to save the same kWh: once directly by the utility (e.g. as an appliance rebate) and a second time in the form of reduced utility bills. These possible distortions in incentives are most effectively represented by a bilevel approach which explicitly models the linkages between utility and its customers.

The model presented here is not intended to replace traditional utility planning models. Data and algorithmic difficulties preclude the development of a version of our model which would be sufficiently detailed for making specific decisions on supply and conservation measures. Rather, the model is to be a tool for "what if" analyses of certain economic issues that are ignored by large-scale models. The results of such analyses could be of great importance to electric utilities which, for example, spend about US\$ 250 million annually on DSM in California alone. For instance, we show that the level of expenditures that are justified under a "least cost" criterion can differ significantly from those that maximize net benefits.

### 3. Structure of the model

A schematic diagram for this model is shown in fig. 1. The lower-level model is that of consumer choice regarding energy consumption and investments in conservation measures. There are  $n$  such models, one for each customer class or

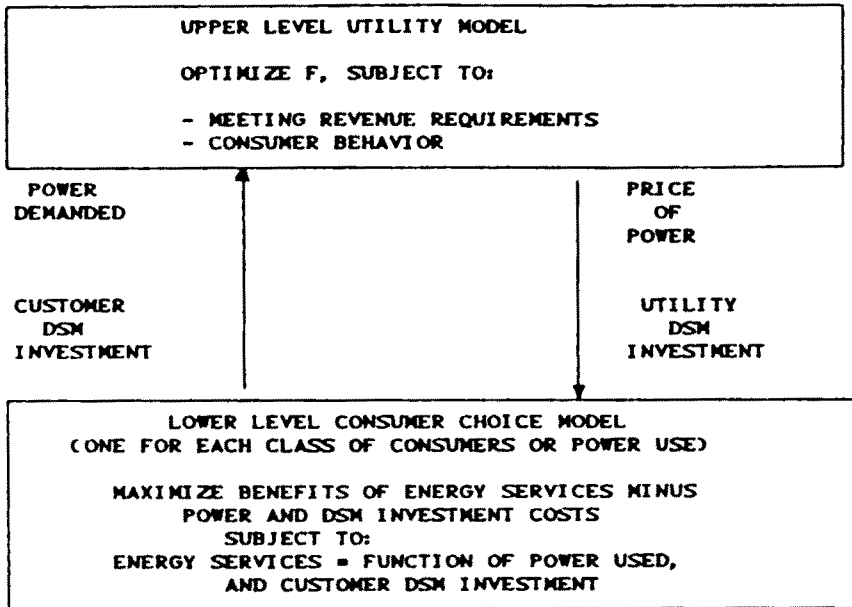


Fig. 1. Structure of bilevel utility model.

particular end-use. The upper-level model optimizes some objective  $F$ , such as net benefits or utility costs. The customer makes electricity use and conservation investment decisions naively, believing that the utility's decisions concerning electric rates and conservation subsidies are immutable. We assume there is no inter-connection among the lower-level models except through the utility model at the upper level.

The bilevel model captures the following essential aspects of the utility planning process:

- Consumers demand energy services such as heat or light; electricity is just one input used to produce those services. Other inputs, such as capital investment in conservation, are partial substitutes for electricity.
- Consumer demands for energy services are not fixed requirements; rather, they are economic demands which are responsive to price.
- Because of the market failures mentioned in section 2, consumers may under-utilize capital in their decisions on how to produce energy services. Distortions in the capital market are modeled by attaching a premium to the consumer's cost of capital.
- Electric rates are set at a level such that all utility costs (including that of DSMs) are recovered. These costs – fixed and variable – cause electric rates to deviate from the marginal cost of production.

#### 4. Model formulation

The following variables represent the decisions to be made by the utility and its customers.

$d$  = annual cost of capital investment [\$/yr] in conservation for class  $i$ . This can be viewed as the incremental investment over the minimum required to provide the energy service; for example, the incremental cost of buying a more efficient air-conditioner. This variable can be divided into conservation investments by the customer  $d_{ci}$ , and DSM investments paid for by the utility  $d_{ui}$ :

$$d_i = d_{ci} + d_{ui}.$$

$p$  = the price of electricity charged by the utility to class  $i$  [\$/kWh]. It is assumed as a simplification that there is one price per class. However, it should be noted that increasing or decreasing block rates are commonplace. Nelson [21] analyzes the case in which rate schedules are designed so that the incremental price of power equals the marginal cost of supply.

$p_{si}$  = the effective price of energy services [\$/unit] equaling the marginal benefit to customer class  $i$  of consumption of those services.

$q$  = amount of energy [kWh/yr] sold by the utility and consumed by customer class or end-user  $i$ .

$q_{si}$  = amount of energy services consumed by class  $i$  [units/yr].

There is one set of such variables for each lower-level model  $i$ . In Nelson [21], a version of the model is presented which accounts separately for on- and off-peak power prices, and electricity use.

##### 4.1. LOWER-LEVEL CONSUMER MODEL

The lower-level model for class  $i$  is the consumer behavior model. It is formulated as a net benefits maximization model, consistent with the theory of welfare economics. Basically, the consumer is assumed to maximize the gross benefits of consumption of energy services (measured by the integral of the demand curve for services), minus the consumer's cost of electricity and capital, subject to a production function relating services provided to electricity and capital inputs. This approach to modeling consumer choice is being used, for example, by Seattle City Light to project responses to home weatherization incentives ([22]; see also [32]).

However, the use of the rational economic model as either an explanatory or predictive model for conservation behavior has been criticized. Komor and Wiggins [23] argue that the cost-minimization model (which results if energy services are held constant in the rational model) fails to adequately explain how decisions are

made. This is because the model excludes nonfinancial goals, assumes consumers are rational, and does not differentiate between perceived and actual costs. Our net benefits model addresses one of their concerns in that it does include the benefits of different levels of energy services.

Komor and Wiggins have summarized alternative models of conservation choice. However, these other models either result in choices which are similar to the rational economic model (the payback model), are unsuitable for quantitative modeling (the diffusion of innovations model), have a poor predictive record (the social psychologist's attitude-behavior model), or require extensive data collection by surveys (the multiattribute marketing model). One field study has shown that a multiattribute marketing model can outpredict the rational economic model [24], if sufficient data are obtained. However, an author of that study has stated that the rational economic model, "like democracy, is not too good, but everything else is probably worse" (P. Komor, private communication). In applying that model, due respect must be paid to the gross uncertainties regarding, for example, discount rates.

For the above reasons, we adopt the net benefits maximizing model. Admittedly, it may not accurately describe the exact processes by which all consumers make decisions. However, we are not claiming that this model *explains* how consumers make decisions. Instead, we argue that it appears to be the most adequate and practical predictor of behavior, which is what matters most in our application.

The lower-level model for class  $i$  is specified as follows. The consumer derives benefits  $b_i(q_{s,i})$  [\$/yr] by the consumption of energy services  $q_{s,i}$ , according to the following function:

$$b_i(q_{s,i}) = \int_0^{q_{s,i}} p_{s,i}(q) dq. \quad (1)$$

The integral of the demand curve  $p_{s,i}(q)$  is a standard measure of the economic benefits of consumption [25]. The demand curve is assumed to be of the constant elasticity form:

$$q_{s,i} = D_i p_{s,i}^{-E_i}, \quad (2)$$

where the parameters  $D_i$  and  $E_i$  (the price elasticity) are constants. Substituting (2) into (1) yields:

$$b_i(q_{s,i}) = K_i + \frac{D_i^{(1/E_i)} q_{s,i}^{(1-1/E_i)}}{(1-1/E_i)}, \quad (3)$$

where  $K_i$  is the constant of integration which can be ignored when solving this model, assuming that  $q_{s,i} > 0$ .



Energy services  $q_{si}$  are assumed to be produced by capital investments and power use according to a Cobb–Douglas production function:

$$q_{si} = Q_i (C_i + d_{ci} + d_{ui})^{(R_i)} q_{ci}^{(1-R_i)}, \tag{4}$$

where  $C_i + d_{ci} + d_{ui}$  is the total capital investment in end use equipment,  $q_{ci}$  the amount of electricity consumed and  $Q_i$ ,  $C_i$  and  $R_i$  are constants. We assume that consumer investment  $d_{ci}$  and utility investment  $d_{ui}$  are completely substitutable.  $C_i$  is the minimum amount of investment that the consumer must undertake in order to produce energy services. For example,  $C_i$  might be the cost of the least expensive refrigerator on the market.

Although eqs. (2) and (4) are simple functions, these are commonly used in economic studies because of their clarity and tractability. For example, the Cobb–Douglas function is also used by Seattle City Light [22]. More complex functions could instead be used, but would not yield additional insights.

The consumer's problem in class  $i$  can now be summarized as:

$$\text{maximize } b_i(q_{si}) - p_{ci} q_{ci} - L_i d_{ci} \tag{5}$$

$(q_{si}, q_{ci}, d_{ci})$

$$\text{subject to } q_{si} = Q_i (C_i + d_{ci} + d_{ui})^{(R_i)} q_{ci}^{(1-R_i)}, \tag{6}$$

$$d_{ci}, q_{ci}, q_{si} \geq 0. \tag{7}$$

The variables  $p_{ci}$  and  $d_{ui}$  are utility control variables which, consistent with the Stackelberg assumption, the customer assumes cannot be changed by his actions. The parameter  $L_i$  represents the bias of consumers against energy-conserving investments.  $L_i$  can be interpreted as the ratio of the annual cost of investment at the implicit interest rate faced by the customers to the annual cost of investment at the market interest rate. A value of  $L_i = 1$  indicates the absence of capital market failures, whereas a value of, say,  $L_i = 8$  signifies that the customer requires a very short payback period due, perhaps, to lack of access to low-cost capital.

Alternatively,  $L_i$  can be viewed as a parameter which captures otherwise unmeasurable influences on decisions. These include social pressures, transaction costs, and the impact of ignorance. The important point is that the parameter is used here to predict how consumers respond to changes in DSM incentives and rates; to do this, it may not be necessary that the model accurately describes all the factors which affect conservation decisions.

Hausmann [26] and Dubin and McFadden [27], among many others, provide estimates of the extent to which the implicit interest rates used by customers exceed market rates. This model can reveal what these estimates imply for utility planning.

Since  $q_{si}$  follows the constant elasticity demand function and  $q_{ci}$  is positive for all values of  $p_{ci}$ , the optimal solution to the customer's problem (5)–(7) automatically

results in  $q_{ei}$ ,  $q_{si} > 0$ . Thus, the only explicit nonnegativity constraint we need is  $d_{ci} \geq 0$ . Substituting (6) into (3) and then (3) into (5) eliminates  $q_{si}$  as a decision variable, leaving only  $q_{ei}$  and  $d_{ci}$ . The Kuhn–Tucker conditions for optimality then yield the following set of equations:

$$D_i^{(1/E_i)} [Q_i (C_i + d_{ci} + d_{ui})^{R_i}]^{(1-1/E_i)} (1 - R_i) q_{ei}^{[(1-R_i)(1-1/E_i)-1]} - p_{ei} = 0; \quad (8)$$

$$d_{ci} \left\{ D_i^{(1/E_i)} [Q_i q_{ei}^{(1-R_i)}]^{(1-1/E_i)} R_i (C_i + d_{ci} + d_{ui})^{[R_i(1-1/E_i)-1]} - L_i \right\} = 0; \quad (9)$$

$$D_i^{(1/E_i)} [Q_i q_{ei}^{(1-R_i)}]^{(1-1/E_i)} R_i (C_i + d_{ci} + d_{ui})^{[R_i(1-1/E_i)-1]} - L_i \leq 0; \quad (10)$$

$$d_{ci} \geq 0. \quad (11)$$

Equation (8) is the only KT equation associated with  $q_{ei}$  that needs to be explicitly considered since  $q_{ei}$  will automatically be positive.

The KT conditions can be interpreted as follows. Equation (8) states that the marginal benefit of electricity must equal its price  $p_{ei}$ . Equations (9)–(11) say that if the consumer invests in conservation, then the marginal benefit of that investment must equal its marginal cost  $L_i$ . However, if the consumer does not invest ( $d_{ci} = 0$ ), then the marginal benefit of investing must be no greater than the marginal cost.

#### 4.2. UPPER-LEVEL UTILITY MODEL

At the upper level, the utility seeks to optimize an objective, subject to how the lower-level model will respond and a revenue recovery constraint which ensures that the utility recovers its costs. The following objectives are considered:

LCP Objective 1, Minimize Total Costs:

$$\text{minimize } F_1 = A + B \sum_i q_{ci} + \sum_i d_{ui}.$$

LCP Objective 2, Minimize Electric Rates:

$$\text{minimize } F_2 = p_e = F_1 / \sum_i q_{ei}.$$

VBP Objective, Maximize Net Benefits:

$$\text{maximize } F_3 = \sum_i b_i(q_{si}) - F_1 - \sum_i d_{ci}.$$

The units of all objectives are in (\$/yr). The first objective assumes that the generation costs are linear in  $q_e$ .  $A$  is the fixed cost component in (\$/yr), whereas  $B$  (\$/kWh) represents variable costs. These costs are assumed to include a fair return on investment for utility stockholders. The second objective is the traditional goal of regulatory commissions in the United States, in which they seek to minimize electric rates which are set equal to average cost. It has been argued that this objective also maximizes consumer surplus if the marginal cost of conservation is equal to the prevailing price of electricity  $p_{ei}$  [13]. However, this assumes that customers are efficient decision makers (i.e. there is no capital cost distortion  $L_i = 1$ ). In the absence of DSM,  $F_1$  and  $F_2$  are generally equivalent, but they diverge if  $d_{ui}$  is a decision variable.

The third objective seeks to maximize net benefits to consumers of energy services. This is the familiar "consumer surplus" of benefit-cost analysis [25], and has been recommended by some analysts for utility planning (e.g. [12, 16, 28]). It equals the gross benefits of power use minus payments to the utility and consumer investment in conservation. Consumer surplus is equivalent to net societal benefits because the utility just recovers its costs, implying that producer surplus is always zero. It is assumed that the social cost of  $d_{ci}$  is 1, not  $L_i$ .

The cost portion of the objective is admittedly simplistic, but it is adequate for the purpose of this paper. A more sophisticated approach would recognize the time-varying nature of demands and marginal costs, and the lumpiness of capital investments in generation plants. For instance, Nelson [21] uses the multiblock load-duration curve approach of Turvey and Anderson [29] to model production costs. To accomplish this, it is necessary to define demand variables for each class and time period, generation variables for each generating unit and period, and constraints relating generation to demand and generating capacity. Generating capacity of various types can also be defined as decision variables in static or dynamic (multi-year) planning models of this type. However, the qualitative conclusions obtained from the more sophisticated model in Nelson [21] do not differ from those of the simple model presented here.

The objectives  $F_1$  through  $F_3$  could also be improved by including a term for the external environmental and social costs of power production. Although reliable estimates for such costs are difficult to obtain, the incorporation of such a term would allow "what if" analyses of the effect of externalities.

The bilevel optimization problem facing the utilities for an  $n$ -class problem is as presented below:

$$\text{optimize } F_k \quad (k = 1, 2 \text{ or } 3) \tag{12}$$

$$\text{subject to } \sum_i p_{ei} q_{ei} = A + B \sum_i q_{ei} + \sum_i d_{ui} . \tag{13}$$

For  $i = 1, 2, \dots, n$ ,

$$\text{maximize } b_i(q_{si}) - p_{ei} q_{ei} - L_i d_{ci} \quad (14)$$

$$\text{subject to } q_{si} = Q_i(C_i + d_{ci} + d_{ui})^{(R_i)} q_{ei}^{(1-R_i)}, \quad (15)$$

$$d_{ci}, d_{ui}, p_{ei}, q_{ei}, q_{si} \geq 0. \quad (16)$$

The objective (12) is optimized subject to consumer behavior (14)–(15) and the constraint that revenues must equal the utility's cost (13). To solve the above problem, the Kuhn–Tucker conditions (8)–(11) for each of the lower-level models are substituted for (14)–(15) as constraints in the upper-level model. This yields a single non-convex mathematical program. The complementary slackness conditions are what makes this optimization problem non-convex. In the single class case ( $n = 1$ ), we deal with the problem of non-convexity by solving two reduced problems – one with  $d_c = 0$  and one with  $d_c > 0$  – thus eliminating (9). A generalized reduced gradient procedure [30] is then used to solve each reduced problem. The optimal solution to the original problem is the better of these two solutions. In theory, the  $n$ -class case requires that  $2^n$  separate reduced problems be solved, one for each possible resolution of the  $n$  constraints (eq. (9)). Research is currently in progress to derive a more efficient algorithm based upon a branch and bound method to lower the number of reduced problems that must be solved.

#### 4.3. MODEL NORMALIZATION

In order to derive conclusions that are as general as possible, it is desirable to minimize the number of independent parameters in the model. Using the following transformations of the variables and coefficients, the normalized model can be derived for the single class case:

$$\text{minimize } F_1 = 1 + q_e + d_u, \quad \text{or} \quad (17)$$

$$\text{minimize } F_2 = p_e, \quad \text{or} \quad (18)$$

$$\text{maximize } F_3 = M(C + d_c + d_u)^U q_e^V - F_1 - d_c \quad (19)$$

$$\text{subject to } p_e q_e = 1 + q_e + d_u, \quad (20)$$

$$VM(C + d_u + d_c)^U q_e^{V-1} - p_e = 0, \quad (21)$$

$$d_c \{UM(C + d_u + d_c)^{U-1} q_e^V - L\} = 0, \quad (22)$$

$$UM(C + d_u + d_c)^{U-1} q_e^V - L \leq 0, \quad (23)$$

$$d_c, d_u, p_e \geq 0. \quad (24)$$

The new variables are related to the variables of the original model as follows:

$$\text{new } q_e = (B/A) * \text{old } q_e,$$

$$\text{new } d_u = \text{old } d_u/A; \text{ new } d_c = \text{old } d_c/A,$$

$$\text{new } p_e = \text{old } p_e/B,$$

$$\text{new } F_1 = \text{old } F_1/A; \text{ new } F_2 = \text{old } F_2/B; \text{ new } F_3 = \text{old } F_3/A.$$

Inverting these relationships enables the user to calculate solutions to the original model. We have reduced the parameter set to  $L$ ,  $U$ ,  $V$ , and  $M$  from the original seven-parameter set for each class, which simplifies the analysis; the latter three parameters are as defined below:

$$M = \frac{\{D^{(1/E)}Q^{(1-1/E)}B^{(1-1/E)(R-1)}A^{-1/E}\}}{(1-1/E)}, \quad (25)$$

$$U = (1-1/E)R, \quad (26)$$

$$V = (1-1/E)(1-R). \quad (27)$$

The meaning of  $M$ ,  $U$ , and  $V$  can be explained as follows.  $U$  and  $V$  represent the relative importance of capital and energy, respectively, in producing benefits from energy services. The larger  $U$  is in absolute value relative to  $V$ , the larger the relative marginal benefit of capital use.  $M$  is a coefficient which rescales the gross benefits of energy services so that both the fixed cost and marginal cost equals 1. If the price elasticity of energy services is less than 1, then  $M$  is negative. The most interesting property of  $M$  is that the more that average cost (= price) exceeds marginal cost (due to either a high  $A$  or low  $B$ ), the smaller  $\log(-M)$  is. In the next section, we vary  $\log(-M)$  in order to examine the effect of increasing divergence of price and marginal cost upon the solutions.

## 5. Results and discussion

In this section, we present a sample of the type of issues that can be analyzed by this model. Table 1 presents a number of numerical solutions to the normalized one-sector model for the case in which the electric rate  $p_e$  is greater than the marginal cost of production  $B$  (which results from  $A > 0$ ). This is a typical situation today, since many utilities have capacity well in excess of their immediate needs. Model solutions for the objective  $F_2$  (minimizing electric rates) are not shown since they are trivial: no DSM is ever justified. This is because if  $p_e > B$ , utility conservation programs would lead to a spreading of fixed costs over reduced sales of electricity, causing higher rates. Table 1 includes:

Table 1  
Results of the one-sector model

(a) No DSM ( $d_{ui} = 0$ )															
$L = 2$				$L = 4$				$L = 8$							
$M$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$
-1.73	0.47	0	0.74	1.47	-5.15	0.66	0	0.41	1.66	-5.39	0.89	0	0.24	1.89	-5.91
-6.92	1.51	0	1.26	2.51	-8.79	1.93	0	0.73	2.92	-9.5	2.41	0	0.43	3.41	-10.6
-43.3	4.81	0	2.9	5.81	-20.3	5.9	0	1.72	6.9	-22.2	7.09	0	1.01	8.09	-25.2
-173.2	9.66	0.92	4.87	11.58	-39.6	12.42	0	3.35	13.42	-43.6	14.91	0	1.99	15.91	-49.4
-4330	48.3	8.6	20.35	57.9	-194	65.5	0	16.5	66.05	-214	77.5	0	9.81	78.5	-245
-69282	193.0	37.55	78.28	231.6	-773	262.4	0	65.86	263.4	-856	312.2	0	39.15	313.2	-979

(b) LCP Objective $F_1$ , minimize total costs															
$L = 2$				$L = 4$				$L = 8$							
$M$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$
-1.73	0.47	0	0.74	1.47	-5.15	0.66	0	0.41	1.66	-5.39	0.89	0	0.24	1.89	-5.91
-6.92	1.51	0	1.26	2.51	-8.79	1.92	0	0.73	2.92	-9.5	1.09	1.09	0	2.09	-9.53
-43.3	4.81	0	2.9	5.81	-20.3	5.28	0.8	0.97	7.08	-22.2	3.05	3.05	0	4.05	-21.3
-173.2	10.33	0	5.66	11.33	-39.6	10.57	2.59	0.95	14.16	-43.4	6.33	6.33	0	7.33	-41.0
-4330	54.59	0	27.79	55.59	-194	53.06	16.64	1.03	70.7	-213	32.67	32.67	0	33.67	-198
-69282	220.6	0	110.8	221.6	-775	212.0	69.83	0.89	282.8	-849	131.4	131.4	0	132.4	-791

(c) VBP Objective $F_3$ , maximize net benefits															
$L = 2$				$L = 4$				$L = 8$							
$M$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$	$q_e$	$d_u$	$d_c$	$F_1$	$F_3$
-1.73	0.47	0	0.74	1.47	-5.15	0.66	0	0.41	1.66	-5.39	0.89	0	0.24	1.89	-5.91
-6.92	1.51	0	1.26	2.51	-8.79	1.92	0	0.73	2.92	-9.5	1.09	1.09	0	3.18	-9.53
-43.3	4.81	0	2.9	5.81	-20.3	5.9	0	1.72	6.9	-22.2	3.05	3.05	0	7.1	-21.3
-173.2	10.33	0	5.66	11.33	-39.6	12.42	0	3.35	13.42	-43.6	6.33	6.33	0	13.66	-41.0
-4330	54.59	0	27.79	55.59	-194	65.05	0	16.5	66.05	-214	32.67	32.67	0	66.34	-198
-69282	220.6	0	110.8	221.6	-775	262.4	0	65.86	263.4	-856	131.4	131.4	0	263.8	-791

- A base case in which the utility does not invest in any DSM ( $d_u$  is deleted as a decision variable).
- Solutions under the LCP objective  $F_1$  (minimize utility costs) and the VBP objective  $F_3$  (maximize consumer surplus).
- Solutions under a number of different values of the parameter  $M$  (eq. (25)), representing cost conditions ranging from average cost (electric rates) much higher than marginal cost (small  $\log(-M)$ ) to average cost close to marginal cost (large  $\log(-M)$ ).
- Solutions for three values of the consumer investment cost multiplier  $L$ :  $L = 2$ ,  $L = 4$  and  $L = 8$ . These values represent increasing degrees of distortions in customer investment decisions. For example,  $L = 8$  implies that the customers are making decisions as if their cost of capital is approximately eight times the market rate. Distortions this high have been reported [23].

The normalized results in table 1 can be translated into results for the original model using the equations presented in section 4.3. As an example, say that we are considering a system with the following parameters:

$A = \$250,000,000/\text{yr}$	(fixed cost of power production)
$B = \$30/\text{MWh}$	(variable cost of power production)
$D = 50,000$	(service demand coefficient)
$Q = 1$	(production function coefficient)
$E = 0.5$	(elasticity of energy services)
$R = 0.5$	(production function exponent)

The resulting parameters of the normalized model using eqs. (25)–(27) are as follows:

$$U = V = -0.5,$$

$$M = -6.92 \quad (\text{corresponding to line 2 of all three sections of table 1})$$

For the case where  $L = 4$ , the least cost solutions corresponding to these parameters are found in table 1. The normalized solutions are reproduced below, together with the results for the original model:

$$\begin{aligned} \text{new } q_e &= 1.92 \Rightarrow \text{old } q_e = 16,000,000 \text{ MWh/yr,} \\ \text{new } d_c &= 0.73 \Rightarrow \text{old } d_c = \$182,500,000/\text{yr,} \\ \text{new } d_u &= 0 \Rightarrow \text{old } d_u = \$0/\text{yr,} \\ \text{new } F_1 &= 2.92 \Rightarrow \text{old } F_1 = \$730,000,000/\text{yr.} \end{aligned}$$

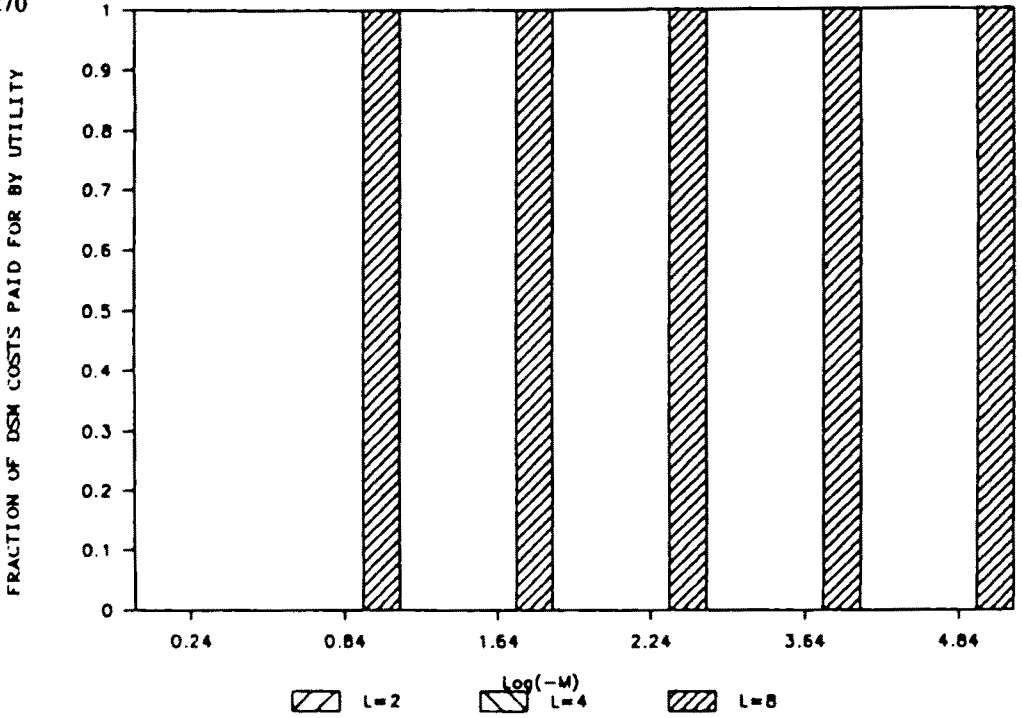


Fig. 2. Fraction of conservation investment paid for by utility  $[d_u/(d_u + d_c)]$  under least cost objective  $F_1$ .

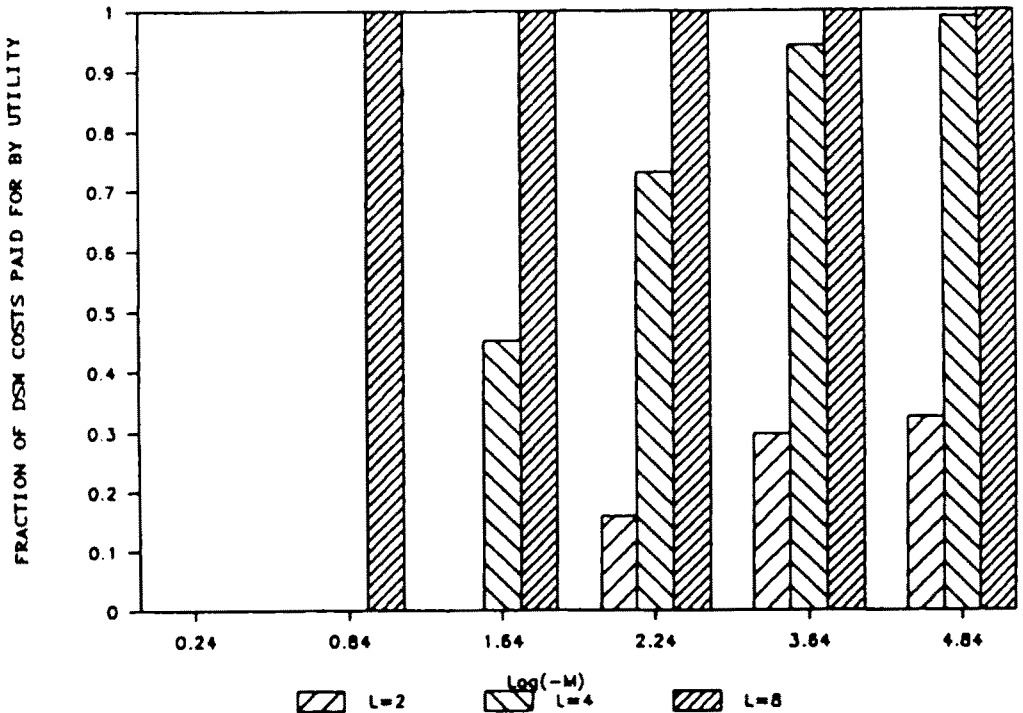


Fig. 3. Fraction of conservation investment paid for by utility  $[d_u/(d_u + d_c)]$  under net benefits objective  $F_3$ .



In this manner, all entries in table 1 can be transformed to yield solutions to the original model. All solutions assume that the coefficients  $U$  and  $V$  both equal  $-0.5$ . These values are consistent with a price elasticity of energy services  $E = 0.5$  and a production function coefficient  $R$  of  $0.5$ , which are arbitrarily assumed. A spreadsheet was utilized to prepare the model, feeding it to a GRG2 optimization package [30]. The GRG2 output was then returned to the spreadsheet for report generation. Each run took less than 10 seconds on an IBM XT with the mathematical coprocessor. For the two-sector models with several distinct power plants developed by Nelson [21], solution times were still on the order of seconds.

Figures 2 and 3 summarize the solutions by showing the fraction of conservation paid for by the utility as a function of the parameters  $M$  and  $L$ . Figure 2 represents the LCP solution ( $F_1$ ), whereas fig. 3 represents the VBP solution ( $F_3$ ). The  $X$ -axis is  $\log(-M)$ ; this value increases with decreasing values of  $A$  and increasing values of  $B$ .

For  $\log(-M)$  sufficiently small, i.e. high fixed costs or low marginal cost of production, DSM programs would not be chosen under either objective even under extremely high values of  $L$ . In the case of objective  $F_3$ , this is because at that level of fixed costs, the loss of consumer benefits caused by average cost pricing ( $p_e \gg B$ ) are more severe than those resulting from suboptimal investment in conservation. Note that VBP in fig. 3 recommends utility subsidies in some cases even when  $L = 4$ , whereas LCP recommends utility subsidies only when  $L = 8$  (fig. 2). When  $L = 4$ , adoption of the VBP objective would lead to higher total costs and rates than LCP but also greater net benefits to consumers.

This last result illustrates a difference between LCP and VBP which is ignored by some observers, who claim that the two objectives are practically the same. However, both the LCP and the VBP models invest in much less conservation than under the following significantly simpler rule, which is widely used by the industry [14]: if a DSM program costs less per kWh saved than the marginal cost of supply, the program should be implemented. The latter rule, which falsely assumes that energy services are fixed, justifies conservation subsidies which can actually increase costs because of free riders and rebound effects. Free riders inflate program costs, while rebound lowers the amount of energy and thus generation costs saved. This points out the advantage of using a bilevel model which explicitly models the interaction of customer behavior and utility decisions.

The above are just a sample of the type of economic issues that can be analyzed using this model. Some important conclusions we draw for this model under the assumed set of parameters are:

- Benefits of DSM under VBP and LCP can be large, but only if customer investment decisions are extremely distorted ( $L \geq 4$ ). The greater the degree of market failure in customer conservation decisions, the more attractive VBP and LCP are.

- The mere presence of distortion (e.g.  $L = 2$ ) does not necessarily justify utility subsidies of DSM, because such subsidies may only worsen the effects of another market failure – average cost-based price regulation.
- VBP, in some cases, results in higher rates and utility costs than LCP, but also higher net benefits for customers. This is because VBP can justify more DSM investments, leading to a greater level of beneficial energy services, even though utility costs are higher.
- Minimization of total utility costs and minimization of rates can lead to very different DSM decisions.
- The free rider and rebound effects are significant, particularly under VBP for low values of  $L$ . Under one set of assumptions, almost 85% of the subsidy goes to free riders. The rebound effect, in one case, decreases the energy savings by nearly half. These impacts are important since they reduce the effectiveness of DSM, and there exists considerable disagreement in the utility industry regarding the extent of these effects [19,20].
- The more the average costs exceed marginal costs, the less attractive DSM programs become.
- It is more difficult to justify DSM if marginal costs are less than average costs than if the opposite is true.
- Forcing the customers to share in the costs of DSM programs could lead to drastic reduction in net benefits under both VBP/LCP. This is modeled by constraining  $d_c$  to be at least a certain fraction of  $d_u$ .

## 6. Closing remarks

This paper has presented a bilevel nonlinear programming model for analyzing economic issues in electric utility resource planning. This model captures certain essential features of utility–customer interactions which are ignored by most existing planning models. These include (1) customer reactions to utility subsidies, (2) distortions in consumer decisions concerning conservation investments, and (3) price distortions caused by setting price equal to the average cost rather than the marginal cost of generation. This model also explicitly provides for analysis of the effects of free riders and rebound on utility planning. Future work will focus on developing a multi-period and multi-class version of the model so that it will be more relevant to particular planning problems. Other types of DSM programs, such as partial subsidies of capital investments and others considered by Wirl [32], will also be investigated.

Another important extension of this Stackelberg-based bilevel approach would be to the analysis of purchases of co-generated power by electric utilities. Hauric et al. [31] point out that under the Public Utilities Regulatory Policies Act, a utility must buy power from qualifying facilities at the utility's avoided cost, but sell

power to those same facilities at average cost-based prices. This asymmetry makes it impossible to describe utility-co-generation interactions by a single convex mathematical program. Instead, Haurie et al. [31] formulate the problem as a two-level optimization problem. However, due to computational difficulties, they did not model it as a Stackelberg game. They viewed that both the utility and the co-generator were Nash players who assumed that their actions would not affect the decisions of other players. Our framework could be extended to model the case in which the utility is the Stackelberg leader who correctly anticipates the reactions of both co-generators and consumers. Such a formulation would present formidable computational difficulties for realistic multi-period, multi-class problems. However, valuable insights might still be gained from formulating and solving simpler problems.

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