International Journal of Infrared and Millimeter Waves, Vol. 16, No. 3, 1995

TO THE PROBLEM OF ENERGY RECUPERATION IN GYROTRONS

V. L. Bratman, G. G. Denisov, and A. V. Savilov

Institute of Applied Physics
Nizhny Novgorod, 46 Ulyanova Str., 603600 Russia

Received December 19, 1994

A relatively high level of the minimal electron energy at the gyrotron output even at very large spread in pitch factor is explained. An estimation of the recuperation efficiency, which can be obtained due to this effect, is given.

Key words: gyrotron, recuperation of electron energy, synchrotron oscillations.

Introduction

The use of a simple recuperation scheme in a high-power gyrotron [1] has made it possible recently to raise the gyrotron efficiency from 30% up to 50%. This achievement is in clear opposition with the known concept about electron energy distribution at the output of the gyrotron operating space. The concept was based on the two correct assumptions:

1) just at the input of the gyrotron interaction space there are electrons with very small longitudinal velocities;
2) at a high electron efficiency there are particles having given to the HF field almost all of their rotatory energy.

0195-9271/95/0300-0459$07.50/0 © 1995 Plenum Publishing Corporation

Using these assumptions the following wrong conclusion was made: at the output from the operating space the particles having very small total energies, would appear. The presence of these particles would significantly complicate recuperation of energies of other electrons.

The error of this statement is in the wrong extension of the second assumption, which is correct for electrons of "central" pitch-angle fractions (which have initial longitudinal velocities close to the rotatory ones), on to electrons with small longitudinal velocities. Numerical simulations [1,6], as well as simple model considerations being given further, show that in optimal regimes the latter particles have a rather large "unspent" output rotatory velocity. This phenomenon makes the effective energy recuperation possible.

Physical Model and Equations of Particle Motion

A gyrotron electron beam is formed in a magnetron injection gun (MIG) with relatively small rotatory velocities of particles. Afterwards, the rotatory velocities grow during the particle motion from the gun to the cavity (Fig. 1) in an adiabatically increasing magnetostatic field $H_0(z)$. Real MIGs form electron beams with almost the same particle energies, but with a significant spread in the pitch angle. Correspondingly, at the input into the operating space of powerful gyrotrons the electron beam contains electron fractions with all possible pitch factors, $g = \beta_\perp / \beta_z$. Naturally, electrons of various fractions interact with the HF field of the cavity with different efficiencies.

Describing interaction of the beam with the HF field inside the cavity, we use the following rather conventional assumptions.

1) The HF field is represented by a $TE_{m,p}$ mode with a fixed longitudinal structure close to the Gaussian distribution,

$$F(z) = F_0 \exp\left[-3\left(\frac{2z}{z_0} - 1\right)^2\right],$$

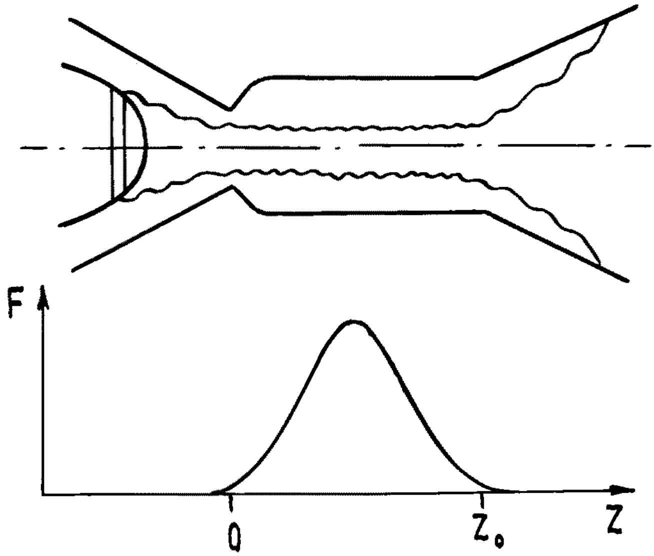

Figure 1. Scheme of a gyrotron, and using approximation of the HF profile $F(z)$ inside the interaction region.

with the characteristic length z_0 and the maximum F_0 being optimal for the "central" (c) velocity fraction having the pitch factor $g_c \sim 1$ (corresponding optimal dimensional values of the amplitude and the length are $\hat{F}_0 = 0.115$, and $\hat{\zeta}_0 = 30$ [2,3]).

2) The magnetostatic field is homogeneous and its strength corresponds to the mismatch $\hat{\Delta} = 0.5$ between the initial cyclotron frequency of electrons, $\omega_H = eH_0/m_0 c\gamma_0$, and the generation frequency, ω, which is also optimal for the central fraction.

3) The longitudinal momenta of particles are supposed to be constant; space-charge effects are considered to be negligible.

Let us study the interaction of various electron fractions with the HF field inside the cavity on the basis of the averaged equations of electron motion [3,4]. In these equations, which are conventional for gyrotrons, we use the following minor modifications. In order to consider particles with different pitch factors we use, as the independent variable, the normalized longitudinal coordinate $\zeta = z\omega/c$, which is the same for all the fractions. Besides, it proves to be convenient to introduce, instead of the transverse electron energy, the normalized total electron energy $u = \gamma/\gamma_0$, where γ and γ_0 are the current and initial Lorentz-factors of a particle, respectively. Then the motion equations can be rewritten as

$$\frac{du}{d\zeta} = F(\zeta)\frac{\sqrt{u^2 - 1 + \beta_\perp^2}}{\beta_z}\sin\theta , \tag{1}$$

$$\frac{d\theta}{d\zeta} = \frac{u-h}{\beta_z} + F(\zeta)\frac{u}{\beta_z\sqrt{u^2 - 1 + \beta_\perp^2}}\cos\theta , \tag{2}$$

with the initial conditions at the input of the interaction region

$$u(0) = 1, \quad \theta(0) = \theta_0 . \tag{3}$$

Here θ is the current electron phase relative to the wave, and $h = \omega_H/\omega$, the initial phases of particles, θ_0, are distributed homogeneously over the interval $[0, 2\pi)$. The maximal HF amplitude F_0 is defined as

$$F_0 = 4\gamma_0^2 \frac{eC}{m_0 c\omega} J_{m-1}\left(\frac{\omega}{c}R\right) ,$$

where C is the amplitude of the Hertz vector, J_m is the Bessel function, R is the beam radius. It is necessary to notice that for the used normalization the HF amplitude, F_0, does not contain parameters $\beta_\perp$ and β_z, and, therefore, it is the same for all the electron fractions. The above parameters are connected with the usual gyrotron parameters, corresponding to the central fraction, as follows:

$$\frac{2F}{\beta_{\perp_c}^3}=\hat{F},\quad \frac{\beta_{\perp_c}^2}{2\beta_{z_c}}\zeta=\hat{\zeta},\quad \frac{2(1-h)}{\beta_{\perp_c}^2}=\hat{\Delta}\,. \tag{4}$$

Peculiarities of Energy Exchange of Various Fractions

In order to analyze Eqs.(1)-(3), it is convenient to write down them in the canonical form [4]

$$\frac{du}{d\zeta}=-\frac{\partial \mathcal{H}}{\partial \theta}\ ,\quad \frac{d\theta}{d\zeta}=\frac{\partial \mathcal{H}}{\partial u}\ ,$$

with the Hamiltonian $\mathcal{H}=\frac{G}{\beta_z}+\frac{(u-h)^2}{2\beta_z}$, where $G=F(\zeta)\sqrt{u^2-1+\beta_\perp^2}\ \cos\theta$. Taking into account that $F(\zeta)$ is a slow function of the longitudinal coordinate, one can interpret the first term in the Hamiltonian as an efficient potential function $G(u,\theta)$, and the second one as an efficient kinetic energy. Such an approach permits us to illustrate the electron motion on the phase plane (u,θ). On this plane (Fig.2), particles of a definite fraction $(\beta_\perp,\beta_z)$ move along the integral curves $\mathcal{H}=\text{const}$, which represent infinite trajectories outside a corresponding separatrix, or finite trajectories inside the separatrix around the point of the equilibrium (the centre), where the electron energy is equal to the synchronous one, $u=h<1$, and the electron phase corresponds to the "zero" of the HF field, $\theta=\pi$. For each of the fractions separatrix $u_s(\theta)$ is the integral curve going through the point

$u=h,\quad \theta=0$:

$\mathcal{H}(u,\theta)=\mathcal{H}(h,0)$.

The separatrix consists of upper and lower curves which are not symmetrical due to the dependence of potential G on electron energy u (which corresponds to the force electron bunching):

$$u_s^{(\pm)}(\theta) = h \pm \sqrt{2\left[G_0 - G\left(u_s^{(\pm)},\theta\right)\right]},$$

where $G_0 = G(h,0) = \beta_z \mathcal{H}(h,0)$.

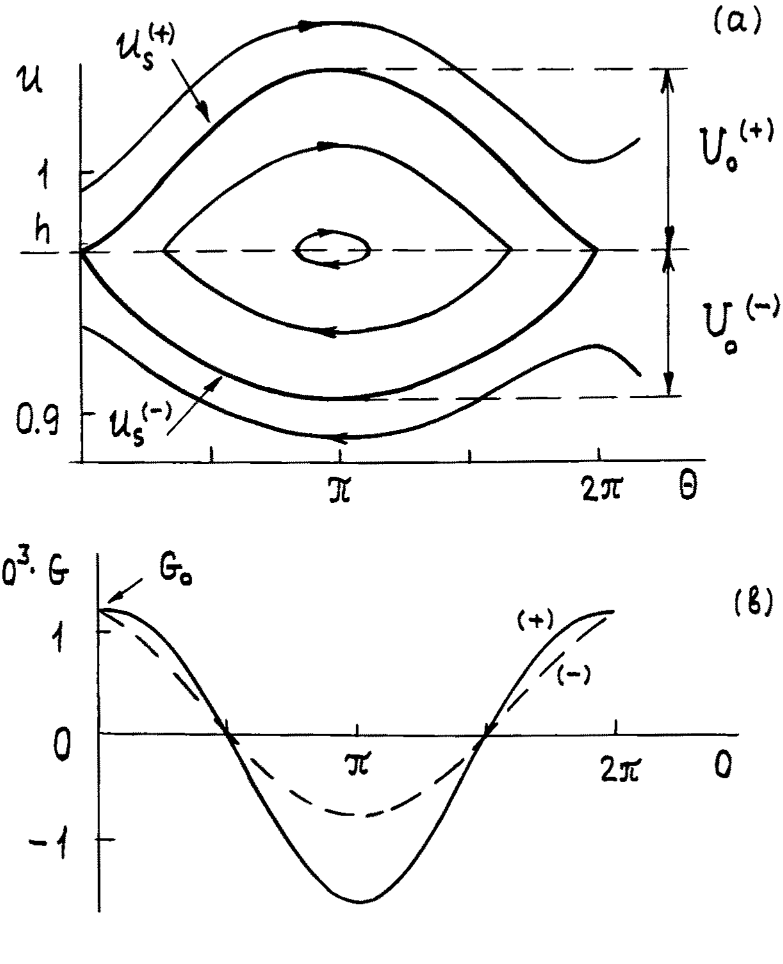

Figure 2. Phase plane for the fraction
with a large pitch factor in the middle of the operation region
for the case of $\hat{\zeta}_0 = 30$, $V_0 = 80$ kV (a), and efficient potential
$G^{(\pm)}(\theta) = G(u_s^{(\pm)},\theta)$ corresponding to the separatrix (b).

Inside the separatrix (the "bucket"), the particles make the so-called synchrotron oscillations around the centre $\left(u=h,\ \theta=\pi\right)$ with amplitudes limited by the upper and lower dimensions of the bucket

$$h-U^{(-)}\le u\le h+U^{(+)} . \tag{5}$$

Here $U^{(\pm)}=\sqrt{2\left(G_0-G(U^{(\pm)},\pi)\right)}$ are the maximal values of the amplitudes, which slowly vary with the coordinate: $U^{(\pm)}(\zeta)\propto\sqrt{F(\zeta)}$. The period of the synchrotron oscillations

$$L_s(\zeta)\approx 2\pi\sqrt{\frac{\beta_z^2}{F(\zeta)\beta_\perp}} \tag{6}$$

also slowly varies with the coordinate.

The synchrotron period $L_{s_0}=2\pi\sqrt{\dfrac{\beta_z^2}{F_0\beta_\perp}}$ and the amplitudes of the synchrotron oscillations $U_0^{(\pm)}$, taken in the middle of the interaction region where the coupling between electrons and the wave is maximal, represents the characteristic length of interaction between particles and the HF field, and the characteristic scale of the electron-wave energy exchange, correspondingly. For electrons of the central fraction, L_{s_0} is close to the interaction region length, $L=z_0\omega/c$. During the motion through the interaction space these particles make only a part of the synchrotron oscillation. For electron fractions with small pitch factors the synchrotron period is much larger than the interaction space length $L_{s_0}>>L$, and the amplitudes $U_0^{(\pm)}$ are small. So, these particles interact with the HF field very weakly.

It is obvious that the maximal spread in output electron energy should be provided by particles with large pitch factors g which have the largest coupling with the wave. For these particles the synchrotron period is much smaller than the interaction space length, $L_{s_0} << L$. It means that such particles make many synchrotron oscillations inside the operating space and, therefore, the description of their motion on the phase plane is the most convenient.

For particles with a large pitch factor, one can distinguish four stages of their motion through the interaction region (Figs.3,4). At its beginning, where the "vertical" bucket dimensions $U^{(\pm)}$ are small and the upper bucket border is lower than the initial electron energy $u = 1$,

$h + U^{(+)} < 1$,

the particles are outside the bucket and oscillate close to the initial energy (Fig.4a). At the second stage, when the upper border of the bucket becomes larger than the initial energy,

$h + U^{(+)} > 1$,

the particles are trapped into the bucket. Inside the bucket they oscillate around the equilibrium energy $u = h$ with amplitude increasing with the coordinate (Fig.4b). At the third stage, when the HF field $F(\zeta)$ begins to

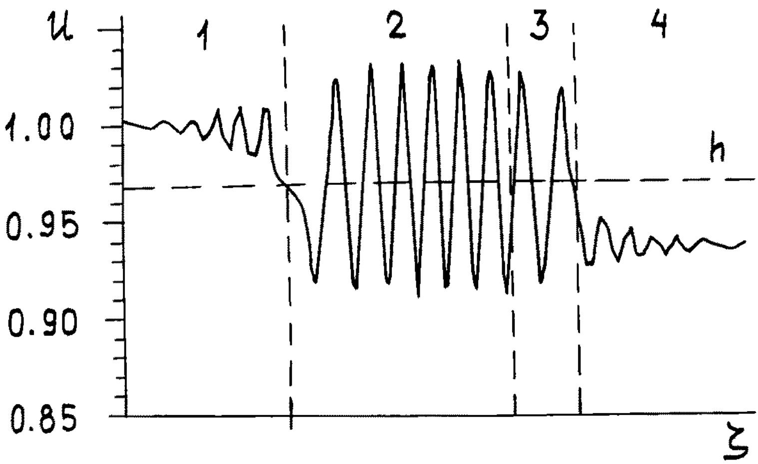

Figure 3. Normalized energy of a particle with a large initial pitch factor as a function of length inside the interaction region. Four stages of the motion are shown (see also Fig.4).

decrease, the bucket is compressed. When a particle meets the bucket border, it leaves the bucket (Fig.4c) with the energy defined by both the synchrotron amplitude and the phase, which the particle had when it met the border. Because during the HF amplitude decrease the bucket is compressed along the coordinate u but not compressed along the coordinate θ, it is easy to understand that particles meet the bucket

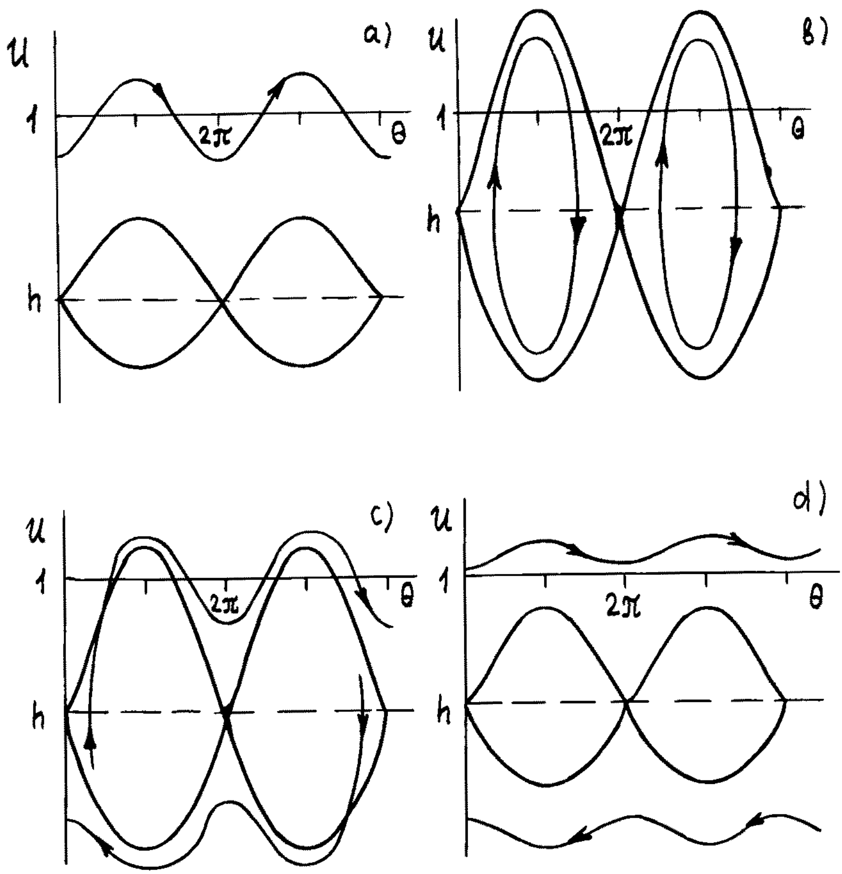

Figure 4. Phase planes corresponding to the four stages of the motion of a particle with a large pitch factor through the interaction region.

border mainly at its either top or bottom parts. Thus, most particles leave the bucket with the energies

$$u \approx h \pm U_0^{(\pm)} . \tag{7}$$

At the fourth stage the particles outside the bucket oscillate close to these energies (Fig.4d) with an amplitude which decreases with coordinate. So, the distribution function of the particles that belong to the fraction with a large pitch factor over output energies should consist of two peaks: the most particles of such fraction leave the interaction region with the energies defined by Eq.(7). It is important to stress that, while the electron-wave coupling coefficient increases infinitely with the increase of the pitch factor, the output energy levels, defined by Eq.(7), are almost the same for all the fractions with relatively large g, because the synchrotron amplitudes grow with the pitch factor increase very weakly. Thus, according to Eq.(7) the minimal output energy is limited,

$$u_{min} \approx h - U_0^{(-)} , \tag{8}$$

and represents the difference between the energy of the precise synchronism and the maximal bottom amplitude of the synchrotron oscillation of particles with large pitch factors.

The above consideration has been done for a sub-relativistic gyrotron. However, it is important to stress that the phase plane approach applied to other cyclotron resonance masers, including cyclotron autoresonance maser, gives similar expressions for the amplitude and the period of synchrotron oscillations [5]. So, in these devices the synchrotron oscillations have similar features and, therefore, the output electron energy also should have a rather high minimal level.

Recuperation Efficiency

According to Eq.(8), the minimal value of the electron energy at the gyrotron output has a low limit and can be relatively high. This effect allows to increase the gyrotron efficiency by means of electron energy recuperation, when after the interaction region the electron beam is decelerated and a part of the rest electron energy is given back to the power supply. It makes it possible to increase the electron efficiency of a gyrotron as follows

$$\eta = \eta_0 \ \eta_{rec} ,$$

where $\eta_0 = \Delta\mathcal{E}/eV_0$ is the efficiency without the recuperation, $\Delta\mathcal{E}$ is the averaged radiated electron energy. The recuperation efficiency η_{rec} represents a factor, which is larger than unity and defined by the input accelerating voltage V_0 and the output decelerating voltage V_{out}:

$$\eta_{rec} = \frac{V_0}{V_0 - V_{out}} .$$

Providing the recuperation, it is very important to avoid the danger of the existence of particles coming back into the interaction region due to the decelerating voltage. So, the maximal permitted value of this voltage has to be defined by the minimal possible electron rest energy

$$V_{out} = \frac{mc^2}{e}\left(\gamma_{min} - 1\right).$$

Correspondingly, one obtains the recuperation efficiency for a sub-relativistic ($\beta^2 = 1 - \gamma_0^{-2} << 1$) gyrotron

$$\eta_{rec} = \frac{\gamma_0 - 1}{\gamma_0 - \gamma_{min}} \approx \frac{\beta^2/2}{1 - u_{min}} . \tag{9}$$

Let us estimate the recuperation efficiency when the HF field is optimal for the central fraction with a pitch-factor $g_c \sim 1$. According to Eqs.(4),(8),(9), the recuperation efficiency is expressed in the following way:

$$\eta_{rec} \approx \frac{\beta^2}{\beta_{\perp c}^2 \hat{\Delta} \ + 2U_0^{(-)}} .$$

As it is seen from Fig.2 (which illustrates the case $V_0 = 80$ kV), the absolute value of the minimum of the potential G for a fraction with a large pitch factor ($\beta_\perp \approx \beta$) is about two times smaller than its maximum $G_0 = F_0\sqrt{h^2 - 1 + \beta^2}$ due to the force electron bunching $G\left(U^{(-)}, \pi\right) \approx -G_0/2$.

Then, using Eq.(4), one can estimate the lower amplitude $U_0^{(-)}$ in the following way:

$$U_0^{(-)} \approx \sqrt{3G_0} \approx \beta_{\perp_c}^2 \sqrt{3\hat{F}_0/2}\left(\frac{\beta^2}{\beta_{\perp_c}^2} - \hat{\Delta}\right)^{1/4}.$$

It gives the estimation for the recuperation efficiency:

$$\eta_{rec} \approx \frac{\eta_{s.p.}^{-1}}{\hat{\Delta} + \sqrt{6\hat{F}_0}\left(\eta_{s.p.}^{-1} - \hat{\Delta}\right)^{1/4}}, \tag{10}$$

where $\eta_{s.p.} = \dfrac{1}{1+g_c^{-2}}$ is the single-particle efficiency of electrons of the central fraction (the maximal part of the energy which can be radiated by a single particle). So, in the sub-relativistic case the recuperation efficiency does not depend on the initial electron energy. According to Eq.(10), this value diminishes with the increase of the pitch factor of the central fraction; it also diminishes with the decrease of the characteristic dimensional length of the interaction region $\hat{\zeta}_0$ because of the increase of the optimal dimensional HF amplitude $\hat{F}_0$. Thus, in the case of the Gaussian structure of the HF field for the optimal length $\hat{\zeta}_0 = 30$ the recuperation efficiency η_{rec} is changed from 1.25 up to 1.42 with the decrease of the pitch factor g_c from 1.2 up to 1. For the shorter length $\hat{\zeta}_0 = 20$, corresponding to the same values of g_c the recuperation efficiency is lower: it is changed from 1.18 up to 1.33 .

Conclusion

The distribution function of electrons over their output energy has a rather high lower level even at a very large initial spread of particles in the pitch angle. This level is defined by the difference between the energy of the precise synchronism and the amplitude of synchrotron oscillations of particles with large pitch angles. In accordance with estimations, given above, and the experiment [1], this effect can be used to essentially increase the efficiency of gyrotrons using the simplest scheme of the electron energy recuperation.

This work is supported by Grant No. R85000 from the International Science Foundation, and Grant No. 9302842 from the Russian Foundation of Fundamental Research.

References

1. K.Sakamoto, M.Tsuneoka, A.Kasugai, K.Takahashi, S.Maebara, T.Imai, T.Kariya, Y.Okazaki, K.Hayashi, Y.Mitsunaka, and Y.Hirata, Digest of 19th Int. Conf. on IR and MM Waves, Sendai, Japan, pp.63-64 (October 1994).

2. G.S.Nusinovich, R.E.Erm, Elektronnaya Tekhnika, Ser.1 Elektronika SVCh., 55, 1972.

3. A.V.Gaponov, V.A.Flyagin, A.L.Gol'denberg, G.S.Nusinovich, Sh.E.Tsimring, V.G.Usov, and S.N.Vlasov, Int.J.Electronics 51, 277, 1981.

4. V.K.Yulpatov, In: *Gyrotrons, Issue 1* (Gorky: Institute of Applied Physics, 1981, Ed. A.V.Gaponov-Grekhov), 26.

5. V.L.Bratman, N.S.Ginzburg, A.V.Savilov, In: *Relativistic High-Frequency Electronics, Issue 7* (Nizhny Novgorod: Institute of Applied Physics, 1992, Ed. A.V.Gaponov-Grekhov), 22.

6. V.N.Manuilov, M.A.Moiseev, In: ITER Technical Meeting, Plasma Heating and Current Drive, Cadarache, France (October 1994; presented by G.G.Denisov).