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The Screen Problem

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The statistical interpretation of the quantum mechanical wave packet contains a gap. The author outlines the problem without offering a solution.

1. INTRODUCTION

It is generally believed that quantum mechanics is backed by firmly established statistical facts which exhaust the probabilistic sense of the "state vectors." This context is most transparent for the Schrödinger's wave packet $\psi = \{\psi(\mathbf{x})\}$, and not less visible for Dirac waves and spin states. Yet, an essential element is missing in the commonly accepted statistical scheme.

The contents of this note correspond to an accidental discussion between Constantin Piron, Göran Lindbladt, and the present author in the Department de Fisique Theorique in Geneve in the summer 1978.

2. THE TEXTBOOK STORY

What I intend to discuss is a dark spot in the perfection of quantum scheme. Whether this spot is cosmetic or profound is up to the reader to decide. Let us recall that the modern (statistical) theory arose to replace the "substantial interpretation" of Schrödinger (of ψ as a physical field). Since then, the wave functions $\psi = \{\psi(\mathbf{x})\}$ have lost any material sense. They became "probability waves," i.e., *mathematical code symbols* com-

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prising our knowledge about the system. The most traditional information to be read from ψ are the *localization probabilities*:

$$p(t, \Omega) = \int_{\Omega} |\psi(\mathbf{x}, t)|^2 \, d_3 x \tag{2.1}$$

meaning the chance that the check of particle position at the time t will find the particle in a space domain $\Omega \subseteq \mathbb{R}^3$. More general rules of decoding the statistical contents[8] of ψ are a direct consequence of (2.1): they are supposed to provide the complete probabilistic information about the particle.

The rules of "reading ψ" would be inconsistent without the axiom about the instantaneous (timeless) reduction of the wave packet. The hypothesis was stated by Dirac, who speaks about the *state jumps*[8]; other authors seem to accept the idea more or less explicitly (see, e.g., J. von Neumann[9]). As the matter of fact, if the particle detection were not associated with an instantaneous reduction, there would be no mechanism preventing a single microparticle from appearing simultaneously on two different screens. In turn, the timeless character of the reduction in the EPR experiments is needed to explain the correlation on two extremes of the EPR arrangement.[10–12] As a result, the concepts of "timeless measurement" and "instantaneous reduction" have become fundamental bricks of quantum theory.

The resulting scheme has some virtues of economy (no empty talk, no conceptual models beyond the operationally verified facts, etc.). It is therefore easy to forget that it, too, faces some challenges, at least as serious as the material doctrine of Schrödinger. They include the paradox of the Schrödinger's cat,[1] the relativistic paradox of Aharonov–Albert,[2–4] and the effect of Zeno.[5–7] Curiously, while the difficulties of the "substantial interpretation" were constantly stressed, the paradoxes of the statistical approach were as constantly overlooked. Generations of physicists have grown accustomed to keep them somewhere in the back of their minds, in a shadowy area of *no thoughts* (or at least no reportable thoughts!). It might thus be worth noticing that the scheme is not so operational as it looks. To the contrary, it balances on the threshold of fiction.

3. PHENOMENOLOGY OF UNPERFORMED EXPERIMENTS

The experimentalists perform a lot of measurements, but none resembling an "instantaneous check of particle position." Trying to design such an experiment, one must come to a rather unreal picture of a sudden particle search (similar to a police race) carried out at a given instant of time, all over a certain simultaneity hyperplane in Galileo (or Minkowski) space-time. The physical models are obscure. Should it be a preprogrammed action of an infinity of detectors, switched on at a given instant of time all over the space?

A method to organize such a search might be to keep the particle in a pot filled with a physical medium (such as, e.g., a Wilson camera, a bubble chamber, etc.) which in a given instant undergoes a phase transition, becoming sensitive to the particle presence. This "phase of awareness" should awake simultaneously all over a certain 3-dimensional space domain, producing a visible spot at the point where the particle is detected. Such a process is basically possible in the nonrelativistic theory, but it would fail in the relativistic case. Indeed, in Minkowski space-time, the internal transformations of a physical medium (caused, e.g., by cooling or heating the pot, etc.), propagate with finite velocity. The particle trapped into the "phase of awareness" will therefore be detected on a characteristic surface marking the progress of the phase transition. Now, except for special situations,(13) the characteristic surfaces are contained inside of the light cones or, at best, can be null (isotropic) surfaces. This might mean that the correct statistical interpretation of the relativistic quantum mechanics should be formulated not for the space-like but rather for isotropic or piecewise isotropic surfaces.(14) Indeed, hints pointing to this direction can be seen in the structure of the relativistic wave equations.(15-17)

Our critique does not mean that the quantities (2.1) have no physical sense. They can certainly express themselves through some remote statistical effects [for instance, if two space-separated parts of a wave packet lead to two excluding physical phenomena far away, the quantities (2.1) then become the respective probabilities]. Yet it is a fact that they are not measured in the known localization experiments. What is measured is something entirely different.

4. THE WAITING SCREEN

The typical measuring apparatus consists of the particle source, some obstacles or fields, and a screen (detector). The particle in an initial state ψ evolves under the influence of the fields (or diffracts on material bodies) and then hits the screen (falls into the detector). The whole scenario (repeated in almost all quantum experiments) has rather little to do with an "instantaneous position check." The main difference is that the screen (detector) does not act suddenly, at the experimenter's wish. Instead, it is just waiting. It is the microparticle, and not the experimenter, which triggers the apparatus to act (and so, in a sense, the wave packet produces its own collapse!)

If one skips all more profound differences, this corresponds to a new space-time scenario, as it seems, carefully avoided in the orthodox descriptions of quantum measurements. Suppose, for simplicity, that the detector is just a flat screen placed at $z = b = \text{const}$. Its 4-dimensional image then is not a "horizontal" spacelike hyperplane $t = \text{const}$ in Galileo or Minkowski space-time, but a "vertical" hyperplane labeled by two space and one time coordinates x, y, t. The particle is thus localized on such a vertical plane. Suppose, now, that an ensemble of particles in a pure state ψ has been prepared on one side of the screen (i.e., $\psi(\mathbf{x}, 0) \equiv 0$ for $z < b$). They propagate in $z < b$ till each of them will (or will not) mark a little spark of absorption. What is the probability density $\rho(x, y, t)$ that the particle prepared in the pure state $\psi(\mathbf{x}, 0)$ will hit the screen at (x, y, b, t)? See Fig. 1.

Curiously, while investing so much care into the formulation of the measurement axioms for a ficticious "instantaneous experiment," quantum mechanics tells nothing about the real measurement of Fig. 1. The only exception is Born's formula for a stationary beam; however, this is precisely the uninteresting part of the story. What picture shall we see in Fig. 1 in the nontrivial case, i.e., for an arbitrary, normalized but nonstationary ψ?

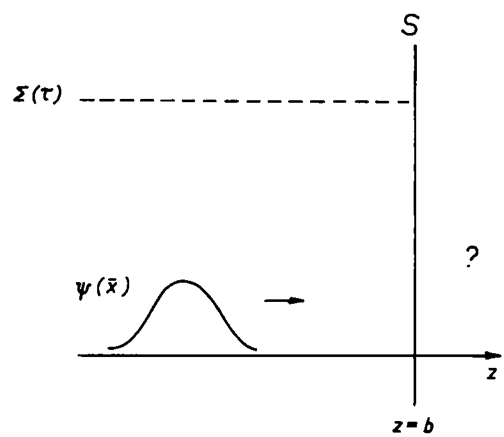

Fig. 1. The missing element of the statistical interpretation: for a normalized wave packet $\psi(\mathbf{x}, 0)$ one ignores the probability of absorption on the surface of a waiting screen. The time coordinate of the event of absorption is not even statistically defined.

5. INADEQUACY OF CURRENT FORMULA

A suggestive idea is to extend the current expression of Born to a nonstationary case. At first sight this might seen fairly easy due to the vanishing of the 4-divergence

$$\partial\rho/\partial t + \nabla \mathbf{j} \equiv 0 \tag{5.1}$$

where $\rho = \psi^+\psi$ is the probability density and $\mathbf{j}$ the corresponding current for the Schrödinger (or Dirac) wave function. Considering the initial packet $\psi(\mathbf{x}, 0)$ with a compact support $\Omega < z$ (left of the screen) and denoting by $\Sigma(t)$ ($t \in \mathbb{R}$) the family of horizontal 1/2-hyperplanes $\Sigma(t) = \{(\mathbf{x}, t)\colon x, y \in \mathbb{R},\ z < b,\ t = \text{const}\}$ and by $S(t)$ the section of the vertical plane $z = b$ between $\Sigma(0)$ and $\Sigma(t)$, one has

$$\int_{\Sigma(t)} \psi(\mathbf{x}, t)^+ \, \psi(\mathbf{x}, t) \, d_3x + \int_{S(t)} \mathbf{jn} \, dx \, dy \, dt = \int_{\Sigma(0)} \psi(\mathbf{x}, 0)^+ \, \psi(\mathbf{x}, 0) = 1 \tag{5.2}$$

i.e., what is lost of the probability on $\Sigma(t)$ (= the probability that the particle escaped detection till the time moment t) is recovered by integrating the current on the vertical section $S(t)$. This might motivate a guess which seems to reflect a widespread subconscious belief:

Hypothesis 1. Let $\psi(\mathbf{x}) = \psi(\mathbf{x}, 0)$ be a Schrödinger's (or Dirac's) wave packet localized left of the screen (i.e., $\psi(x, y, z, 0) \equiv 0$ for $z \geqslant b$). Let then $\psi(\mathbf{x}, t)$ be the result of the free propagation of $\psi(\mathbf{x}, 0)$, i.e.,

$$\psi(\mathbf{x}, t) = e^{-iH_0 t}\psi(\mathbf{x}, 0) \tag{5.3}$$

where H_0 is the free Hamiltonian of Schrödinger (or Dirac). Then $|\psi(\mathbf{x}, t)|^2$ (for $z < b$) gives the probability density for the particle localization left of the screen at the time t, and, simultaneously, the probability current j_z of the wave (5.3) for $z = b$ defines the probability density for the particle absorption on the vertical plane.

Since the traditional expression (2.1) enjoys some universality (i.e., holds not only in vacuum but also for wave packets propagating in the presence of any fields or obstacles), one might also venture to formulate the more general:

Hypothesis II. To determine the detection probability for a microparticle on the surface of a waiting screen of any form and in the presence of any external fields, one has to determine the wave function $\psi(\mathbf{x}, t)$ propagating under the influence of these fields; the corresponding current

$\mathbf{j}(\mathbf{x}, t)$ then determines the probability density for the particle detection on the 3-dimensional screen surface S in either Galileo or Minkowski space-time.

Unfortunately, Hypothesis II turns out to be wrong! This is immediately seen if the screen is not flat. Suppose, for example, the screen is cylindrical (Fig. 2). Then, for a freely propagating wave packet, the currents $\mathbf{j}(\mathbf{x}, t)$, in general, penetrate the cylinder in both the in and the out directions. The normal component $\mathbf{nj}$ is negative on some parts of the surface (Fig. 2), and so it cannot define the probability. Hypothesis II simply neglects the fact that the microparticle, if initially outside, never penetrates into the cylinder. Thus, the returning (negative) current $\mathbf{j}(\mathbf{x}, t)$ of freely propagating $\psi(\mathbf{x}, t)$ is unphysical. However, it would not help to take just the positive part of $\mathbf{nj}$, as this would destroy the probability conservation.

The same difficulty arises if there is a field (or an obstacle) behind the screen. The wave packet (5.3) calculated ignoring the screen must then contain an unphysical part refracted by the obstacle and returning from behind the screen with negative $\mathbf{nj}$. The difficulty again is impossible to eliminate by any *ad hoc* assumption, such as for example, taking only the positive part of $\mathbf{nj}$.

One might hope that the difficulty vanishes for flat and concave screens (which permit no "self-screening" of the screen surface). Unfortunately, the trouble persists even for flat screens and even in the absence of fields. This fact was first noticed by Kijowski.[18] His observation concerns the Fourier components, but can be easily reformulated to describe the configuration of Fig. 1. One has:

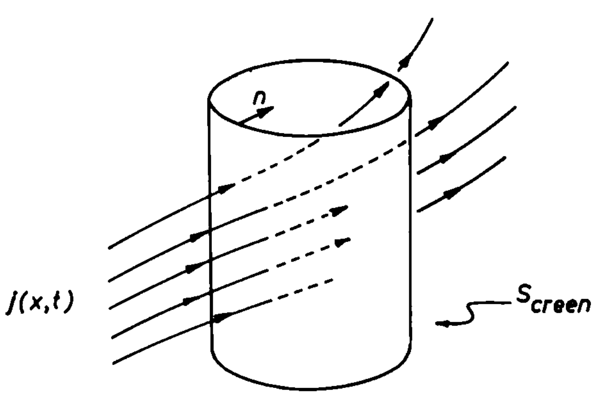

Fig. 2. The currents of a freely propagating wave packet cannot determine the probability of absorption on the cylindrical screen.

Lemma. For any open, bounded domain $\Omega \subset \{\mathbf{x} \in \mathbb{R}^3: z < b\}$, situated "to the left" of the hyperplane $S = \{\mathbf{x}: z = b\}$ and for any $\tau > 0$ there exists a free Schrödinger's wave packet $\psi(\mathbf{x}, t)$ with the initial support in Ω (i.e., $\psi(\mathbf{x}, 0) = 0$ for $\mathbf{x} \neq \Omega$) which at $t = \tau$ produces the probability currents $\mathbf{j}$ crossing S in the negative z-direction [i.e., with $j_z(x, y, b, \tau) < 0$].

Proof. Let Γ be a surrounding on the x, y-plane and (β_1, β_2) an open interval, $\beta_1 < \beta_2 < b$, such that $\Gamma \otimes (\beta_1, \beta_2) \subset \Omega$. Consider a Schrödinger's wave packet $\psi(\mathbf{x}, t) = K(x, y, t)\, \Psi(z, t)$, where (1) $K(x, y, 0)$ has a compact support in Γ and $K(x, y, t)$ obeys the free evolution equation in $L^2(\mathbb{R}^2)$; (2) $\Psi(z, 0)$ has a compact support in (β_1, β_2) and $\Psi(z, t)$ obeys the free evolution equation in $L^2(\mathbb{R})$. We shall show that $\Psi(z, t)$ can generate a $j_z < 0$ at $z = b > \beta_2 > \beta_1$ and $t = \tau$. Without lose of generality, we may put $\tau = 1$. The integral form of the Schrödinger's evolution operator in 1 dimension implies

$$\Psi(z, 1) = \int_{\beta_1}^{\beta_2} \psi(\beta, 0)\, e^{i(z-\beta)^2/2}\, d\beta = e^{iz^2/2} f(z) \tag{5.4}$$

where

$$f(z) = \int_{\beta_1}^{\beta_2} \Phi(\beta)\, e^{-i\beta z}\, d\beta \tag{5.5}$$

and

$$\Phi(\beta) = \psi(\beta, 0)\, e^{-i\beta^2/2} \tag{5.6}$$

where we have taken $m = \hbar = 1$. Since $\psi(z, 0)$ is an arbitrary function with a support in (β_1, β_2), so is $\Phi(z)$ in (5.5)–(5.6). Let now

$$f(z) = R(z)\, e^{i\alpha(z)} \tag{5.7}$$

where $R(z) \geqslant 0$ and $\alpha(z) \in \mathbb{R}$. The current component orthogonal to the screen is

$$\begin{aligned} j_z(b, 1) &= (1/2i)[\psi^*(\partial/\partial z)\,\psi - \psi(\partial/\partial z)\,\psi^*] \\ &= |K(x, y, 1)|^2\, R(b)^2 [\alpha'(b) + b] \end{aligned} \tag{5.8}$$

and its sign depends on $\alpha'(b) + b$, where $\alpha'(z) = d\alpha/dz$ measures the "velocity of phase variations" of (3.5)–(3.7). Now, putting $\Phi(\beta) = g(\beta) \exp(i\beta b)$ with $g(\beta)$ real, one has

$$f(z) = \int_{\beta_1}^{\beta_2} g(\beta)\, e^{-i\beta(z-b)}\, d\beta \tag{5.9}$$

and therefore

$$\alpha'(b) = (1/2i)[f'(b)/f(b) - f'(b)^*/f(b)^*]$$
$$= -\frac{\int_{\beta_1}^{\beta_2} \beta g(\beta)\, d\beta}{\int_{\beta_1}^{\beta_2} g(\beta)\, d\beta} \tag{5.10}$$

The numerator and denominator of (5.10) are two independent mathematical moments of $g(\beta)$ in (β_1, β_2). Choosing now a sign changing $g(\beta)$ with a small, positive integral (denominator) but finite positive first moment (nominator), one gets $\alpha'(b)$ negative, of arbitrarily great absolute value, giving $\alpha'(b) + b < 0 \Rightarrow j_z(x, y, 1) < 0$. □

The above lemma means that there must be an error in Hypothesis I. For a nonstationary $\psi(\mathbf{x}, t)$ the currents $\mathbf{j}(\mathbf{x}, t)$ do not provide the probabilities of detection even on the flat screens! Worse, this simultaneously means that the integral

$$\int_{z<b} |\psi(\mathbf{x}, t)|^2\, d_3x$$

is not a decreasing function of t, and so it cannot provide the probability that the particle remains undetected somewhere left of the screen till the time moment $t > 0$.

6. BLIND ALLEY OF THE ZENO PARADOX

The trouble indicates that the measurement of Fig. 1 is not reducible to the simple reading of probability currents. The screen does something else. As the packet approaches, its "free evolution cues" (which should penetrate to the other side of the screen), most evidently, are canceled, and cannot contribute to the returning (negative) currents. When trying to formalize the idea, one is tempted to divide the evolution interval $[0, t]$ into a sequence of subintervals $[kt/n, (k+1)\,t/n]$, $(k = 1,..., n)$, and then postulate a stepwise process consisting in free propagation incidents and reduction acts. Thus, at the beginning of each time interval $[kt/n, (k+1)\,t/n]$ the wave packet (localized left of the screen) starts to propagate freely, always producing a little cue to the right of the screen. However, the screen is infinitely sensitive: the absence of a visible spot on its surface means the certainty that the particle is still to the left. In agreement with the doctrine that the state can be reduced "without touching the object," by the simple absence of a visible effect (see, e.g., Dicke[19]; more radically Elitzur and Vaidman[20]), this certainty reduces the cue of the packet to nonexistence at $t=(k+1)\,\tau$. An analogous process is repeated in the next time intervals $[t/k+1, t/k+2]$, etc., causing a slow decrease of the packet norm, until the particle is finally absorbed. As a result of that process, the norm squared of the gradually absorbed packet gives the probability that the particle still escaped detection, while the reduced "free propagation cues" define the absorption probabilities in the subsequent time intervals $t/k+1$, $t/k+2$, etc. (see Fig. 3). The model is promising, but deceptive.

If the screen is infinitely sensitive, it "watches" the particle (or particle absence) all the time, thus performing the reduction permanently and not only in selected time moments. To describe the phenomenon properly, one should take the limit $n \to +\infty$. Then, however, one faces the mechanism of the Zeno paradox[5-7] (also "watched pot effect"): when $n \to +\infty$, the norms squared of the "lost cues" tend to zero as quickly as $o(1/n^2)$ and the total probability of the particle absorption in $[0, t]$ vanishes as $o(1/n)$. The process, in the limit, is described by the unitary operator

$$e^{-iPH_0P} \tag{6.1}$$

(where P is the projector onto the 1/2-space $z<b$), generating a norm-conserving evolution of the packet inside of the space domain $z<b$. One is thus led to a surrealistic conclusion that the particle can never be absorbed by the screen!

By contemplating the argument, we could find in it a singular kind of beauty. In reality, there are no infinitely sensitive and dense screens! Each

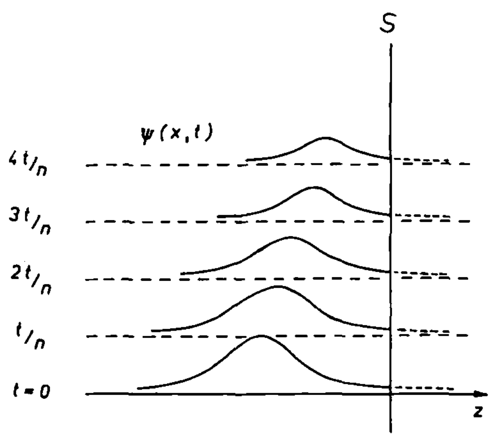

Fig. 3. A frustrated attempt at representing the particle detection (absorption) on the screen as a gradual collapse of the wave packet.

screen has a certain finite depth, density, and "awareness": it leaves some small but nonvanishing probability that the particle, instead of being detected, may penetrate into it or even sneak out through it, due to the mechanism of the tunnel effect. Could it be precisely due to this imperfection that the screen can act at all? Perhaps, a perfect screen, if it existed, could not work: its unlimited sensitivity would cause the total wave packet reflection instead of the detection.

The argument seems profound, but is deceptive again. In fact, if there is a link between the screen sensitivity and the (undesired) ability to refract particles, this link should manifest itself in the most conventional scattering experiments, where Born's approximation holds. It should affect the pictures on too sensitive screens (e.g., preventing too good photographic plates to catch pictures!). Northing like that happens: as far as we know, there is no screen sensitivity parameter which would play this kind of destructive role. It is also worth noticing that the certainty of the detection (i.e., the "perfection" of the screen) depends not only on the quality of the screen itself, but also on the type of the microobject. It must increase together with the object mass and charge. (While the neutrinos quite easily penetrate the best screens, the heavy ions or molecules have no such chance). Thus, if the certainty of the detection is what enforces the "Zeno effect" for an electron, the same mechanism should be much more efficient for heavier molecules, or semimacroscopic bodies, preventing them from hitting screens. If such a mechanism existed, in the classical limit it should also prevent the cannon balls from hitting the walls, which is unfortunately not the case. See Fig. 4

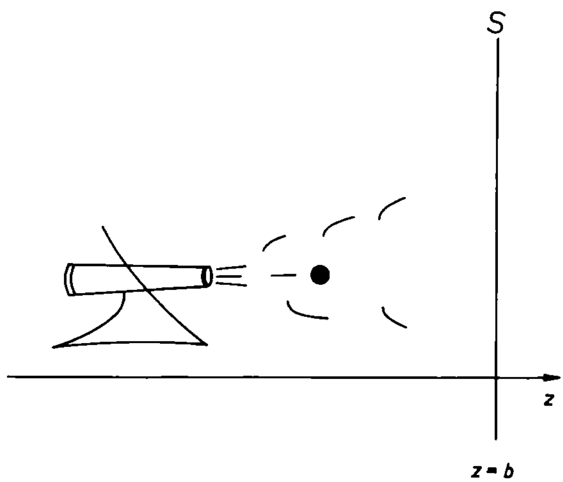

Fig. 4. Extreme consequence of the Zeno effect: the macroscopic particle is unable to hit an infinitely sensitive screen.

Our last argument is visibly unfair. One might answer that for sharply localized wave packets of heavy particles, when they approach the screens, the Zeno time[5] is too short to generate the Zeno effect. Yet, the trouble might be deeper than that. There is something evidently unreal in the whole reduction axiom, including its supposed instantaneity, as well as the Dicke doctrine of *reduction in absence*" (reduction without interaction). As a matter of fact, one feels that the activity (or rather inactivity) of the screen has very little to do with a "dense sequence of instantaneous measurements" necessary to produce the Zeno effect (see Peres[21]). What happens looks rather as if the *micro*object and not the *apparatus* caused the measurement. Before this happens, the screen does nothing, knows nothing, and has no responsability for the mental processes of the theoretical physicist who invented the reduction axiom to find peace of mind without solving the problem. All this might be fortunate for the existence of pictures on photografic plates and monitor screens, but still gives no hint about the most elementary monitor picture of Fig. 1.

7. ALGORITHM OF KIJOWSKI AND PIRON'S WAVE MECHANICS

The unique numerical guess as to the probability density on the flat screen (Fig. 1) was stated in 1978 by Kijowski.[18] His hypothesis was motivated by some general axioms [but indeed is a free inspiration of the current formula; cf. (5.8)]. Put $m = 1$. For the screen placed at $z = b$, Kijowski considers a free packet $\psi(\mathbf{x}, t)$ composed of plane waves "propagating to the right" (i.e., in the positive p_z-direction):

$$\psi(\mathbf{x}, t) = \int_{p_z > 0} c(\mathbf{p})\, e^{i(\mathbf{px} - Et)}\, d_3 p \tag{7.1}$$

where $E = E(\mathbf{p}) = \mathbf{p}^2/2$ are energies of the Fourier components. A wave of that form belongs to the positive spectral domain of p_z, in which $\sqrt{p_z}$ is a real, positive operator. Now, if one skips axiomatics, the Kijowski prescription reduces to defining a new (freely propagating) wave packet:

$$\phi(\mathbf{x}, t) = \sqrt{p_z}\, \psi(\mathbf{x}, t) = \int_{p_z > 0} \sqrt{p_z}\, c(\mathbf{p})\, e^{i(\mathbf{px} - Et)}\, d_3 p \tag{7.2}$$

One can show that the norm of ϕ is conserved on the "vertical planes" $z = \text{const}$. Moreover, the screen integral of $tj_z(\mathbf{x}, t)$ (with $j_z(\mathbf{x}, t)$ of indefinite sign!) coincides with the t-average calculated for the positive function $|\phi|^2$:

$$\int_{z=b} tj_z(\mathbf{x}, t)\, d_2x\, dt = \int_{z=b} t\, |\phi(\mathbf{x}, t)|^2\, d_2x\, dt \tag{7.3}$$

Kijowski infers that $|\phi(\mathbf{x}, t)|^2$ is the probability density for the particle detection on the "vertical planes." The hypothesis posesses some elegance; one might thus be tempted to solve similarly the problem for the curved screens (i.e., by substituting $\sqrt{p_z}$ in (7.2) by $\sqrt{p\xi}$, where $\xi = \text{const}$ defines the screen surface and $p\xi$ is the corresponding canonical momentum). However, the idea backfires.

Indeed, suppose the hypothesis is correct. The integral

$$\int_{S(\tau)} |\phi(\mathbf{x}, t)|^2\, dx\, dy\, dt \tag{7.4}$$

(i.e., the probability of the particle detection on the screen for $t \leqslant \tau$) is (and should be) a monotonic, nondecreasing function of τ. Henceforth, the complementing probability that for $t = \tau$ the particle is still undetected (left of the screen) must be monotonic, nonincreasing. Unfortunately, the negative $j_z(\mathbf{x}, t)$ means that the integral

$$\int_{z \leqslant b} |\psi(\mathbf{x}, \tau)|^2\, d_3x \tag{7.5}$$

is not a monotonic (decreasing) function of τ, and henceforth it cannot represent the probability of the particle survival (nonabsorption) until $t = \tau$. It seems like a specific sarcasm of the scheme: now, when we have almost found the vertical probability, we have lost the horizontal one!

The trouble does not end here. Indeed, if the integral (7.5) is not the detection probability on $\Sigma(\tau)$, then $\psi(\mathbf{x}, \tau)$ is not the correct particle state at $t = \tau$ (it cannot, since it has no correct statistical interpretation!). If so, then why in the first place should it be taken as a starting point to determine the vertical amplitude $\phi(\mathbf{x}, t)$?

Our critique is methodological and does not yet prove that the formula (7.2) is wrong. However, some other attributes make it rather unlikely. In the first place, the construction seems handicapped by "thinking in terms" of Fourier transforms, inconvenient in the configuration space. Note that the wave packet composed only of the positive p_z part has no compact support; in general, it extends on both sides of the screen. Hence, it is rather arbitrary to insist that it "propagates toward the screen" (and not, for example, that it "escapes from the screen" toward the positive z's!).

The physically interesting, continuous packet $\psi(\mathbf{x})$ with a compact support $\Omega < b$ is never "propagating forward." Its Fourier transform

$$c(\mathbf{p}) = \int_\Omega \psi(\mathbf{x})\, e^{-i\mathbf{px}}\, d_3x$$

$$= \sum_{n=0}^{\infty} \frac{(-i\mathbf{p}_z)^n}{n!} \int_\Omega \psi(x, y, z)\, z^n\, dx\, dy\, dz \tag{7.6}$$

is an analytic function of p_z with an infinite radius of convergence. Henceforth, $c(\mathbf{p})$ cannot vanish just for p_z negative, except if $c(\mathbf{p}) \equiv 0$. Applied "to the letter," the Kijowski construction thus gives no hint about the experiment of Fig. 1 (i.e., for the initial packet localized on one side of the screen). To overcome the difficulty, one might try to decompose an arbitrary $\psi(\mathbf{x}, t)$ into the positive and negative p_z parts:

$$\psi^{(\pm)}(\mathbf{x}) = \int_{\mathbb{R}^3} \vartheta(\pm p_z)\, c(\mathbf{p})\, e^{i\mathbf{px}}\, d_3p \tag{7.7}$$

where $\vartheta(\xi) = (1 + \operatorname{sgn} \xi)/2$, and extend the algorithm, using $\psi^{(+)}(\mathbf{x})$ to define the "vertical amplitude"[8]:

$$\phi(\mathbf{x}, t) = \sqrt{p_z}\, \psi^{(+)}(\mathbf{x}, t) \tag{7.8}$$

Then, however, $\phi(\mathbf{x}, 0)$ does not share the compact support of $\psi(\mathbf{x}, 0)$. In particular, taking

$$\psi(\mathbf{x}) = f(x, y)\, g(z) \tag{7.9}$$

with $g(z) \equiv 0$ for $z \geqslant b$, one has

$$\phi(x, y, z, 0) = f(x, y)\, G(z) \tag{7.10}$$

where

$$G(z) = \sqrt{p_z}\, g^{(+)}(z) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} \sqrt{p}\, \tilde{g}(p)\, e^{ipz}\, dp \tag{7.11}$$

If now

$$g(z) = g(-z) = g(z)^* \Rightarrow \tilde{g}(p) = \tilde{g}(-p) = \tilde{g}(p)^* \Rightarrow G(-z) = G(z)^* \tag{7.12}$$

then $G(z)$ must be nonzero for sequencies of z-values tending to both $\pm\infty$. Choosing now a point $(x, y, z = b, t = 0)$ with $G(b) \neq 0$, sufficiently far away from the support Ω of ψ, one arrives at a strange conclusion that the initial wave packet $\psi(\mathbf{x}, 0)$ localized in $\Omega < b$ has a nonzero probability density to be captured on an (arbitrarily distant) screen $z = b$. This occurs at the very moment $t = 0$ when $\int_{\Omega} |\psi(\mathbf{x}, 0)|^2 \, dx = 1$, i.e., when the particle is certainly and totally in Ω! The conclusion is questionable even in Schrödinger's quantum mechanics (the localized probability with an instantaneous capacity to appear on arbitrarily distant screens!). Its analog for the relativistic particle (indeed attempted in Ref. 18) would imply a manifest causality violation (the packet detectable with a nonzero probability outside of the future light cone determined by its support at $t = 0$; see Fig. 5).

It thus seems that the problem is still open. As a matter of fact, this concerns not only the "vertical" probability: for a microparticle propagating in the presence of an absorbing screen, both vertical and horizontal probabilities are missing. A similar problem exists but is approached differently in an alternative solution proposed by Piron.

7.1. Piron's Quantization

In 1978 Constantin Piron proposed a "vertical variant" of Schrödinger's wave mechanics describing a generalized optical bench.[22] The "states" of a particle are defined on a family of vertical planes (given by $z = \text{const}$) and are described by a wave function $\phi(\mathbf{x}, t)$ normalized to unity:

$$\int_{\mathbb{R}^3} |\phi(\mathbf{x}, t)|^2 \, dx \, dy \, dt = 1 \tag{7.13}$$

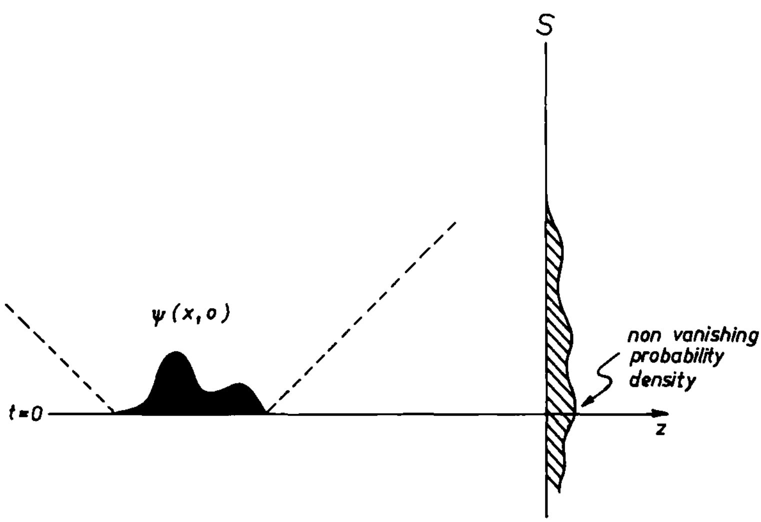

Fig. 5. The algorithm of Kijowski leads to a nonzero detection probability on arbitrarily distant screens. Note the causality problem in the relativistic case.

The concept of the "evolution" now refers to the wave packet changes from one vertical plane to another (so, it might be called the "z-evolution" instead of the "time evolution"). Piron assumes additionally that the wave packet $\phi(\mathbf{x}, t)$ is composed of the Broglie waves propagating in the positive z-direction and postulates the evolution equation

$$-i\,\partial\phi/\partial z = \mathscr{H}\phi \tag{7.14}$$

with the "vertical Hamiltonian":

$$\mathscr{H} = [2mi\partial/\partial t - \partial^2/\partial x^2 - \partial^2/\partial y^2]^{1/2} \tag{7.15}$$

The evolution (7.14) conserves the "vertical normalization" (7.13); moreover, ϕ fulfills the traditional Schrödinger's equation. It is Piron's guess that its square $|\phi(\mathbf{x}, t)|^2$ defines the probability density for the particle localization on the family of waiting screens $z = \text{const}$. Although Eq. (7.14) was already known,[18] the very idea emanates a "spirit of rebellion." Piron desists to specify the interrelation between $\phi(\mathbf{x}, t)$ and the traditional wave packet $\psi(\mathbf{x}, t)$, and he offers a justification of some philosophical elegance. In the traditional (instantaneous) measurements, says Piron, the *time* of the measurement is strictly controlled; henceforth, it becomes a superselected variable (parameter). If, however, the particle is to be absorbed on a waiting screen, the time t of the event is uncertain, whereas z is strictly controlled (superselected). The familiar scheme of quantum mechanics for such a particle fails, and the quantization procedure must be carried out from the very beginning: z is now an "evolution parameter," while t and $i\partial/\partial t$ are the genuine quantum mechanical observables. Both pictures, according to Piron, are incompatible. Piron's theory permits one therefore to construct the probability amplitude for the screen at $z = b$ if one knows the similar probability amplitude on another vertical screen (e.g., on $z = 0$), but gives no hint how to determine $\phi(x, y, b, t)$ in terms of $\psi(x, y, z, 0)$. As a structural proposal, Piron's doctrine is close to atypical quantum schemes[23] and/or theories where the superselection rules assume an unconventional form (anticipating the Aharonov–Albert idea of quantization on a family of monitoring hyperplanes.[2] Comparable also with Sudarshan's systems,[24] where z is an observable while $-i\partial/\partial z$ is not). Should the quantum theory take further steps in this direction, Piron's *Nouveau principe d'évolution*[22] might mark one of its turning points.

However, what about our original problem of Fig. 4? Despite all efforts, the solution, apparently, is not yet at hand.

In some interpretational schools one hears that quantum mechanics, for some fundamental reasons, is inadequate to answer certain questions, as, for example, involving the time duration and time control of quantum measurements. One even derives some sort of "austere satisfaction" from the fact that some answers are denied (for a number of reasons, e.g., the questions cannot be *well stated*, the experiments *properly programmed*, or else, the underlying thoughts are *no thoughts*, etc.). It seems, however, that quantum theory is too constrained by a variety of "don't think principles" (from the diplomacies of the Copenhagen school to "Hawking's gun"[(25)] reducing to nonexistence wide classes of problems. Moreover, this time at least, the method fails. The question is quite well stated: the microparticle can exist in the presence of an absorbing screen in a similar way as it can exist in the presence of an external potential. The experiment can be easily designed, and the collision times of the pure ensemble particles (for an arbitrary initial wave function) can be registered. They will certainly show some statistical pattern. One of the crucial statements of quantum mechanics is that the state vector contains complete noncontradictory information about the system.[(8)] So, where is the information about the results of the experiment represented in Fig. 1?

NOTE ADDED IN PROOF

The argument of Sect. 6 illustrated by Fig. 3 has been very nicely discussed by L. E. Ballentine [*Found. Phys.* **20**, 1329 (1990)].

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