CALCULATION OF NONAXISYMMETRIC THERMALLY STRESSED STATE OF DISCRETE HOMOGENEOUS BODIES OF REVOLUTION WITH ORTHOTROPIC LAYERS

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The nonaxisymmetric thermally stressed state of laminar bodies of revolution made of isotropic and orthotropic materials is considered. The procedure used is based on the semi-analytical finite-element method and the method of successive approximations. The results from calculations for the stressed state of a structural element are reported. The structure has layers of isotropic and orthotropic materials with principal axes of anisotropy that coincide with the directions of the axes of the cylindrical and Cartesian coordinate systems.

In [2-4] we proposed procedures for investigating the nonaxisymmetric temperature fields and the stressed–strained state of laminar bodies of revolution consisting of inelastically deforming isotropic materials and elastic anisotropic materials under a heat and force load. As anisotropic materials we considered curvilinearly orthotropic materials [2] and rectilinearly orthotropic materials [3, 4]. In the anisotropic materials one principal direction of anisotropy of thermophysical and mechanical characteristics coincides with the circular coordinate φ, and two mutually perpendicular directions lie in the plane of the meridional section of the body (the angle between those directions and the z and r axes of the cylindrical coordinate system varies in each layer, depending on its structural features). In the orthotropic materials [3, 4], one of the principal axes of anisotropy coincides with the axis of revolution of the body.

As in the case of an isotropic material [5], the stress–strain relation for an orthotropic material is written in the form of Hooke's law for a homogeneous material with some additional terms, which takes into account the thermal strain, the temperature dependence of the elastic characteristics of the anisotropic materials, and their variation in the circumferential direction. Taking the temperature and components of the displacements to be the main unknowns, we construct the problem of determining the thermal and stressed state on the basis of the pertinent variational equations in the form of trigonometric series in the circular coordinate

$$
\begin{gathered}
T(z,r,\varphi,t)=\sum_{m=0}^{\infty} T_m(z,r,t)\cos m\varphi+\sum_{m=1}^{\infty} \bar{T}_m(z,r,t)\sin m\varphi,\\
u_z(z,r,\varphi,t)=\sum_{m=0}^{\infty} u_z^{(m)}(z,r,t)\cos m\varphi+\sum_{m=1}^{\infty} \bar{u}_z^{(m)}(z,r,t)\sin m\varphi,\quad (z,r),\\
u_\varphi(z,r,\varphi,t)=\sum_{m=1}^{\infty} u_\varphi^{(m)}(z,r,t)\sin m\varphi+\sum_{m=0}^{\infty} \bar{u}_\varphi^{(m)}(z,r,t)\cos m\varphi.
\end{gathered}
\tag{1}
$$

With this approach, the initial three-dimensional problem reduces to the solution, in the meridional section of the body, of the ensemble of two-dimensional variational problems of each harmonic separately:

when solving the heat problem

$$
\begin{gathered}
\delta\Bigg\{\int_V \frac{\lambda_{zz}^0}{2}\left(\frac{\partial T_m}{\partial z}\right)^2+\frac{\lambda_{rr}^0}{2}\left(\frac{\partial T_m}{\partial r}\right)^2+\frac{m^2}{2r^2}\lambda_{\varphi\varphi}^0 T_m^2+\lambda_{zr}^0\frac{\partial T_m}{\partial z}\frac{\partial T_m}{\partial r}+c\rho\frac{\partial T_m}{\partial t}T_m-\\
-q_z^{*(m)}\frac{\partial T_m}{\partial z}-q_r^{*(m)}\frac{\partial T_m}{\partial r}+\frac{m}{r}q_\varphi^{*(m)}T_m\Big]r\,dz\,dr+\int_S\frac{\alpha T_m}{2}(T_m-2\Theta_m)r\,ds\Bigg\}=0;
\end{gathered}
\tag{2}
$$

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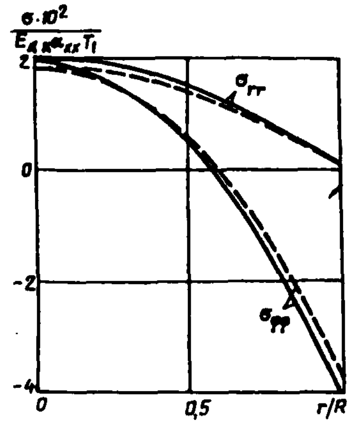

Fig. 1

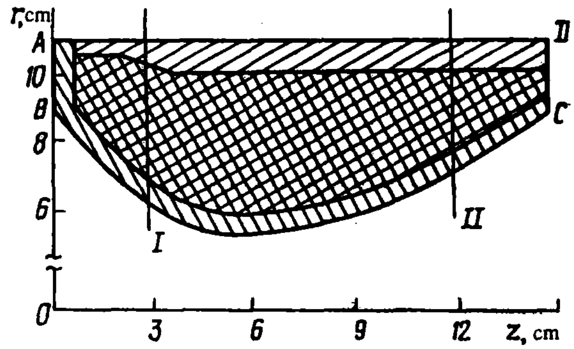

Fig. 2

when solving the problem of determining the stressed–strained state

$$\delta\left\{\int_{V}\left[\frac{1}{2}(A_{11}^{0}\varepsilon_{zz}^{(m)2}+A_{22}^{0}\varepsilon_{rr}^{(m)2}+A_{33}^{0}\varepsilon_{\varphi\varphi}^{(m)2})+2(A_{44}^{0}\varepsilon_{zr}^{(m)2}+A_{55}^{0}\varepsilon_{z\varphi}^{(m)2}+A_{66}^{0}\varepsilon_{r\varphi}^{(m)2})+A_{12}^{0}\varepsilon_{zz}^{(m)}\varepsilon_{rr}^{(m)}+A_{13}^{0}\varepsilon_{zz}^{(m)}\varepsilon_{\varphi\varphi}^{(m)}+A_{23}^{0}\varepsilon_{rr}^{(m)}\varepsilon_{\varphi\varphi}^{(m)}-\sigma_{ij}^{*(m)}\varepsilon_{ij}^{(m)}-K_{i}^{(m)}u_{i}^{(m)}\right]r\,dz\,dr-\int_{s}t_{ni}^{(m)}u_{i}^{(m)}r\,ds\right\}=0\quad(i,j=z,r,\varphi),\tag{3}$$

which are discretized in each approximation on the basis of the finite-element method, using triangular elements with linear variation of the desired coefficients in the series (1).

In particular, we obtained the following recurrence formula for determining the coefficients T_m, $\bar{T}_m$ at nodes in the meridional section [2, 3]:

$$T_{mi}(t+\Delta t)=T_{mi}(t)+\frac{\Delta t}{\sum\limits_{q=1}^{M}\langle c\rho\rangle_{q}H_{i}^{(q)}}\sum_{q=1}^{M}\{A_{ij}\theta_{mi}+B_{ij}\theta_{mj}-[D_{ij}+m^{2}N_{ii}+A_{ij}]T_{mi}(t)-[D_{ij}+m^{2}N_{ij}+B_{ij}]T_{mj}(t)-[D_{ik}+m^{2}N_{ik}]T_{mk}(t)+L_{i}\langle q_{z}^{*}\rangle+P_{i}\langle q_{r}^{*}\rangle-mR_{i}\langle q_{\varphi}^{*}\rangle\}_{q},\quad(i=1,2,\ldots,N).\tag{4}$$

This formula can be used to calculate the values of the coefficients at a time $t+\Delta t$ for a given distribution at the time t.

TABLE 1

T/θ	σ_i, MPa					$\alpha_T \cdot 10^5$
	$\varepsilon = 0$ %	0,04	0,1	0,2	2,0	
0	0	156	330	580	2080	0,66
0,85	0	156	200	220	260	0,55
1,0	0	156	159	162	170	0,55

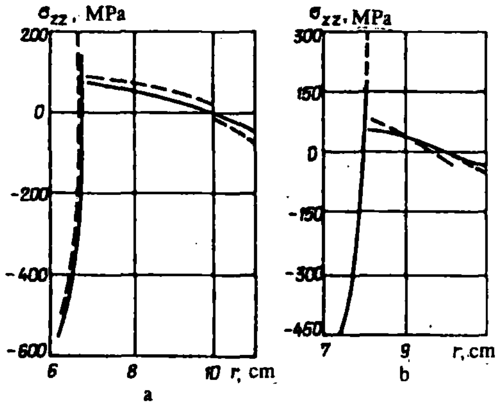

Fig. 3

To determine the coefficients $u_\alpha^{(m)}$, $\bar{u}_\alpha^{(m)}$ $(\alpha = z, r, \varphi)$ at the vertices (i, j, k) of triangular elements of the meridional section of the body in trigonometric series (1) in each approximation we obtain a system of $3N$ linear algebraic equations

$$\sum_{q=1}^{M} B^{\rho i(q)}_{\alpha c} u_{\alpha c} = D_{\rho i}; \ \langle \rho = z, r, \varphi;\ i = 1, 2, \dots, N \rangle,\ (\alpha = z, r, \varphi; c = i, j, k), \tag{5}$$

where summation is carried out between the indicated limits over the repeating indices α and c.

Expressions for the coefficients in (4) and (5) as well as the algorithms for the given procedure for solving specific problems on determining the temperature fields and stressed state in structural elements in the form of laminar bodies of revolution can be found in [2-5]. The reliability of the results obtained by using this procedure to solve the problem for bodies of revolution made of a rectilinear orthotropic material under a force load was checked on the example of a revolving solid orthotropic disk, for which there is an analytic solution in the case of a plane stressed state [1].

The convergence of the algorithm for the problem for unevenly heated bodies of revolution with orthotropic layers is demonstrated with the example of a thin rectangular orthotropic disk, whose temperature varies as

$$T(r) = T_0 + T_1 \cdot r^2/R^2, \tag{6}$$

where T_0 is the initial temperature and R is the radius of the disk.

In the case of a plane stressed state, when $E_x = E_y$ and $\alpha_{xx}^T = \alpha_{yy}^T$, an axisymmetric stressed state arises in the disk. In the cylindrical coordinate system the latter state is determined by [6]

$$\sigma_{rr} = \beta \frac{E_x \alpha_{xx}^T T_1}{1 - \nu_{xy}} \left(1 - \frac{r^2}{R^2}\right), \quad \sigma_{\varphi\varphi} = \beta \frac{E_x \alpha_{xx}^T T_1}{1 - \nu_{xy}} \left(1 - 3\frac{r^2}{R^2}\right), \tag{7}$$

where

$$\beta = (1 - \nu_{xy}) / \left(3 - \nu_{xy} + \frac{E_x}{2G_{xy}}\right). \tag{8}$$

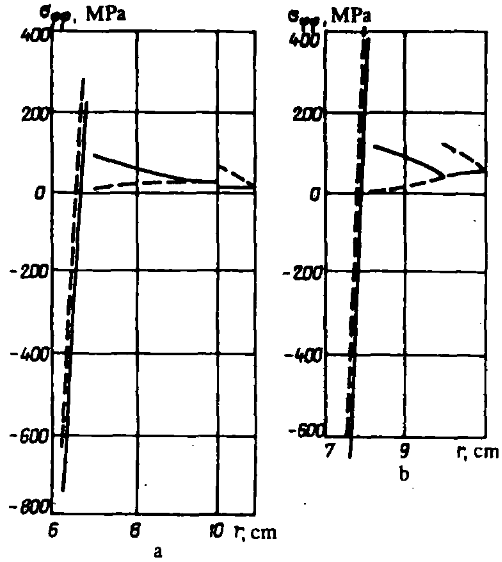

Fig. 4

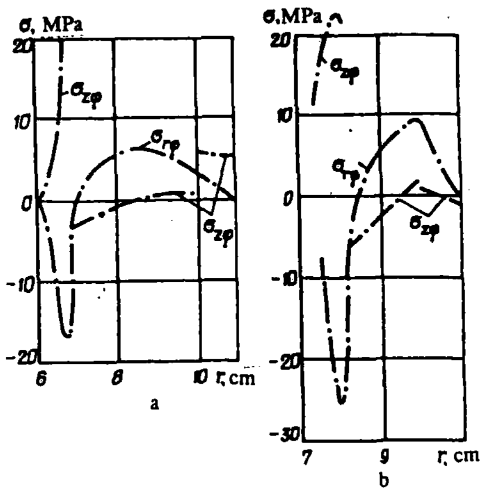

Fig. 5

The calculated results for the stressed state of a rectilinearly orthotropic thin disk for $E_x = E_y = E_z$, $G_{zx} = G_{zy} = G_{xy}$, $\alpha_{xx}^T = \alpha_{yy}^T = \alpha_{zz}^T$, $\nu_{xx} = \nu_{zy} = \nu_{xy} = 0.05$, and $E_x/G_{xy} = 100$ are shown in Fig. 1. The solid lines represent the results obtained by the procedure described and the dashed lines, the analytical results. With this choice of material for the disk, the coefficient (8) in the relations for the components of the stresses is 0.01794 in contrast to the isotropic case, when $\beta = (1 - \nu)/4 = 0.2375$, i.e., if the material is assumed to be isotropic the stress components are 13.2 times those given in Fig. 1. Comparison shows that the results are in satisfactory agreement; this means that the procedure developed can be used to calculate the thermally stressed state of a specific structural element.

As an example we give the results of calculation of the thermally stressed state of a structural element in the form of a three-layer body of revolution; half of its meridional section is shown in Fig. 2, where the z axis is the axis of revolution of the body. The part of the body shown by cross-hatching was made of carbon–carbon composite [6], with a coating of isotropic material on its internal surface. The outer cylindrical shell was made of curvilinearly orthotropic material with principal axes of anisotropy in the directions of the cylindrical coordinate system. It was assumed in the calculations that the component parts of the body are fastened together without interference and they deform without slip and detachment. At the time $t = 0$ the body is heated by the ambient medium at the temperature θ. The part ADC of the surface of the body (Fig. 2) is assumed to be thermally insulated. The coefficient α of heat transfer between the ambient and the coating material is 0.35 W/cm^2.

The carbon–carbon composite has nonaxisymmetric mechanical properties since it is made by weaving a three-dimensional skeleton of mutually orthogonal carbon fibers and then saturating the space between them. The temperature fields formed during axisymmetric heating, therefore, cause a considerable three-dimensional stressed state in such a body. Moreover, taking the influence of the shear modulus in the xy plane into account in calculations for such bodies is extremely important, since such materials usually have small values of that modulus. In calculations for a given structural element we also assume that this material is homogeneous and rectilinearly orthotropic with the z, x, and y axes as the principal axes of anisotropy, thus ignoring its heterogeneous structure.

The following thermophysical and mechanical characteristics of the materials are assumed for the layers:

– for an isotropic coating material the thermal conductivity $\lambda = 0.222$ W/cm·K, the product of the heat capacity c and the density ρ of the material is $c\rho = 2.75$ J/cm³·K, Poisson's ratio $\nu = 0.17$, and the $\sigma \sim \varepsilon$ diagram of the linear thermal expansion coefficient α_T for three values of the relative temperature T/θ are given in Table 1:

– for a rectilinearly orthotropic material

$$E_z = 4{,}5 \cdot 10^4 \text{ MPa}, \; E_x = E_y = 3{,}1 \cdot 10^4 \text{ MPa}, \; \nu_{zx} = \nu_{zy} = \nu_{xy} = 0{,}23,$$
$$G_{zx} = G_{zy} = 1{,}5 \cdot 10^3 \text{ MPa}, G_{xy} = 2 \cdot 10^3 \text{ MPa}, \alpha^T_{zz} = \alpha^T_{xx} = \alpha^T_{yy} = 0{,}12 \cdot 10^{-5} \; 1/\text{K},$$
$$c\rho = 2{,}5 \text{ J/cm}^3\cdot\text{K}, \quad \lambda_{zz} = \lambda_{xx} = \lambda_{yy} = 0{,}067 \text{ W/cm}\cdot\text{K};$$

– for the material of the outer cylindrical shell

$$E_z = E_r = E_\varphi = 2{,}57 \cdot 10^4 \text{ MPa}, \quad \nu_{zr} = \nu_{z\varphi} = \nu_{r\varphi} = 0{,}23,$$
$$G_{zr} = G_{z\varphi} = G_{r\varphi} = 5{,}22 \cdot 10^4 \text{ MPa}, \; \alpha^T_{zz} = \alpha^T_{rr} = \alpha^T_{\varphi\varphi} = 0{,}4 \cdot 10^{-5} \; 1/\text{K},$$

$$\lambda_{zz} = 0{,}067 \text{ W/cm}\cdot\text{K}; \quad \lambda_{rr} = \lambda_{\varphi\varphi} = 0{,}0069 \text{ W/cm}\cdot\text{K}; \quad c\rho = 1{,}407 \text{ J/cm}^3\cdot\text{K}$$

In the calculations, the meridional section of the body was divided by 680 nodes into 1280 triangular elements and nine terms in the trigonometric series (1) were used for the sought temperature and the components of the displacements, i.e., in each approximation the problem came down to solving nine linear systems of algebraic equations of the 2040-th order. The stressed state was determined on the basis of the algorithm, when the system (5) was solved completely once in the first approximation, and in all subsequent approximations only the right sides were calculated and the reverse was done when the Gauss method was used.

The results of the stressed state calculations for a body of revolution for $z = 2.75$ cm (section I in Fig. 2) and $z = 12.875$ cm (section II) after heating for 2.5 sec are shown in Figs. 3-5 (a and b, respectively). Figures 3 and 4 show the radial variation of the normal stresses σ_{zz} and $\sigma_{\varphi\varphi}$ for two values of the circular coordinate $\varphi = 0$ (solid lines) and $\varphi = \pi/4$ (dashed lines). The radial variation of the tangential stresses $\sigma_{z\varphi}$ and $\sigma_{r\varphi}$ for the circular coordinate $\varphi = \pi/8$ (dash–dot lines) are shown in Fig. 5. Analysis of the results shows that the variation of the mechanical characteristics in the circular direction in the carbon–carbon composite under axisymmetric heating leads to a nonaxisymmetric stress distribution in the body. The resulting tangential stresses $\sigma_{z\varphi}$ and $\sigma_{r\varphi}$ reach about 10% of the normal stresses σ_{zz} and $\sigma_{\varphi\varphi}$.

In summary, the results reported here indicate that the method developed for solving three-dimensional thermoplasticity problems for laminar bodies of revolution from isotropic and orthotropic materials makes it possible to effectively study the kinetics of the stressed–strained state in crucial structural elements during high-temperature heating.

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