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The VFractal: a new estimator for fractal dimension of animal movement paths

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Abstract

Fractal measurements of animal movement paths have been used to analyze how animals view habitats at different spatial scales. One problem has been the absence of error estimates for fractal d estimators. To address this weakness, I present and test 4 new estimators for measuring fractal dimension at different spatial scales, along with estimates of their variation. The estimators are based on dividing the movement path into pairs of steps, forming V's, and then estimating various statistics from each V.

I measured the performance of these estimators by comparing them to the traditional divider d method, using data generated by two different animal movement models. The estimator based on the net distance between the two steps and the cos turning angle was most accurate, giving estimates similar to those of the traditionally-used divider d method. Precision increased with longer and straighter paths.

Strengths of this new estimator are that it can estimate fractal d at different spatial scales, give an estimate of variation, and combine data from many separate path segments which have been gathered at various spatial scales.

1. Introduction

Animals select habitats for many reasons: food, protection from predators, shelter, density of competitors, mating, etc. All of these processes act at different spatial scales, and act at different scales for different animals. For example, Least Flycatchers (*Empidonax minimus*) and American Redstarts (*Setophaga ruticilla*) select similar areas at large scales, but the two species of birds are not found together in small 4-ha plots (Sherry and Holmes 1988). The response to scale also depends on the type of animal. For example, at lower spatial scales, specialist predatory mites aggregate, and at higher ones they distribute themselves randomly, but generalist mites do the opposite (Zhang and Sanderson 1993). Thus, we need to know at what spatial scales animals view their habitat in order to understand how and why they select habitats (Kotliar and Wiens 1990).

We can measure at what spatial scales animals view habitats by doing extensive or intensive studies: extensively, by analyzing the spatial patterns of the locations of many individual animals at different spatial scales, and intensively, by measuring changes in tortuosity of movement paths of individual animals. Movement path tortuosity, or crookedness, has most commonly been assessed using fractal dimension (fractal d). Fractal d is the continuous analogue of discrete geometric dimensions (Mandelbrot 1967; Milne 1991); for example, lines have a geometric dimension of 1 and planes a dimension of 2. The fractal d for movement paths lies between 1 and 2 – *i.e.* d is 1 when the path is straight and a maximum of 2 when the path is so tortuous as to completely cover a plane.

Typically, biologists estimate fractal d of animal paths by using the dividers method (Mandelbrot 1967; Milne 1991). The length of the path is measured by walking a pair of dividers of a certain size along the path. If this is done for larger and larger dividers, then the slope of the plot for log(path length) vs log(step size) is 1–d, yielding one overall estimate for fractal d over a range of scales.

At large scales this is slightly modified (With 1994b) because estimates of path length vary depending at which of the data points the large steps were started. This variation is typically minimized

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by walking the dividers over the path many times, each time beginning at different starting points and thereby finding a mean path length.

In order to measure how animals use habitats at different scales it is important to measure not only overall fractal d but also to measure how fractal d changes with scale. For example, Benhamou (1990) showed that at a smaller scale wood mice (*Apodemus sylvaticus*) travel in a directed path toward individual bushes, but at a larger scale they move from bush to bush randomly. At the smaller scale the directed path would show a constant d but at the larger scales the correlated random walk would show an increasing d.

The traditional divider method can be adapted to measure d over different ranges of scales by the following method (Krummer *et al.* 1987; Sugihara and May 1990). First, choose a narrow range of step sizes and measure fractal d over that interval by estimating the slope of log(path length) vs log(step size) over that range. Then shift the range of step sizes along the x axis a bit and again estimate d. Repeating this will give estimates of fractal d at different scales. However, although now you can see how fractal d changes with scale, you do not know which changes in d are real, because there are no confidence intervals around the d estimates. Thus a method is needed to statistically test for changes in d.

One potential test for changes in d would be to test for change in slope in the log(path length) vs log(step size) relationship. However, the statistical tests for nonlinearity usually employed in regression analyses are not valid to test for a change in slope because it is not known whether the points on the graph are independent. For example, you could measure log(path length) with 10 step sizes separated by one unit (*e.g.* 1, 2, 3, ...), or do the same with 100 step sizes of 0.1 apart, or 1000 step sizes 0.01 apart. Thus the number of points on the graph is arbitrary and at some step spacing the log(path length) measurements are no longer independent from one step size to the next. Some other method is needed to statistically test for changes in d with scale.

I present a new set of estimators for fractal dimension, called VFractals, that solve this problem. They estimate fractal d at different spatial scales, give a measure of variance, and can combine data from many separate path segments which have been gathered at various spatial scales. I test four versions of these estimators with two different simulation models for movement paths and compare the accuracy and precision of the VFractals to that of the divider method.

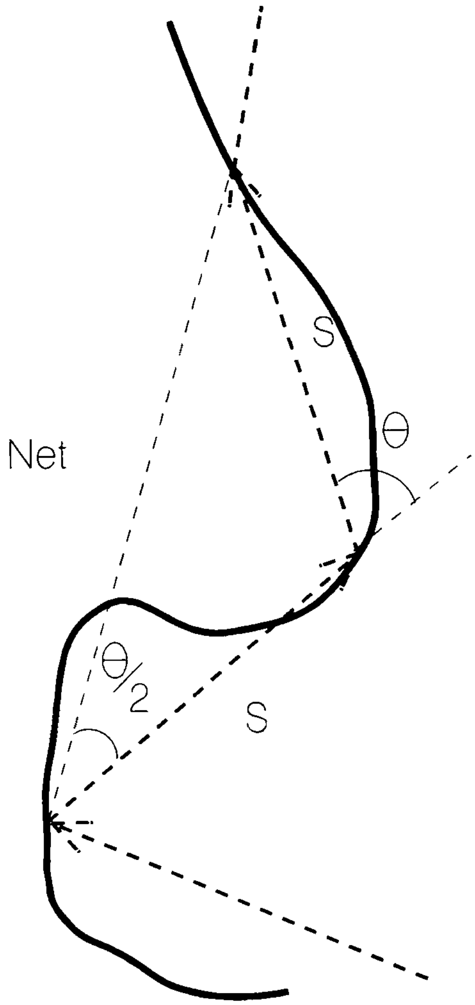

Fig. 1. One individual V from an animal's path. The thick line is the path. The thick dotted lines represent steps of equal size S along the path, and the two steps form the V. θ represents one turn angle between two steps, and Net is the net distance from the start of one to the end of the second step.

2. The VFractal estimators

We need an estimate of fractal d at different spatial scales. For each scale consider an animal's movement path to be a series of turning angles. For each

V formed by turning angle I will calculate various statistics, and then estimate fractal d from the mean of these statistics over the whole path.

Start by considering the divider method for estimating fractal dimension. If the movement path is fractal, then for any size of divider (Mandelbrot 1967),

$$\text{Gross Distance} = k\ \text{size}^{1-d}; \qquad (1)$$

for some constant k, where d = fractal dimension.

This relationship should apply at all path segments and scales, including down to individual step sizes. I will apply this relationship to the individual V's delineated by any two consecutive steps of size s (Fig. 1):

let Net = net distance from start of one step to the end of the second

θ = the turning angle between the two steps

First, use s for the divider size. Applying equation (1) at the scale of s we get that

$$\text{Gross Distance} = 2\ s = k\ s^{1-d}. \qquad (2)$$

Next use a different scale – instead of s, use Net for the divider size. Now when we apply equation (1) we find that

$$\text{Gross Distance} = \text{Net} = k\ \text{Net}^{1-d}. \qquad (3)$$

We cancel the k's by combining (2) and (3) and rearranging to get

$$d = \frac{\log(2)}{\log\left(\frac{\text{Net}}{s}\right)}. \qquad (4)$$

This equation gives us an estimate of fractal d for the V formed by one pair of steps. This d is a function of Net. We can also estimate d as a function of the angle (θ) between the steps as follows. Based on the geometrical relationship between Net and θ (Fig. 1),

$$\cos\left(\frac{\theta}{2}\right) = \frac{s}{\left(\frac{\text{Net}}{2}\right)} \qquad (5)$$

Since for all θ,

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{\cos\theta + 1}{2}}, \qquad (6)$$

then combining (4), (5) and (6) we get that

$$d = \frac{\theta}{1 + \log_2(\cos\theta + 1)}. \qquad (7)$$

Equations (4) and (7) give an estimate for fractal d of only this one V, at the scale of s to Net. However, we need an estimate for the whole path. Based on these equations, I propose 4 different estimators for fractal d of the whole path at the scale of s to Net:

(i) *Dmean:* Estimate d for each V, and then find the mean d for the whole path.
(ii) *Net:* Estimate the mean Net over the whole path, and then estimate overall fractal d by equation (4), but with using $\overline{\text{Net}}$ instead of Net.
(iii) *Cos:* Estimate the mean cos(θ) over the whole path, and then estimate overall fractal d by equation (8), but with using $\overline{\cos(\theta)}$ instead of cos(θ).
(iv) *NetCos:* The mean of the estimates (ii) and (iii).

One problem with the above estimates is the behavior of d at large turn angles (Fig. 2). Overall, fractal d should lie between 1 and 2, but equation (7) overestimates: for turn angles greater than 90°, it lies outside of that range. To accommodate this problem for Dmean, instead of just using equation (7) for each V, d was truncated to:

$$d = \begin{cases} \dfrac{\theta}{1 + \log_2(\cos\theta + 1)}; & \theta < 90^\circ \\ 2 & ; \theta \geq 90^\circ \end{cases} \qquad (8)$$

The Net and Cos estimates were also similarly truncated when estimating d from $\overline{\text{Net}}$ and $\overline{\cos(\theta)}$.

2.1. *Error estimates*

Unlike the traditional divider d method, we can estimate confidence intervals for fractal d with these VFractal estimators. All of the estimators are based on means, and means are normally distrib-

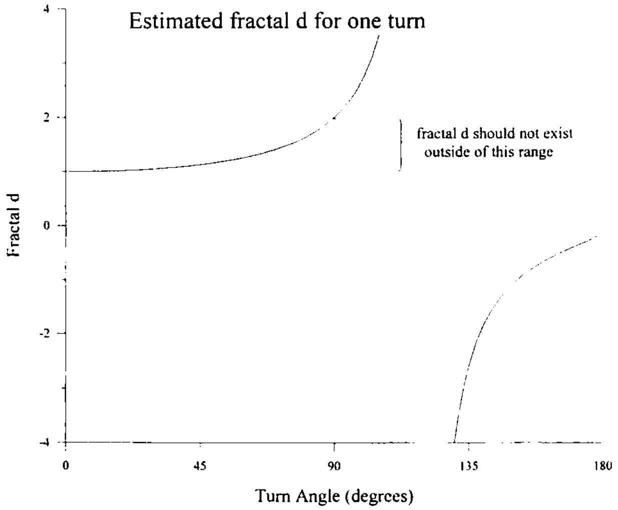

Fig. 2. Fractal d estimated for one turn angle. Note that at turn angles > 90°, d is outside of [1,2]. Fractal dimension is undefined outside of that range.

uted. Therefore even if the resulting estimator is not normally distributed, one can still calculate confidence intervals, as follows:

(i) *Dmean:* Consecutive pairs of steps are independent of each other, so this estimate is just a mean of n independent samples and is normally distributed. Thus, simply estimate the standard deviation of the mean d over all pairs of steps at each step size, and calculate a confidence interval from that.

(ii)–(iv) All of these estimates are nonlinear functions of means, and thus are not normally distributed. However, since the means are normally distributed, first estimate confidence intervals for the means (as done for Dmean). Then apply equations (4) and (7) to the lower and upper bounds of the confidence intervals.

There are, however, some problems with the error estimates calculated from the above equations. First, because estimates (ii)–(iv) are not normally distributed, they give asymmetrical confidence intervals and no estimate of variance to be used in further analyses. Second, they do not take into account the variation at large scales caused by initial starting position of the dividers.

These problems can be solved by applying a bootstrapping procedure (Efron and Tibshirani 1991). Bootstrapping allows one to calculate estimates that are normally distributed from any estimate of a sample of independent points. The following example for NetCos shows how to apply it to these estimators.

Suppose there are n turning angles in the path when it is viewed at a certain scale. Randomly select n of these angles, with replacement. Since these are being chosen with replacement, some no doubt will be duplicates. Then estimate NetCos from these n angles. Repeat this m times (each is called a bootstrapping replicate), each time selecting n new angles and estimating NetCos from them. The series of m estimates for NetCos will be normally distributed with a mean of

$$\overline{\mathrm{NetCos}} = \frac{1}{m} \sum \mathrm{NetCos}_j;$$

$$j = \text{the } j^{\text{th}} \text{ bootstrapping replicate} \tag{9}$$

and an estimate of variance of the mean of

$$s^2_{\mathrm{NetCos}} = \frac{1}{m-1} \sum (\mathrm{NetCos}_j - \overline{\mathrm{NetCos}})^2 \tag{10}$$

Since it is normally distributed, estimate 95% confidence intervals by:

$$95\% \text{ c.i.} = \pm\, 1.96\, s_{\mathrm{NetCos}}. \tag{11}$$

In order to minimize variation due to initial placement of the steps (With 1994b), I modified this procedure as follows. For each bootstrapping replicate I randomly chose a starting point within distance of one step from the beginning, then walked the steps to the end of the path. The data points for each replicate were from the successive V's along the path.

The 4 resulting estimates (Dmean, Net, Cos, NetCos) are normally distributed (because of the bootstrapping; Efron and Tibshirani 1991), the confidence intervals are symmetrical, and error due to initial placement of steps is both minimized and included in the estimate.

3. Testing the estimators

3.1. Methods

I tested the 4 estimators with simulated animal movement paths. To ensure that choice of the optimum estimator was not dependent on the movement model, I used 2 different models. The first was the correlated random walk, in which an animal walks in discrete steps. Each step the animal turns in a random direction that is circularly normally distributed and centered in the direction the animal is heading. The second was the fractal walk, in which an animal walks towards a fixed point in a fractal environment. For the correlated random walk the fractal d increases with scale while for the fractal walk the fractal d is constant.

For each model I used a range of parameters to simulate path fractal d's ranging from 1.0 to 1.9 (1 = a straight line, 1.9 = almost filling the plane). For each parameter combination I simulated 100 movement paths, and for each path I estimated all four estimators of fractal d, along with their error estimates, using 100 bootstrap replicates for each.

The VFractal estimates estimate fractal d over a narrow range of scales. I compared these estimates to the divider fractal d estimated over a narrow range of scales (Krummer *et al.* 1987; Sugihara and May 1990). For the divider d, in order to compensate for the random effect of placement of the first step for large step sizes (With 1994b), path length at each step size was averaged over 10 iterations. For each iteration the step was started at a random point along the path and the dividers were walked in both directions to each end of the path[1].

3.2. Results

3.2.1. Accuracy

To measure accuracy I calculated the difference between each VFractal estimate and the divider d estimate. For both movement path models, the most accurate is the NetCos estimator (Fig. 3), with the poorest being the Dmean estimator. The Dmean estimator strongly underestimates d at larger d values (likely because of the truncation of the equation estimating d at turn angles greater than 90°; although the other estimators use the same truncated equations, the equations are applied to $\overline{\text{Net}}$ and $\overline{\cos(\theta)}$, not Net and cos(θ) of each turn angle). All the estimators have poor accuracy for fractal d > 1.5, but again, NetCos is still the most accurate. Thus I chose NetCos as the VFractal estimator and only considered that one in these further analyses.

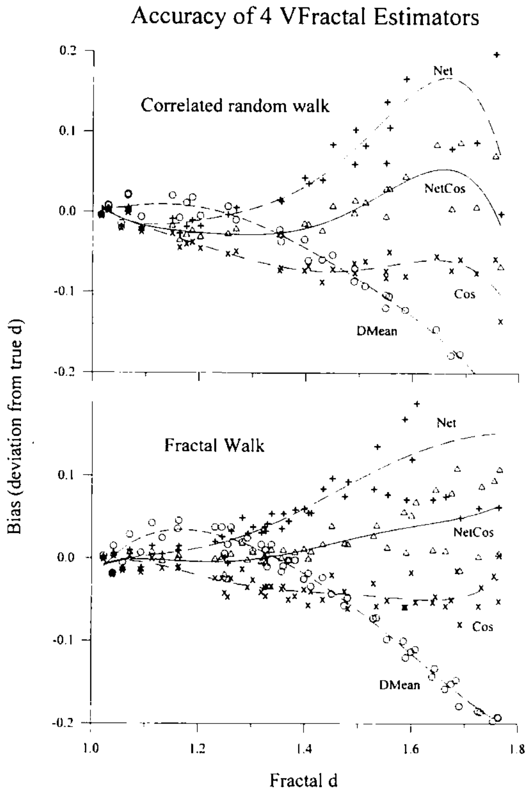

Fig. 3. Accuracy of the four VFractal estimators: mean difference between the estimators and divider d. NetCos is most accurate, but all are most accurate at fractal d < 1.5. Dmean is least accurate, underestimating fractal d quite severely.

3.2.2. Precision

Longer and straighter paths gave more precise estimates of fractal d (Fig. 4). For fractal d < 1.5, the variance decreased in proportion to the increase in path length – as would be expected from a normally distributed statistic – whereas $\sqrt{}$variance increased in proportion to the increase in fractal d. These are likely general relationships that are not dependent on the type of movement path, since both simulation models gave similar results.

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[1] A computer program to estimate the VFractals and the divider d can be obtained from the author.

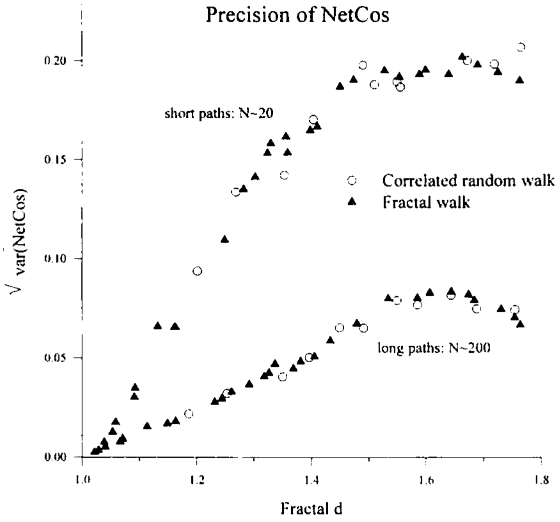

Fig. 4. Precision of NetCos estimator versus fractal d for two path lengths, for both types of movement path simulation models. Precision is measured by $\sqrt{\text{var(NetCos)}}$.

4. Discussion

4.1. Accuracy

The VFractal estimator is biased for more tortuous paths (d > 1.5; Fig. 3). Although this seems like a major limitation, since that is half of the [1.0–2.0] scale for fractal d, practically this will not create a large problem. First, d = 1.5 represents a very tortuous movement path that is rarely seen in nature – fractal d's of animal movement paths are much generally smaller (Table 1). Second, the bias is not hidden – we know over which range of d values to discard the VFractal estimator.

Finally, the main purpose of the VFractal estimator is to see how animals use different spatial scales. This requires comparing fractal d's at different spatial scales for each animal, not fractal d's among different types of animals (for comparing different animals, the traditional divider method is more useful because it gives an overall measure of fractal d). Since the biases are much more constant within than among types of movement paths (*e.g.* for d > 1.5, the ratio of variance among:variance between, $F_{39,2586} = 11.27$; $p < 0.001$), they should only minimally affect comparisons at different spatial scales within the same movement paths.

Table 1. Fractal d measurements of animal movement paths. These are considerably within the [1,1.5] range of unbiased values for the VFractal NetCos estimator, showing that in most cases the bias will not be a problem.

Type of animal	Fractal d (mean ± s.d.)	Source
grasshoppers (*Opeia obscura*)	1.15 ± .07	With 1994a
various acridid	1.10 ± .14	
grasshopper species	1.16 ± .11 1.09 ± .05	With 1994b
Wood turtles (*Clemmys insculpta*) – adult males	1.06 ± .06	McCurdy, D. unpublished data
Blandings turtles (*Emydoidea blandingii*) day one hatchlings	1.13 ± .04	Standing, K.L., Hurlburt, D., and Herman, T. unpublished data
marten (*Martes americana*)	1.09 ± .17	Bourgeois, M. unpublished data
coyotes (*Canis latrans*)	1.11 ± .04	Brodie, H. unpublished data
two-spotted spider mite *Tetranychus urticae*	1.10 ± .06	Dicke and Burrough 1988
Eleodes beetles	1.16 ± .08	Crist *et al.* 1992

4.2. Precision and sample size

As spatial scale increases, precision of VFractal estimates decrease (*e.g.* Fig. 5), because precision decreases with both increasing fractal d and decreasing sample size (Fig. 4). One way to increase precision at larger scales is to increase sample size by combining data from several path segments gathered at different spatial scales. This can be done since the VFractal estimator treats both individual turns and spatial scales independently.

For example, if mice and voles are tracked by the very fine-scale method of fluorescent dusting (Lemen and Freeman 1985; Goodyear 1989), then one can get locations every 1 cm, but only for path lengths of 15–30 m long. This represents an N of 1500–3000 at the scale of 1 cm, but only 15–50 at the scale of 1 m. For the larger scales one might use a coarser method of sampling, such as threading (Boonstra and Craine 1986), to gather data at scales of 10 cm for path lengths of 100–150 m long, or radiotelemetry to gather data at scales of 1 m for path lengths of several km long. All of these can be then combined into one analysis of fractal d vs scale.

At each scale a different number and combina-

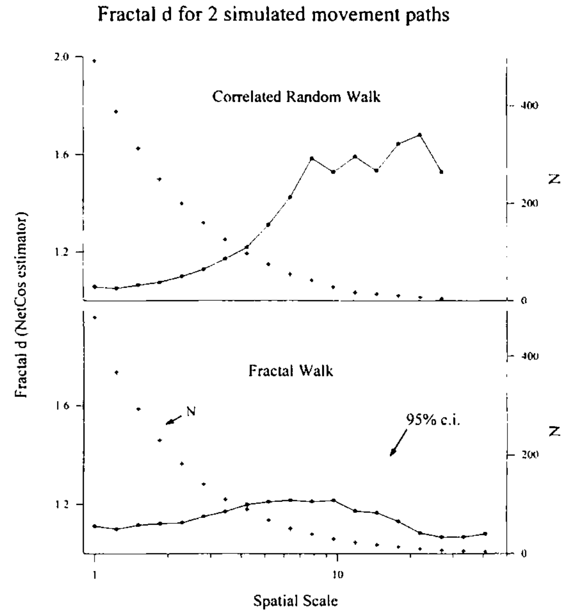

Fig. 5. Example of d (estimated with the VFractal estimator) vs scale for two movement paths. N is the length of the path in numbers of steps.

tion of path segments would be used. For example, to estimate fractal d at a scale of 20 cm, all the path segments tracked by fluorescent dusting and threading would be used, since their locations were gathered at less than 20 cm. First, one would find turn angles at a scale of 20 cm by running dividers of size 20 cm along all the path segments. Then, since the analysis treats all turn angles independently, all of the turn angles would be grouped together, and samples for the bootstrapping estimate drawn from that group. This whole process would be repeated for each scale, to show overall changes in fractal d vs scale.

4.3. Statistical sampling units

When comparing movement paths of different species, the sampling unit should be an individual, not a movement path, with only one fractal d estimate at each scale for each individual (Wiens *et al.* 1993). Thus the VFractal estimator can be used to combine all movement paths for one individual, but the error estimates should not be used in this case. The main purpose of the error estimates is to

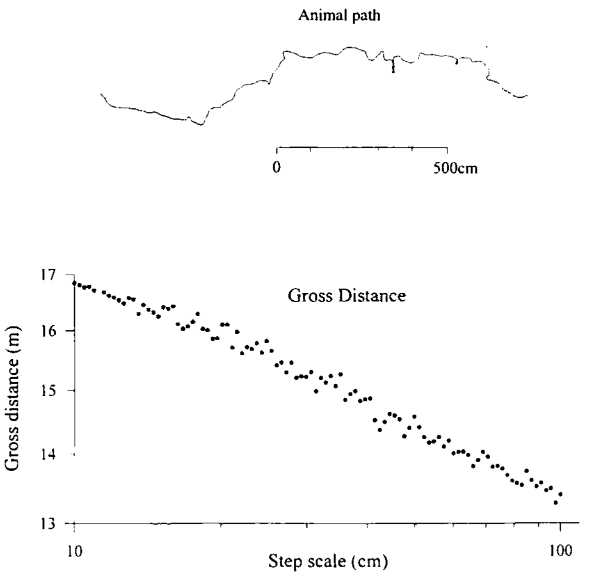

Fig. 6. (a) A movement path from a male red-backed vole. (b) Plot of log(Gross Distance) vs log(step size). 100 intervals of step size were used in the plot.

allow one to decide whether fractal d is constant or varying for each individual animal.

5. An example

At what scale do red-backed voles (*Clethrionomys gapperi*) view habitats? To answer this I followed the movements of voles near Truro, Nova Scotia, Canada. These mouse-sized herbivorous mammals live in the boreal forest, with home ranges of about 50 × 50 m^2. They live on the forest floor and in burrows just under the forest floor.

The sample data comes from an adult male vole (25 grams) that was tracked by following a thread spool attached to its back (Boonstra and Craine 1986). As the thread plays out, it snags in grass and other vegetation on the forest floor, thus retaining the shape of the movement path. The track was 50 m long and data points were recorded with an accuracy of ± 5 cm (Fig. 6a).

Overall, the fractal dimension is 1.11, indicating quite a straight path. Fractal d is calculated from 1 minus the slope of the plot of log(Gross Distance) vs log(step size) (Fig. 6b). This slope seems to be linear, suggesting that fractal d is constant with scale. However, when we estimate d at different

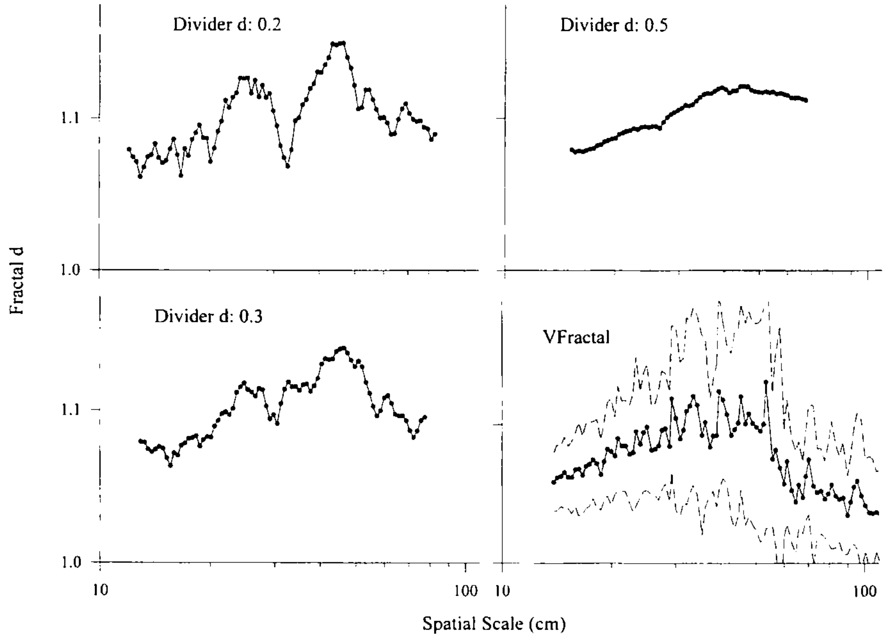

Fig. 7. Fractal dimensions from the movement path of the vole in Fig. 6. (a) Fractal d estimated by the divider method in a range of ± 0.20 of the scale. (b) Fractal d estimated in a range of ± 0.30 of the scale. (c) Fractal d estimated in a range of ± 0.50 of the scale. (d) Fractal d estimated using the NetCos VFractal estimator. Dashed lines represent 95% confidence intervals.

spatial scales, we find some variation. I estimated fractal d using the divider method and three different interval ranges in step size: 0.2, 0.3 and 0.5 (Fig. 7a–c). For example, a range of 0.2 means that at each scale, the interval used to estimate fractal d was scale × (1–0.2) to $\frac{\text{scale}}{1-0.2}$ (a linear interval in the log scale). The wider the interval range in step size, the smoother the plot of fractal d vs scale. However, it is difficult to know by looking at these plots which of the fluctuations in d are real and which are due to sampling variation. Thus we next use the VFractal estimator, which includes error estimates (Fig. 7d). The 95% confidence intervals of the VFractal show that most of the fluctuations in d are not real. The real patterns are the gradual increase in d until 50 cm, then a decrease to a low d.

To interpret these results biologically, consider first that the increase in d from 10 to 50 cm is what is seen in a correlated random walk. At scales larger than 50 cm, the d decreases and remains constant. The decreases means that the path is straighter. The constant d is what is seen in the fractal walk, which is a directed walk, where the animal walks to a certain point but has to avoid occasional obstacles that occur at all scales. Thus this vole is wandering about randomly in patches of size < 50 cm, but on a larger scale is going in a certain direction.

When we now look back to the map of the path (Fig. 6), we can see these results. At a large scale, the path is straight and direct. On a smaller scale, the path is more crooked, suggesting that the animal is searching in patches of about 50 cm in size.

6. Summary

The VFractal and the traditional divider d estimates each have different strengths. The divider d is best for one overall estimate of d if the data come from one continuous movement path. Both estimators can estimate fractal d at different spatial scales, but the VFractal also gives a measure of error. Most importantly, the VFractal can combine data from many separate path segments which have been gathered at various spatial scales.

With these tools we can analyze how animals view their environment at different spatial scales. This should allow us to examine what causes these differences, addressing such questions as, is the topology of the movement path determined by the topology of the habitat, or solely by the behavior of the animal? If by the topology of the habitat, then what habitat features are used by the animal? Do all individuals of a species view the habitat at similar scales?

7. Acknowledgements

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