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Inversion of Dynamical Indicators in Quantitative Basin Analysis Models. I. Theoretical Considerations[1]

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Present-day observed downhole quantities, which a dynamical model of basin evolution should account for, include: total depth drilled, formation thicknesses, variations of porosity, permeability and total fluid pressure with depth, and depths of unconformities. Following a line of logic previously employed with multiple thermal indicators, it is shown how the observed quantities can be used in a nonlinear inverse sense to determine, or at least constrain, parameters and functions entering quantitative models of dyanmical sedimentary evolution. A procedure is given so that the inverse methods can be used: (a) with single well data; (b) with multiple well data; and (c) simultaneously with thermal indicator data, which have already been previously successfully inverted using a tomographic procedure. Parameters that can be evaluated using the dynamical indicator inversion (dynamical tomography) include, but are not limited to, values dealing with geological events (such as unconformity timing and amount of material eroded, the "openness" or "shutness" of faults; critical fracture pressure, etc.), as well as values dealing with intrinsic, or assumed, lithologic equations of state (such as power law values in connections between permeability and void ratio, or between frame pressure and void ratio). The dynamical tomography procedure can be used with or without weighting the data and/or the dynamical indicators; is guaranteed to produce a closer correspondence between predicted and observed behaviors at each nonlinear iteration; and is guaranteed to keep all parameters within any chosen domain. When used in a multiple well setting, the dynamical tomography method enables an assessment to be made of the assumed invariance to spatial location of parameters in equations of state, as well as allowing geologic process parameters to vary with well location. The procedure also automatically incorporates the ability to determine precision, resolution, sensitivity, and uniqueness of any or all parameters, both associated with equations of state and associated with geological processes. Thus, a sharper understanding is achieved of the trustworthiness and uncertainty of quantitative basin analysis models in respect of: (I) intrinsic assumptions of a model; (ii) implicit or explicit parameter dependences for both geological events and imposed functional dependences of variables; (iii) resolution with respect to finite sampling and measurement error or uncertainty in the quality and quantity of observed data.

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INTRODUCTION

The dynamical evolution of sediments, and their compaction, are the underpinning keystones on which rests the ability to model thermal history of a basin, and hydrocarbon generation, migration, and accumulation histories. In the last decade or so, numerical frameworks have been constructed which permit data-driven models to be analyzed for fluid-flow and compaction in sediments (Yukler, 1979; Welte and Yukler, 1981; Bethke, 1985; Cao et al., 1986; Lerche, 1990a, b). These models, by and large, include auxiliary procedures for handling other phenomena allied to the evolution of fluids and compaction in sediments, such as chemical cementation and dissolution (Bethke, 1985; Cao et al., 1986), faulting and fracturing (Cao et al., 1989; Wei and Lerche, 1988), erosion events (Armagnac, 1985; Armagnac et al., 1989), thermal and hydrocarbon histories, and so on. But the dominant support to analysis of these subsidiary processes is provided by the burial history of the sediments.

Elsewhere (Lerche, 1988a,b; He and Lerche, 1989; Pantano and Lerche, 1990), we have provided a scheme such that multiple thermal indicators, which carry the impressed signature of a thermal record, can be inverted to yield information concerning paleoheat flux variation with time, unconformity events, physical and chemical parameters of the individual thermal indicators, stratigraphic ages, and overthrust events (Kuckelkorn et al., 1990). The essence of the method used to invert present-day downhole thermal indicator data was based on thermal indicator tomography—itself an offshoot of seismic tomography.

It seemed to us that it might be possible to generalize the tomographic procedure to allow the burial history of sediments at single and multiple well sites to be subject to a similar procedure. The rationale for wishing to obtain such a scheme is the plethora of seemingly freely variable parameters and functions which can enter a basin analysis burial history calculation. For instance, the quasi-empirical relation between permeability, κ, and void ratio, e, for shales is often written in the form

$$\kappa = \kappa_*(e/e_*)^B \tag{1}$$

where κ_* is the permeability on $e = e_*$, e_* is the surface void ratio (void ratio, e, is related to porosity ϕ, through $e = \phi/(1 - \phi)$), and B is a number, usually of order 3 ± 3 for most shales (Cao et al., 1986; Dutta 1989). This equation of state for shales, therefore, contains three freely adjustable parameters. Equally, the equation of state relating frame pressure (effective stress), P_f, and void ratio for shales is often written as the quasi-empirical relation

$$P_f = P_{f*}(e/e_*)^{-A} \tag{2}$$

where, again, P_{f*} and A are freely adjustable parameters.

Each lithology has its own equations of state relating permeability and frame pressure to void ratio, and each of these equations has its own adjustable parameters. In addition, there can be included different equations of state for the same lithology under different conditions. For instance, a shale may have an anisotropic permeability, which is different parallel and perpendicular to bedding.

Thus, from the point of view of using observed data to provide constraints on parameters in the equations of state, the advantages in having available a dynamical inversion scheme are clear.

Equally, the burial histories of sediments are influenced by dynamical events that occurred in the past. Thus, the development of a fault can have a major impact on the on-going evolution of the basin. If the throw of the fault is significant, then different lithologic units can be brought into juxtapositional butting, leading to a change in the paths allowed thereafter for migration of fluids. In turn, this change will impact the evolution of the dynamical compaction of the sediments. Likewise, whether a fault is open to hydrodynamic fluid flow or closed to such flow has a significant impact on the loss of fluid and consequent fluid overpressure in the basin, as has been shown for the Pinedale Anticline, Green River Basin, Wyoming by Wei and Lerche (1988).

The development of fracturing and the critical pressure at which fracturing can take place also have roles to play in controlling the evolution of sediment compaction and fluid loss in a basin (Cao and Lerche, 1989).

Similarly, the presence or absence of diagenetic cementation and dissolution, the timing of such events, the degree of cementation or dissolution, and the formations subject to such effects, all influence the rate, amount, and timing of fluid loss in sediments.

In addition, the presence of erosional events and whether they are depositional hiatae or represent deposition followed by erosion (in which case the thickness and lithology of the eroded units and the timing of such erosion are the critical unknown parameters influencing the burial history) are of concern in attempting to assess the consequences to sedimentary burial history of such events.

We see then two sorts of parameters entering burial history calculations:

1. Parameters associated with equations of state of lithologic units.
2. Parameters associated with geologic events.

The determination from observational data of those two sorts of parameters is not independent in the sense that one cannot first determine, say, the parameters relevant to the equations of state, and then the geological event parameters. The reason is that one set of parameter values impacts the other. Thus, if the shale scaling permeability, κ_*, is arbitrarily chosen to be very small, then the loss of fluids from shale layers is extremely slow. Thus, an erosional event may then have little consequence for the shale. On the other hand, if κ_* is larger, so that fluids can escape relatively quickly, then a depositional/erosional event may have a significant impact on the loss of fluid from the underlying shale. Clearly, the problem is one of determining *simultaneously* the sets of parameters for both equations of state and for geological events which are most consistent with present-day data. In addition, the resolution of each parameter must be determinable.

Supposing such a scheme to have been devised, it would then appear that burial histories most consistent with observed data, and with known resolution and uncertainty, are constructed. However, as has been noted elsewhere (Lerche, 1990a), both thermal conductivities, fluid viscosities, chemical diagenesis, and conversion of kerogen to mobile and soluble oil and gas phases are temperature-dependent, as is the thermal expansibility of fluids, the pressure of gases, and the solubility of methane and ethane in water and oil. Thus, the burial history evolution is dependent to a greater or lesser extent on the thermal evolution of the sediments and on the hydrocarbon evolution within the sediments.

Thus, it would appear that the thermal history of the system must be known simultaneously with the dynamical burial history of the sediments. This point is reinforced when it is also recognized that thermal indicators contained information concerning the impressed signature of the temperature history they have been exposed to, which, in turn, owes some allegiance to the burial paths of sedimentary units as well as to intrinsic variations in paleoheat flux. Effects of unconformities (Armagnac, 1985; Armagnac et al., 1989), of stratigraphic age uncertainties (Pantano and Lerche, 1990), and so on, are recorded with some resolution in thermal indicators. Conversely, observed present-day thermal indicator values can be used to determine the degree to which such geological events can be determined. But then these events, whether determined from dynamical burial path information or from thermal indicators, should have the same parameter values. Hence, it should be possible to use both dynamical information and thermal indicator information simultaneously to better determine event parameters. Such a procedure for multiple thermal indicators has already been used (He and Lerche, 1990) to better constrain not only paleoheat flux variations but also physical and chemical parameters for each thermal indicator, since all thermal indicators must have been exposed to the same paleoheat flux—which information, therefore, acts as a constraint on allowed variations of other parameters.

This paper sets up the formal quantitative development for handling all of these problems on an equal footing in order to illustrate the underlying mathematical framework of the procedure. Case histories are considered in the next papers in the series.

In the second section we present the methodology for handling the dynamical inversion scheme when only single-well information is available. The third section then provides a procedure for generalizing the method of the second section to allow for multiple-well information. The fourth section shows how the dynamical inverse tomographic scheme can be combined with the thermal indicator tomographic procedure already developed (Lerche, 1988a,b).

DYNAMICAL TOMOGRAPHY IN ONE-DIMENSION

The arguments put forward by He and Lerche (1989) for dealing with thermal indicator tomography are valid in like manner when dealing with dynamical tomography. Indeed, both the beholdenness to seismic tomography and the general procedures developed in He and Lerche (1989) are transportable almost wholesale to the dynamical tomography problem. Despite the closeness in general development, nevertheless, it is appropriate to spell out some of the detailed reasons underlying the rationale, as well as the appropriate generalizations necessary to carry through the machinations for dynamical tomography. These developments seem appropriate for two main reasons: first, so that one does not have to have the earlier papers on thermal indicator tomography and so this paper stands on its own in a complete form; second, because there are interesting technical differences of procedures and control functions for dynamical tomography which, when combined with thermal indicator tomography (as we shall do later in the paper), lead to a general procedure for handling both dynamical and thermal parameters on the same footing. For these reasons we believe it is important to construct the dynamical tomography procedure from first principles.

GENERAL REMARKS

As with thermal indicator tomography, dynamical tomography is a nonlinear inverse problem. The nonlinearity arises because the parameters and/or functions to be determined influence the burial paths of the sediments, the thicknesses of stratigraphic units, the porosity of formations, and the fluid paleo-overpressure. The essence of most tomographic procedures devised to date from many disciplines has been to construct ''almost linear'' solutions. Thus, the true nonlinear behavior is linearized around initial estimates for the parameters, the linearized equations are then solved to obtain linearized corrections to the parameters, the new parameters are then used to linearize again around the new nonlinar estimate, and the process iterated (Aki and Lee, 1976; Hawley et al., 1981; Menke, 1984; Lerche, 1988a,b). In constructing a thermal indicator tomography inverse scheme we originally (Lerche, 1988a,b) followed this tried and proven path. However, there are several difficulties that arise that make it desirable to devise a more general procedure. First, as noted by Menke (1984), a damping constant has to be introduced to stabilize inversion of matrices in the linearized equations. The value of this damping constant is arbitrary, but tests against many synthetic data sets have shown that a value of about 5–10% of the main diagonal elements of the relevant matrices produces acceptable inverse solutions. But in any particular problem there is no way of knowing, short of trial and error, the sensitivity of the inverse solutions to the value chosen for the damping constant. (We would prefer to avoid the introduction of an arbitrary constant, we have enough parameters to determine already.) Second, it can, and does, happen that the linearized corrections to the initial parameters can cause the up-dated (initial plus linearized correction) parameters to land in non-physical space (e.g., a correction to a small initial erosional unconformity may give a negative amount of erosion). Such facts can cause rapid numerical instability of the linearized iteration scheme. It is, of course, possible to rewrite parameters to enforce the physical requirements, but it would be preferable if the tomographic procedure would ensure such correct physical behaviors automatically. Third, it can happen that the initial estimate of parameter values is very far from the best parameter values so that many iterations are necessary to approach the minimum mismatch between predicted and observed data. Such effects can cause massive computer run time. We would prefer to have a procedure that would automatically find initial parameter values close to the ''best'' values. Fourth, there is no guarantee that, once a local minimum mismatch is found, there is not some other, more global, minimum within the physically acceptable domain. We would prefer to have a procedure which could readily be used to search quickly for such a potential minimum. Fifth, it can happen that the system is not sensitive to the precise value of a parameter, or a set of parameters, in some range. When this occurs, the implication is that the system cannot resolve such parameters, and attempts to find determined values should then be discontinued. We would prefer a procedure which would rapidly sort out such insensitive parameters from sensitive parameters which can be determined.

As a consequence of such difficulties with the conventional procedure, we have gradually abandoned the linearized procedure outlined above over the last few years in favor of a nonlinear scheme which does not suffer from the above drawbacks, but which is still remarkably fast numerically. We consider this framework in the next subsection.

Control Functions and Nonlinear Tomography

In a single well the information usually available is: (i) the depth, Z_T, measured from the sediment-water interface to the total depth (TD) drilled; (ii) the thicknesses, τ_i, and lithologies and ages of each formation in an ordered sequence $i = 1, 2, \ldots, I$. The sum of the thicknesses should be equal to the total depth drilled

$$Z_T = \sum_{i=1}^{I} \tau_i. \tag{3}$$

(iii) a set of porosity measurements ϕ_j measured at depths, z_j, with $j = 1, 2, \ldots, J$; (iv) a set of total fluid pressure measurements, P_k, measured at depths z_k, with $k = 1, 2, \ldots, K$; (v) occasionally, but not always, there will also be a set of permeability measurements, κ_l, measured at depths z_l, with $l = 1, 2, \ldots, L$; (vi) if unconformities have occurred, their depth positions, Z_n, ($n = 1, 2, \ldots, U$) are also noted. One of the purposes of dynamical burial history modeling is to devise time-dependent, fluid-flow, compaction models which will honor the above data.

As we have remarked already, in the construction of burial history models a suite of parameters is specified relating both to equations of state and to geological events. Let the vector, $\mathbf{p}$, denote the complete set of all such parameters. We recognize that the parameters are often of different dimensions which, in the nearly linear procedure discussed above, requires a renormalization to dimensionless form prior to initiating a tomographic procedure (Menke, 1984). As we shall see, the same is true with the nonlinear tomographic procedure to be exhibited below.

The argument is now relatively straightforward. A quantitative burial history model constructs dynamically evolving values for total depth, $Z_M(\mathbf{p}, t)$, for porosity $\phi_M(z, \mathbf{p}, t)$, for total fluid pressure $P_M(\mathbf{p}, t, z)$, for formation thicknesses $\tau_M(z, \mathbf{p}, t)$, for permeability $\kappa_M(z, \mathbf{p}, t)$ and will also insert unconformities with depth and time so that the modeled unconformities occupy positions $Z_M(\mathbf{p}, t, z)$. Here t is time, z is depth below the water base, and $\mathbf{p}$ is the parameter vector to show explicitly that all modeled quantities depend on the parameters invoked.

At the present-day ($t = 0$), the modeled quantities take on the values $Z_M(\mathbf{p})$, $\phi_M(z, \mathbf{p})$, $P_M(\mathbf{p}, z)$, $\tau_M(z, \mathbf{p})$, $\kappa_M(z, \mathbf{p})$, and $Z_M(z, \mathbf{p})$. For consistency, all of the modeled values should be in agreement with the present-day measured values. Criteria used to asscess the degree of mismatch between predicted and observed values are least-squares control functions as follows:

1. For TD,

$$X_{\mathrm{TD}}^2(\mathbf{p}) = (Z_M(\mathbf{p}) - Z_T)^2 \tag{4a}$$

2. For formation thicknesses,

$$X_T^2(\mathbf{p}) = I^{-I} \sum_{i=1}^{I} (\tau_i - \tau_M(\mathbf{p}, z_i))^2 \tag{4b}$$

3. For porosity variations,

$$X_\phi^2(\mathbf{p}) = J^{-1} \sum_{j=1}^{J} (\phi_j - \phi_M(z_j, \mathbf{p}))^2 \tag{4c}$$

4. For fluid pressure variations

$$X_p^2(\mathbf{p}) = K^{-1} \sum_{k=1}^{K} (P_k - P_M(z_k, \mathbf{p}))^2 \tag{4d}$$

5. For permeability variations

$$X^2_\kappa(\mathbf{p}) = L^{-1} \sum_{l=1}^{L} (\ln \kappa_l - \ln \kappa_M(z_l, \mathbf{p}))^2 \tag{4e}$$

6. For unconformity positions,

$$X^2_u(\mathbf{p}) = U^{-1} \sum_{n=1}^{U} (Z_n - Z_M(z_n, \mathbf{p}))^2 \tag{4f}$$

The reason for using the natural logarithm of the permeability in the least-squares control function (4e) is that the observed permeabilities of shales are, typically, several orders of magnitude smaller than those of sands (Glezen and Lerche, 1985; Dutta, 1989), with the lower shale permeabilities having dominant control on the escape of fluids from the subsurface (Cao and Lerche, 1990; Lerche, 1990a,b). The logarithmic dependence is appropriate to describe this dominance of shales, as has also been noted by Warren and Price (1961), Freeze (1975), Palcianskas and Domenico (1980), and others.

Note that the individual control functions in Eq. (4a) through (4f) have different dimensions so they are not immediately comparable.

Suppose, for the moment, that some initial estimate has been chosen for the parameter vector $\mathbf{p}$. Let this value be $\mathbf{p}_0$. (We shall return later to a procedure for determining the initial parameter vector $\mathbf{p}_0$). Then construct the dimensionless quantities:

$$\begin{aligned} y_1^2(\mathbf{p}) &= X^2_{\mathrm{TD}}(\mathbf{p})/X^2_{\mathrm{TD}}(\mathbf{p}_0); \quad y_2^2(\mathbf{p}) = X^2_T(\mathbf{p})/X^2_T(\mathbf{p}_0) \\ y_3^2(\mathbf{p}) &= X^2_\phi(\mathbf{p})/X^2_\phi(\mathbf{p}_0); \quad y_4^2(\mathbf{p}) = X^2_p(\mathbf{p})/X^2_p(\mathbf{p}_0) \\ y_5^2(\mathbf{p}) &= X^2_\kappa(\mathbf{p})/X^2_\kappa(\mathbf{p}_0); \quad y_6^2(\mathbf{p}) = X^2_u(\mathbf{p})/X^2_u(\mathbf{p}_0) \end{aligned} \tag{5}$$

On $\mathbf{p} = \mathbf{p}_0$, each of the y's is unity. Clearly, a smaller mismatch to the observations occurs when each of the y's tends to zero. Now construct a global control function $Y^2(\mathbf{p})$ as follows:

$$Y^2(\mathbf{p}) = \sum_{r=1}^{6} a_r y_r^2(\mathbf{p}) \tag{6}$$

where the a_r are positive fractional weight coefficients whose sum is unity. The weight coefficients can be chosen to emphasize one aspect of the model and agreement between observations and predictions over others. For instance, if no permeability measurements are available, it would be inappropriate to compare model predictions of log permeability against nonexistent observations. In that case, one might set $a_5 = 0$. Or again, it might be thought that porosity measurements are less reliable than, say, formation thickness measurements, in which case a_3 might be made smaller than the remaining weight coefficients.

Before we can construct the nonlinear tomographic procedure, one further factor needs to be addressed. The parameter vector, **p**, is a conglomerate of scalar parameters, all of which have different dimensions. In order to construct a simple nonlinear tomographic procedure, it is best to normalize parameters. To this end, suppose that a scalar parameter is considered to be physically constrained to lie between a minimum p_{min}, and a maximum p_{max}. Then write

$$a = (p - p_{min})/(p_{max} - p_{min}) \tag{7a}$$

so that $a = 0$ on $p = p_{min}$ and $a = 1$ on $p = p_{max}$. Perform a similar normalization on each component of the parameter vector, **p**, so that for each component we have

$$p = p_{min} + a(p_{max} - p_{min}), \qquad 0 \leq a \leq 1 \tag{7b}$$

Regard a vector **a**, of identical dimensionality to the vector **p**, as being fundamental with the connection between each component of **a** and each component of **p** being given by Eq. (7b). The vector **a** then has entries which are dimensionless for each component and are in the range $\{0, 1\}$. Regard Y^2 as a direct function of the vector **a**, with **p** being determined parametrically through the component representations as in Eq. (7b). Thus, an initial estimate for each component **p**, in the range $\{p_{min}, p_{max}\}$, provides an initial estimate of a_0, through Eq. (7a), and so an initial estimate for the vector $\mathbf{a}_0$, directly related to the initial estimate of the vector $\mathbf{p}_0$.

Consider then the following nonlinear iteration scheme for each scalar component, $a(j)$ of **a**, to be run through N times:

$$a(n + 1) = a(n) \exp\left[-\alpha_i \frac{\partial Y^2(\mathbf{a}(n))}{\partial a_i(n)}\right] \tag{8}$$

where

$$\alpha_i = \left|\frac{\partial Y^2(\mathbf{a}_0)}{\partial a_i(0)}\right|^{-1} \ln\,[1 + (Na_i(0))^{-1}] \tag{9a}$$

with $a_i(0) = a_0$, and the derivative calculated numerically from

$$\frac{\partial Y^2(\mathbf{a}(n))}{\partial a_i(n)} = [Y^2(a_1(n), a_2(n) \ldots a_i(n) + 0.1a_i(n), a_{i+1}(n) \ldots)$$
$$- Y^2(\mathbf{a}(n))]/(0.1a_i(n)) \tag{9b}$$

The nonlinear iteration scheme given by Eq. (8) has several interesting characteristics. First, it guarantees that if a_i is positive at any iteration (including the initial estimate), then $a_i(n)$ will be positive at every iteration. Second, it guarantees that $Y^2(\mathbf{a})$ will always be smaller or the same at each iteration if the partial derivatives are calculated exactly. [If the derivatives are only calculated numerically approximately, as by Eq. (9b), then small localized oscillations around a minimum can occur.] Thus, Y^2 is always decreased away from its initial value so the procedure guarantees that a minimum will be found. (The questions of resolution, precision, uniqueness, and sensitivity are deferred to the papers dealing with case histories.)

Third, it is a relatively simple matter, once a minimum is achieved, to "freeze" all parameters except one at the values corresponding to the minimum, and to then perform a linear or random search between minima and maxima. The procedure is repeated for each and all parameters in turn. In this way, a relatively confident position can be taken that no minima have been overlooked.

Fourth, in respect of the choice of initial estimates for the parameters, it now becomes clear how to ensure that they will be close to a minimum position. Choose the search range of each parameter (i.e., p_{min}, p_{max}). Then perform a linear or random search on each within the parameter range to find the minimum of $Y(\mathbf{p}_0)^2$, thereby identifying the rough value of the parameter. Choose that value as the estimate of p_0. In practice this procedure works extremely effectively. Indeed, when a rough estimate of the initial parameter vector has been found, the linear search procedure can be iterated to provide closer approximations to the minimum value of $Y(\mathbf{p}_0)^2$, and so a correspondingly better estimate of the parameter vector.

Fifth, as a tactical maneuver, a more rapid approach to a minimum value is obtained if N nonlinear iterations are done, the up-dated parameter vector, $\mathbf{p}_N$, at the end of the iterations is used to replace $\mathbf{p}_0$, and then N more nonlinear iterations performed. This tactic is far superior to doing $2N$ nonlinear iterations around the original estimate, $\mathbf{p}_0$.

To date, numerical experience on single well studies and on synthetic data has shown that:

1. The nonlinear inversion generally converges rather rapidly in 5–10 iterations.
2. The pragmatic procedure: linearly estimate $\mathbf{p}_0$, nonlinearly invert, linearly estimate, nonlinearly invert, is usually more than enough to exhaust the precision of the input data, so that the model arrived at after the inversion procedure is a significantly better fit to data than the results of the first linear search estimate.

MULTIPLE WELLS IN A BASIN, 2-D DYNAMICAL TOMOGRAPHY

In the case of a single-well study, there is little need to discriminate between parameters which are associated with equations of state from parameters which are, or may be, associated with geological events. However, this is not the case in multiple-well studies because geological events can change from spatial location to spatial location while, presumably, parameters in equations of state should be invariant. Exceptions have to be provided for: thus, the anisotropy in shale permeability parallel and perpendicular to bedding can be influenced by the local depositional and compactional history of a system, and so is location-dependent. But frame pressure–void ratio and permeability–void ratio rules should, presumably, not be dependent on location within a given basin.

Two fundamentally different nonlinear inverse procedures are available, depending upon the assumptions made concerning the parameter vector. On the one hand, there is the extreme end-member decision that every component of the parameter vector may be location-dependent for reasons not completely understood. On the other hand, there is the extreme end-member strategy which regards only geological event parameters as location-dependent, and any and all equation-of-state parameters are considered independent of location.

In either event, there exists a procedure for determining the validity of the extreme end-member scenarios. First, consider the situation in which *all* components of the parameter vector $\mathbf{p}$ are considered dependent on location x_i ($i = 1, 2, \ldots, X$) as well as on depth, z. Write $\mathbf{p} = \mathbf{p}(x)$ to express this dependence. Then suppose that at position x_i, thicknesses, TD, porosity, fluid pressure, permeability, and unconformity depths have been measured. Construct, as above, $Y_r(\mathbf{p}(x_i))^2$ for each quantity which now depends on the parameter vector, $\mathbf{p}$, at lateral location x_i. The evolution and compaction of sediments in the basin are now laterally connected because fluids can flow horizontally as well as vertically. Thus, consistency of predicted and observed behaviors at a given location does not guarantee a minimum discord at other positions in the basin. To obtain a minimum mismatch at all positions in the basin introduce

$$y_r(\mathbf{p}(x_1), \mathbf{p}(x_2), \ldots)^2 = X^{-1} \sum_{i=1}^{X} y_r(\mathbf{p}(x_i))^2 \tag{10}$$

Then proceed as noted above. Construct

$$Y^2 = \sum_{r=1}^{6} a_r y_r(\mathbf{p}(x_1), \mathbf{p}(x_2), \ldots)^2 \tag{11}$$

and minimize Y^2 with respect to the total set of parameter values, both geological event and equation of state, each which is considered location-specific.

Then perform the linear or random search routine for each parameter, thereby constructing a range of sensitivity for each parameter around the minimum value of Y^2.

Now repeat the analysis but hold parameters thought to be independent of geological events at common, location-independent values. Again minimize Y^2, again construct the sensitivity range for each parameter around the minimum value of Y^2. Compare the degrees of resolution and sensitivity of this end-member situation with the previous analysis above. If, to within some imposed degree of tolerance either in respect of intrinsic data resolution or in respect of predicted and observed mismatch weighted or unweighted, there is no difference in the parameter values, then justification can be given for arguing that parameters representing equations of state are indeed location-independent. A significant departure of the sensitivity curves one from the other in the two cases implies that, to within the resolution limit set, the parameter under investigation *is* location-dependent. Thus, the procedure in the 2-D case is a simple generalization of the 1-D procedure with only minor variations in technique—but providing the potential for assessing the validity of intrinsic assumptions of different extreme end-member behaviors.

COMBINED DYNAMICAL AND THERMAL TOMOGRAPHY

Information on the dynamical evolution of sedimentary basins is imparted not only by present-day downhole data such as TD, formation thicknesses, porosity, permeability, total fluid pressure, and unconformity depths, but some information related to dynamical evaluation has also been impressed on thermal indicators throughout their evolution (Lerche, 1990a,b). Indeed, elsewhere we have used multiple present-day thermal indicator data in an inverse manner to determine not only paleoheat flux variations and physical and chemical parameters associated with the thermal indicators, but also parameters associated with geological events (such as unconformity thicknesses, stratigraphic ages, emplacement time of overthrusts, insertion temperature and time or igneous overthrusts) (Lerche, 1988a,b; Pantano and Lerche, 1990; He and Lerche; 1989; Lerche, 1990a,b).

Clearly, parameters for geological events and equations of state are making their effects felt on both dynamical tomography (in a direct sense) and on thermal indicator tomography (in an indirect sense through the effects on temperature experienced by thermal indicators). Thus, a combined dynamical and thermal indicator tomography should better resolve, or constrain, common parameters influencing both types of indicator.

To allow for such a situation simply introduce the υ^{th} thermal indicator, $TI_\upsilon(Z_s)$, measured at the set of depths Z_s, $s = 1, 2, \ldots, S$. Then, following the procedure laid down elsewhere (Lerche, 1988a,b) for a thermal indicator with a given kinetic description, we can write

$$U_\upsilon^2(\mathbf{p}) = S^{-1} \sum_{s=1}^{S} (TI_\upsilon(Z_s, \mathbf{p}) - TI_\upsilon(t_s, \mathbf{p}))^2 \tag{12}$$

where $TI_\upsilon(t_s, \mathbf{p})$ is the predicted value of the υ^{th} thermal indicator with time t_s corresponding to present-day depth Z_s. To combine a set of different multiple thermal indicators, in a manner similar to that set out in the second section above, we introduce

$$u_\upsilon^2(\mathbf{p}) = U_\upsilon(\mathbf{p})^2 / U_\upsilon(\mathbf{p}_0)^2 \tag{13}$$

and

$$u^2(\mathbf{p}) = V^{-1} \sum_{v=1}^{V} b_r u_v(\mathbf{p})^2 \tag{14}$$

where V is the total number of thermal indicators, and the b_r are positive, fractional weight factors. In the case of thermal indicators not only is the parameter vector **p** made up of components due to geological events and equations of state, but also components due to paleoheat flux and physical/chemical parameters associated with each thermal indicator. Absorbing these "extra" parameters into the definition of **p**, we can then combine the dynamical and thermal indicator control functions to write

$$W(\mathbf{p})^2 = Y(\mathbf{p})^2 + u(\mathbf{p})^2 \tag{15}$$

and perform the nonlinear inversion procedure of the second section on the combined control function $W(\mathbf{p})^2$.

This completes the formal mathematical analysis because, with both the thermal and dynamical components successfully mated as above, we have available a procedure for assessing parameters of geological processes which affect the evolution of sedimentary basins in both a local and a regional sense.

The combined determination of structural and dynamical properties of petroliferous basins and of quantitative paleoheat flux variations, as well as physical/chemical parameters of equations of state and thermal indicators, together with quantitative measures of their uncertainties consistent with downhole data, offers the greatest probability of accurately assessing not only the timing of hydrocarbon production and migration, but also the relationship to the timing and development of structural and stratigraphic traps which serve as oil reservoirs in a basin.

DISCUSSION AND CONCLUSION

The development of dynamical indicator tomography closely parallels the corresponding development for thermal indicator tomography. In essence, the idea is to use observed, or inferred, present-day data to constrain parameters or functions occurring in dynamical models of paleo-evolution of sediments. The logic for assessing the degree to which model parameters can be constrained, or determined, by present-day data is founded on a nonlinear inversion or least-squares control functions in which total depth, formation thicknesses, porosity, fluid pressure, permeability variations with depth, and depths to unconformities act to constrain model behaviors.

This paper has spelled out the main theoretical procedures for enabling quantitative numerical codes to be constructed in order to examine the resolution, precision, sensitivity, and degree of uniqueness with which information can be extracted concerning the dynamical aspects of compaction and evolution of sedimentary basins.

The main factor which controls the evolution of compaction is the rate at which fluid can escape from the sediments. That loss is dominated by both the permeability variation and the excess fluid pressure gradient of shales, although other factors (such as overburden load, unconformities, cementation, and so on) play roles.

The inverse, nonlinear, dynamical tomography scheme set up here is designed to address directly the questions of which parameters control the detailed, dynamical development, the degree to which parameters can be determined or constrained, and the degree to which different hypotheses (e.g., open/closed faults; erosion/hiatus; equation of state parameters dependent/independent of spatial location) can be evaluated for their veracity.

As well as demonstrating that the design and implementation of a dynamical indicator tomography procedure are eminently practical goals for a single well (one-dimensional system), we also showed that both a two-dimensional generalization as well as a combined dynamical/thermal indicator tomographic procedure were implementable.

The advantage to having available the above method is that trial-and-error searches do not have to be undertaken for each and every parameter, something that is both time-consuming and very difficult to implement systematically. The tomographic nonlinear inversion automatically incorporates a systematic method for determining multiple parameters and their sensitivity, while guaranteeing to stay within any pre-set ranges.

Underlying the development of the multiple dynamical and/or combined thermal indicator methodology is the idea that multiple types and quantities of dynamical downhole information act as powerful constraints to provide checks and balances of one dynamical indicator against another—such as, for example, formation thickness, porosity with depth, and fluid pressure with depth.

The point here is that different dynamical indicators spanning the same depth range in the same well must have been exposed to precisely the same burial path behavior and to the same variations of fluid pressure or fluid flow with time. Each dynamical indicator (TD, formation thickness, porosity, permeability, fluid pressure, unconformity depth) must then produce the same value for a parameter common to all indicators. By demanding such agreement, and since each dynamical indicator has its own particular dependence on time and fluid flow, the inverse procedure can extract those parameters which are most consistent with the downhole data of a particular dynamical indicator, and also with the geological parameters determined by inversion from other downhole dynamical indicators.

Potential applications of the dynamical tomography procedure on its own, in one or two dimensions, and in combination with the thermal indicator tomography procedure developed previously (He and Lerche, 1989) are legion.

For example, as we have remarked already, the determination of parameters associated with permeability is a crucial ingredient if we are to more accurately determine the temporal loss of fluid from a sedimentary basin. In turn, the loss of fluid impacts the evolution of paleo-overpressure, which is linked to both rock fracturing as well as to increased temperature (through retention of fluid), thereby influencing thermal indicator evolution and hydrocarbon generation.

Or again, the presence of an erosional unconformity and the determination of both the amount of sediment eroded as well as the timing of erosion influences the dynamical evolution of sediments in a basin, the thermal history of the sediments, and the potential for hydrocarbon retention.

Many such cases can be investigated for the ability of present-day dynamical and thermal indicator information to provide knowledge of the paleo-evolution of a basin. Indeed, it could be argued that *all* of quantitative basin analysis requires such a treatment, leading to the exciting possibility of treating every sedimentary basin by such techniques.

In the next paper in this series, we present case histories illustrating and exemplifying applications of the theoretical development given here.

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