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Describing the Macroscopic World: Closing the Circle within the Dynamical Reduction Program

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With reference to recently proposed theoretical models accounting for reduction in terms of a unified dynamics governing all physical processes, we analyze the problem of working out a worldview accommodating our knowledge about natural phenomena. We stress the relevant conceptual differences between the considered models and standard quantum mechanics. In spite of the fact that both theories describe systems within a genuine Hilbert space framework, the peculiar features of the spontaneous reduction models limit drastically the states which are dynamically stable. This fact by itself allows one to work out an interpretation of the formalism which makes it possible to give a satisfactory description of the world in terms of the values taken by an appropriately defined mass density function in ordinary configuration space. A topology based on this function and which is radically different from the one characterizing the Hilbert space is introduced, and in terms of it the idea of similarity of macroscopic situations is precisely defined. Finally, the formalism and the interpretation are shown to yield a natural criterion for establishing the psychophysical parallelism. The conclusion is that, within the considered theories and at the nonrelativistic level, one can satisfy all sensible requirements for a completely satisfactory macro-objective description of reality.

1. INTRODUCTION

A quite natural question which all scientists who are concerned about the meaning and the value of science have to face is whether one can elaborate a worldview which can accommodate our knowledge about natural phenomena. Such a program has been appropriately denoted by

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A. Shimony[1] as *closing the circle*. As is well known, this desideratum does not raise particular problems within classical physics for various reasons which have been lucidly pointed out by J. Bell:[2]

> Of course it is true that also in classical mechanics any isolation of a particular system from the world as a whole involves approximations, but at least one can envisage an accurate theory of the universe, to which the restricted account is an approximation,

and moreover

> ... even a human observer is no trouble (in principle) in classical theory—it can be included in the system (in a schematic way) by postulating a psychophysical parallelism—i.e., by supposing his experience to be correlated with some function of the coordinates.

The situation is quite different in quantum mechanics, due to the specific peculiarities of the formalism. In particular, the linear nature of the Hilbert space description of the states of physical systems gives rise to well-known difficulties with macroscopic objects. The theory allows linear superpositions of macroscopically different states which render problematic to attribute definite properties to the systems they describe. In particular, as is well known, this situation occurs in measurement-like processes in which, after the system–apparatus interaction is over, one has, in general, a linear superposition of macroscopically distinguishable apparatus states.

Various solutions to this puzzling situation have been proposed; for our purposes it turns out not to be relevant to discuss their specific features and/or to comment on their pros and cons. What has to be pointed out is that now everybody agrees that one needs a reinterpretation or a modification of the formalism (such as breaking the von Neumann chain, introducing hidden variables, limiting measurability, or modifying the evolution law) which does not appreciably alter quantum predictions for microsystems but implies or makes it legitimate to state that macro-objects have definite macroproperties.

All these attempts attribute to positions a privileged role in the description of the macroscopic world.[4] This is quite natural since the definiteness, the particularity of the world of our experience, derives from our perceiving physical objects in definite places, and this is also why the prescriptions for establishing the psychophysical correspondence usually involve positions.

[4] Obviously, a satisfactory description of the macro-world also requires taking into account how positions change with time.

This paper is devoted to show how, within the context of the recently introduced nonrelativistic models[3,4] of spontaneous dynamical reduction, one can give a consistent description of the universe satisfying all previous requirements in terms of mass density in ordinary space.

In Section 2 we briefly sketch CSL, the most elaborate nonrelativistic model of dynamical reduction worked out till now.[4] We also discuss how this model, while yielding a solution to the measurement problem, requires one to consider a question which has been raised in Refs. 5, 6 and which will be referred to as *the problem of the tails* of the wavefunction. This alleged difficulty will be shown to find a natural solution when one adopts the point of view which we are going to propose in the paper.

Sections 3 and 4, which in a sense represent the core of the paper, deal with a reinterpretation of the wavefunction allowing one to describe the macroscopic world in terms of an objective mass density in ordinary space.

Section 5 is devoted to sketching a possible way of establishing the psychophysical correspondence and to prove its consistency.

2. A CONCISE REVIEW OF DYNAMICAL REDUCTION MODELS

Models have recently been developed which, by considering nonlinear and stochastic modifications of Schrödinger's dynamics, imply, without entailing any violation of established experimental facts, wave packet reduction with fixed pointer positions in measurement processes and, more generally, forbid the persistence of linear superpositions of macroscopically distinguishable states.[3,4]

The first model of this kind, QMSL, is based on the assumption that, besides the standard evolution, physical systems are subjected to spontaneous localization occurring at random times and affecting their elementary constituents. Such processes, which we will call "hittings," are formally described in the following way. When the ith constituent of the system suffers a hitting, the wave function changes according to

$$\begin{aligned} \Psi(\mathbf{r}_1,..., \mathbf{r}_N) &\rightarrow \Psi_x(\mathbf{r}_1,..., \mathbf{r}_N) = \Phi_x(\mathbf{r}_1,..., \mathbf{r}_N)/\|\Phi_x\| \\ \Phi_x(\mathbf{r}_1,..., \mathbf{r}_N) &= (\alpha/\pi)^{3/4}\, e^{-(\alpha/2)(\mathbf{r}_i-\mathbf{x})^2}\, \Psi(\mathbf{r}_1,..., \mathbf{r}_N) \end{aligned} \tag{2.1}$$

Such processes occur at randomly distributed times with a mean frequency $\lambda = 10^{-16}\,\text{sec}^{-1}$. The probability density of the process occurring at point $\mathbf{x}$ is given by $\|\Phi_{\mathbf{x}}\|^2$. The localization parameter $1/\sqrt{\alpha}$ is assumed to take the value 10^{-5} cm.

The QMSL mechanism does not respect the symmetry properties of the wave function in the case of identical constituents. Its generalization satisfying such a requirement, the continuous spontaneous localization model (CSL), has been presented and discussed in various papers.[4]

2.1. The CSL Model

The model is based on a linear stochastic evolution equation for the state vector. The evolution does not preserve the norm but only the average value of the square norm. The equation, in the Stratonovich version, is

$$\frac{d\,|\Psi_w(t)\rangle}{dt}=\left[-\frac{i}{\hbar}H+\sum_i A_i w_i(t)-\gamma\sum_i A_i^2\right]|\Psi_w(t)\rangle \tag{2.2}$$

In Eq. (2.2), the quantities A_i are commuting self-adjoint operators, while the quantities $w_i(t)$ are c-number stochastic processes with probability of occurrence satisfying

$$P_{\text{Cook}}[w(t)]=P_{\text{Raw}}[w(t)]\;|\;\||\Psi_w(t)\rangle\|^2 \tag{2.3}$$

In Eq. (2.3) $P_{\text{Raw}}[w(t)]$ is equal to

$$P_{\text{Raw}}[w(t)]=\frac{1}{\mathcal{N}}\,e^{-(1/2\gamma)\sum_i\int_0^t d\tau\, w_i^2(\tau)} \tag{2.4}$$

$\mathcal{N}$ being a normalization factor, i.e., to the probability density of a white noise process satisfying

$$\langle\!\langle w_i(t)\rangle\!\rangle=0,\qquad \langle\!\langle w_i(t)\,w_j(t')\rangle\!\rangle=\gamma\delta_{ij}\delta(t-t') \tag{2.5}$$

To clarify the physical meaning of the model, let us assume, for the moment, that the operators A_i have a purely discrete spectrum and let us denote by M_σ their common eigenmanifolds and by P_σ the associated projection operators.

Then we make the following precise assumption: if a homogeneous ensemble (pure case) at the initial time $t=0$ is associated to the state vector $|\Psi(0)\rangle$, then the ensemble at time t is the union of homogeneous ensembles associated with the normalized vectors $|\Psi_w(t)\rangle/\||\Psi_w(t)\rangle\|$, where $|\Psi_w(t)\rangle$ is the solution of Eq. (2.2) with the assigned initial conditions and for the specific stochastic process $w(\tau)$ which occurred in the interval $(0, t)$. The probability density for such a subensemble is that given by Eq. (2.3).

One can prove[4] that the map from the initial ensemble to the final ensemble obeys the forward time translation semigroup composition law. It is also easy to prove that the evolution, at the ensemble level, is governed by the dynamical equation for the statistical operator

$$\frac{d\rho(t)}{dt} = -\frac{i}{\hbar}[H, \rho(t)] + \gamma \sum_i A_i \rho(t) A_i - \frac{\gamma}{2}\left\{\sum_i A_i^2, \rho(t)\right\} \tag{2.6}$$

from which one immediately sees that, if one disregards the Hamiltonian evolution, the off-diagonal elements $P_\sigma \rho(t) P_\tau$ $(\sigma \neq \tau)$ of the statistical operator are exponentially damped.

For our concerns, the relevant feature of the dynamical process (2.2) with the prescription (2.3) is that it drives the state vector of each individual member of the ensemble into one of the common eigenmanifolds of the operators A_i, with the appropriate probability. To clarify this, we consider[4] a simplified case in which only one operator A appears in Eq. (2.2). The solution of this equation corresponding to the particular initial condition (involving only two eigenmanifolds of A with eigenvalues α, β)

$$|\Psi(0)\rangle = P_\alpha |\Psi(0)\rangle + P_\beta |\Psi(0)\rangle \tag{2.7}$$

when the Hamiltonian is disregarded,[5] is

$$|\Psi_B(t)\rangle = e^{\alpha B(t) - \alpha^2 \gamma t} P_\alpha |\Psi(0)\rangle + e^{\beta B(t) - \beta^2 \gamma t} P_\beta |\Psi(0)\rangle \tag{2.8}$$

Here $B(t)$ is the Brownian process

$$B(t) = \int_0^t d\tau \, w(t) \tag{2.9}$$

Taking into account Eq. (2.8) and the cooking prescription, one gets the cooked probability density for the value $B(t)$ of the Brownian process at time t:

$$P_{\text{Cook}}[B(t)] = \|P_\alpha |\Psi(0)\rangle\|^2 \frac{1}{\sqrt{2\pi\gamma t}} e^{(1/2\gamma t)(B(t) - 2\alpha\gamma t)^2} + \|P_\beta |\Psi(0)\rangle\|^2 \frac{1}{\sqrt{2\pi\gamma t}} e^{(1/2\gamma t)(B(t) - 2\beta\gamma t)^2} \tag{2.10}$$

[5] In Eq. (2.8) and following we have changed the notation for the state vector from the one labeled by the white noise symbol w as in Eq. (2.2) to the one labeled by the Brownian motion symbol B, to stress the fact that, under our assumptions, the state at time t does not depend on the specific sample function $w(\tau)$ in the interval $(0, t)$ but only on its integral, Eq. (2.9).

From (2.10) it is evident that for $t \to \infty$, the process $B(t)$ can assume only values belonging to an interval of width $\sqrt{\gamma t}$ around[6] either the value $2\alpha\gamma t$ or the value $2\beta\gamma t$. The corresponding probabilities are $\|P_\alpha |\Psi(0)\rangle\|^2$ and $\|P_\beta |\Psi(0)\rangle\|^2$, respectively. The occurrence of a value "near" to $2\alpha\gamma t$ for the random variable $B(t)$ leads, according to Eq. (2.8), to a state vector that, for $t \to \infty$, lies in the eigenmanifold corresponding to the eigenvalue α of A. In fact, one gets

$$\frac{\|P_\beta |\Psi_B(t)\rangle\|^2}{\|P_\alpha |\Psi_B(t)\rangle\|^2} \cong e^{-2\gamma t(\alpha-\beta)^2} \frac{\|P_\beta |\Psi(0)\rangle\|^2}{\|P_\alpha |\Psi(0)\rangle\|^2} \xrightarrow[t \to \infty]{} 0 \tag{2.11}$$

Analogously, when the random variable $B(t)$ takes a value "near" to $2\beta\gamma t$, for $t \to \infty$, the state vector is driven into the eigenmanifold corresponding to the eigenvalue β of A.

It is then clear that the model establishes a one-to-one correspondence between the "outcome" (the final "preferred" eigenmanifold into which an individual state vector is driven) and the specific value (among the only ones having an appreciable probability) taken by $B(t)$ for $t \to \infty$, a correspondence irrespective of what $|\Psi(0)\rangle$ is.[7] In the general case of several operators A_i, a similar conclusion holds for the "outcomes" α_i of A_i and the corresponding Brownian processes $B_i(t)$.

This concludes the exposition of the general structure of the CSL model. Obviously, to give a physical content to the theory one must choose the so-called preferred basis, i.e., the eigenmanifolds on which reduction takes place or, equivalently, the set of commuting operators A_i. The specific form that has been presented and shown to possess all the desired features is obtained[(4)] (in accordance with the remarks of Section 1 about the privileged role played by positions) by identifying the discrete index i and the operators A_i of the above formulas with the continuous and discrete indices $(\mathbf{r}, k)$ and the operators

$$N^{(k)}(\mathbf{r}) = \left(\frac{\alpha}{2\pi}\right)^{3/2} \sum_s \int d\mathbf{q}\, e^{-(\alpha/2)(\mathbf{q}-\mathbf{r})^2} a_k^+(\mathbf{q}, s)\, a_k(\mathbf{q}, s) \tag{2.12}$$

Here $a_k^+(\mathbf{q}, s)$ and $a_k(\mathbf{q}, s)$ are the creation and annihilation operators of a particle of type k (e.g., $k =$ electron, proton, ...) at point $\mathbf{q}$ with spin component s, satisfying the canonical commutation or anticommutation

[6] Note that, even though the spread $\sqrt{\gamma t}$ tends to ∞ for $t \to \infty$, its ratio to the distance $2(\alpha - \beta)\gamma t$ between the two considered peaks of the distribution tends to zero.

[7] Obviously $|\Psi(0)\rangle$ enters in a crucial way in determining the probability of occurrence of the Brownian processes $B(t)$.

relations. Correspondingly one has a continuous family of stochastic Gaussian processes satisfying

$$\langle\langle w_k(\mathbf{r}, t)\rangle\rangle = 0, \qquad \langle\langle w_k(\mathbf{r}, t)\, w_j(\mathbf{r}', t')\rangle\rangle = \gamma\delta_{kj}\delta(\mathbf{r}-\mathbf{r}')\,\delta(t-t') \quad (2.13)$$

The parameter α is assumed to take the same value ($10^{10}\,\mathrm{cm}^{-2}$) as in the case of QMSL, while γ is related to the frequency $\lambda = 10^{-16}\,\mathrm{sec}^{-1}$ of that model according to $\gamma = \lambda(4\pi/\alpha)^{3/2}$.

2.2. How Does Dynamical Reduction Work?

Due to the choice of the parameters for QMSL and the corresponding ones for CSL, the considered dynamics has the following nice features:

— In the case of microscopic systems the non-Hamiltonian terms have negligible effects.

— On the contrary, in the macroscopic case the reduction mechanism is extremely effective in suppressing linear superpositions of states in which a macroscopic number of particles are discplaced by more than the characteristic localization length.

This can be easily seen both for QMSL and for CSL. Within QMSL, a localization of any one of the displaced particles yields the suppression of the linear superpositions. Thus, one could roughly describe the situation by stating that the hitting frequency is amplified in proportion to the number of particles.

To discuss the decoherence features of CSL ensuing from the choice (2.12), even though the reduction processes occur at the individual level, one can limit his considerations to the evolution equation for the statistical operator:

$$\frac{d\rho(t)}{dt} = -\frac{i}{h}[H, \rho(t)] + \gamma\sum_k \int d\mathbf{r}\, N^{(k)}(\mathbf{r})\,\rho(t)\,N^{(k)}(\mathbf{r}) - \frac{\gamma}{2}\sum_k \left\{\int d\mathbf{r}\, N^{(k)2}(\mathbf{r}), \rho(t)\right\} \quad (2.14)$$

For simplicity's sake here we will further restrict ourselves to a simplified version of CSL obtained by disregarding the Hamiltonian term and discretizing the space. This allows us to derive in a straightforward way the main consequences which are of interest for the subsequent discussion.

We divide the space into cells of volume $(\alpha/4\pi)^{-3/2}$ and we denote by $N_i^{(k)}$ the number operator counting the particles of type k in the ith cell. As follows from the discussion of the preceding subsection in the considered case, the dynamical evolution drives the state vector into a manifold such that the number of particles present in any cell is definite. The simplified equation for the statistical operator reads

$$\frac{d\rho(t)}{dt} = \gamma\left(\frac{\alpha}{4\pi}\right)^{3/2} \sum_k \left(\sum_i N_i^{(k)}\rho(t)\, N_i^{(k)} - \frac{1}{2}\left\{\sum_i N_i^{(k)2}, \rho(t)\right\}\right) \quad (2.15)$$

In accordance with the relation of Subsection 2.1, we will often use the QMSL frequency parameter λ in place of the expression $\gamma(\alpha/4\pi)^{3/2}$. If we denote by $|n_1^{(k)}, n_2^{(k)},..., n_i^{(k)},...\rangle$ the state with the indicated occupation numbers for the various types of particles and for the various cells, the solution of Eq. (2.15) reads, in the considered basis

$$\begin{aligned}&\langle n_1^{(k)}, n_2^{(k)},... |\rho(t)|\, m_1^{(k)}, m_2^{(k)},...\rangle \\ &\quad = e^{-(\lambda/2)\sum_k \sum_i (n_i^{(k)} - m_i^{(k)})^2 t} \langle n_1^{(k)}, n_2^{(k)},... |\rho(0)|\, m_1^{(k)}, m_2^{(k)},...\rangle \end{aligned} \quad (2.16)$$

Equation (2.16) shows that linear superpositions of states containing different number of particles in the various cells are dynamically reduced to one of the superposed states with an exponential time rate depending on the expression $(\lambda/2) \sum_k \sum_i (n_i^{(k)} - m_i^{(k)})^2$.

The amplification process in going from the microscopic to the macroscopic case and the preferred role assigned to position make it clear how such models overcome the difficulties of quantum measurement theory. In fact in measurement processes one usually assumes that different eigenstates of the measured micro-observable trigger (through the system–apparatus interaction) different displacements of a macroscopic pointer from its "ready" position. The unique dynamical principle of QMSL or CSL leads then,[3,4] in extremely short times, to the dynamical suppression, with the appropriate probability, of one of the terms in the superposition, i.e., to the emergence of an outcome.

2.3. "Outcomes" in Dynamical Reduction Models

For the following analysis and with reference to the fundamental issue of the objectification of properties, i.e., of the dynamical emergence of outcomes, it is important to deepen the discussion of the formal and physical aspects of the theory and to mention some peculiar situations which may occur. To this purpose we confine our considerations to the case in which only two *macroscopically different* outcomes α and β which we identify with the eigenvalues of an operator A can occur. We consider the CSL model with A taking the place of the operators A_i of Eq. (2.2) and we assume that the initial state vector has the form (2.7). As repeatedly remarked, when one disregards the Hamiltonian evolution, the CSL dynamics, for $t \to \infty$, drives the state vector either within the eigenmanifold M_α or within M_β. However, it is important to stress that for any finite time t, no one among the states evolved from the considered initial state vector can be exactly an eigenstate of A. As discussed at the end of the previous subsection, after a characteristic reduction time Δt (defined through $(e^{-2\gamma \Delta t(\alpha-\beta)^2} \ll 1)$, for all values of the Brownian process $B(\Delta t)$ which have an appreciable probability of occurrence (i.e., those for which $B(\Delta t) \approx 2\alpha\gamma \Delta t$ or $B(\Delta t) \approx 2\beta\gamma \Delta t$) the normalized state vector describing an individual system will have a negligible component on one of the two eigenmanifolds. Since one wants outcomes to emerge in the characteristic reduction time Δt, one is compelled to attribute[(7)] to the system, for example, the "definite outcome α" also when $\|P_\alpha |\Psi\rangle\|^2/\| |\Psi\rangle\|^2$ is extremely close but not exactly equal to one.[8] This attitude of attributing to the individual physical system the value α of A even when the state vector describing it has a very tiny component on the eigenmanifold associated to the value β, has been criticized by A. Shimony[(5)] and by D. Z. Albert and B. Loewer[(6)] on the basis of the probabilistic interpretation of the wavefunction. In particular, in Ref. 5 the author claims that, within a dynamical reduction model, *one should not tolerate tails in wavefunctions which are so broad that their different parts can be discriminated by the senses, even if very low probability amplitude is assigned to them.* In the next section we will show how, when the state vector is reinterpreted according to the lines we are going to propose, the problem of the tails will find a natural solution.

We come now to the discussion[(8)] of some peculiar aspects of the theory. The first one derives from the fact that, in principle, also in the macroscopic case it could happen that even for a time larger than Δt no outcome has emerged. In fact, if one considers for the Brownian process $B(\Delta t)$ the value $(\alpha + \beta)\gamma \Delta t$ whose probability density, although very small, is not zero, one can easily show that Eq. (2.8) leads to a state vector which coincides, apart from a normalization factor, with the initial one. In other words, no reduction has taken place and no outcome has been obtained. Since, as already remarked, the probability of such a peculiar event is extremely small, its occurrence cannot be considered as a drawback of the theory.

Another peculiar situation can occur, namely the "reversal" of a macroscopic outcome. To see this, suppose one has a normalized state

[8] It is useful to remark that, also within standard quantum mechanics with the reduction postulate, since outcomes are usually related to positions of macroscopic pointers and no wavefunction can have, in general, compact support in configuration space, one is unavoidably led to adopt an analogous criterion for the attribution of "outcomes".

vector $|\Psi\rangle$ which "almost" belongs to the eigenmanifold M_α, i.e., for which $\|P_\alpha|\Psi\rangle\|^2$ is extremely close but not identical to 1. One can then state that the definite outcome α has occurred. However, according to the theory, there is a very small probability $\|P_\beta|\Psi\rangle\|^2$ that in the far future the Brownian process $B(t)$ takes a value such that in the state vector (2.8) the norm of the second term becomes overwhelmingly large with respect to that of the first. Correspondingly, one would be led to conclude that the outcome β has emerged. This shows that, though with extremely low probabilities, even definite macroscopic outcomes can (spontaneously) change.[9] The analysis we have just performed and the conclusions we have reached hold for QMSL too. The above considerations are useful for some points which we will discuss in what follows.

3. HOW TO DESCRIBE THE MACROSCOPIC WORLD WITHIN A DYNAMICAL REDUCTION CONTEXT

In this section we will show how, by taking advantage of the specific features of the dynamical reduction mechanisms, one can give a description of the world in terms of the mean values $\mathcal{M}(\mathbf{r}, t)$, at different places and at different times, of appropriately defined mass density operators. The procedure will involve the following steps. First we will show how, if one does not restrict the set of all possible states of the Hilbert space of "our universe," one unavoidably meets situations which cannot be consistently described in terms of the function $\mathcal{M}(\mathbf{r}, t)$. Fortunately, one can show that the universal dynamics of the reduction models does not consent to the persistence for[(9)] *more than a split second* of the unacceptable states, thus allowing one to use the function $\mathcal{M}(\mathbf{r}, t)$ as the basic element for the description of the world. In terms of it one can then define an appropriate "topology" which is the natural candidate for establishing a satisfactory psychophysical correspondence.

3.1. Relating Reductions to the Mass Density

In this subsection we consider a CSL type dynamics for the state vector in which, in place of the operators $N^{(k)}(\mathbf{x})$ considered previously, we introduce the mass density operators

$$M(\mathbf{r}) = \sum_k m_k N^{(k)}(\mathbf{r}) \tag{3.1}$$

[9] To avoid misunderstandings we consider it appropriate to stress that, when one is dealing with an entangled state, this "reversal," if it takes place, preserves the correlations implied by the state vector.

where m_k is the mass of the particles of type k. The Stratonovich stochastic evolution equation for the state vector is

$$\frac{d\,|\Psi_w(t)\rangle}{dt}=\left[-\frac{i}{\hbar}H+\int d\mathbf{r}\,M(\mathbf{r})\,w(\mathbf{r},t)-\frac{\gamma}{m_0^2}\int d\mathbf{r}\,M^2(\mathbf{r})\right]|\Psi_w(t)\rangle \quad (3.2)$$

where m_0 is a reference mass and γ is the parameter appearing in standard CSL of Section 2. We identify the mass m_0 with the nucleon mass; in this way the reduction rates for macroscopic objects are practically equal to those of the standard CSL model. With this choice the decoherence is governed by the mass of the nucleons in ordinary matter, the contribution due to electrons being negligible.

As usual the corresponding equation for the statistical operator is easily obtained:

$$\frac{d\rho(t)}{dt}=-\frac{i}{\hbar}[H,\rho(t)]+\frac{\gamma}{m_0^2}\int d\mathbf{r}\,M(\mathbf{r})\,\rho(t)\,M(\mathbf{r})-\frac{\gamma}{2m_0^2}\left\{\int d\mathbf{r}\,M^2(\mathbf{r}),\rho(t)\right\} \quad (3.3)$$

One of the main motivations to replace the number density operators $N^{(k)}(\mathbf{r})$ in the CSL dynamics with the mass density operators $M(\mathbf{r})$ derives from the desire to relate reductions to gravity as suggested by various authors[10] (a model with analogous characteristics has been presented in Ref. 11). Another important feature of the above choice has been pointed out by P. Pearle and E. Squires[12]: while the reduction rates for macro-objects are practically the same as those of the standard CSL model, the probabilities of excitation or dissociation of microscopic bound systems turn out to be depressed by large factors,[12,13] thus leading to a smaller disagreement with the predictions of quantum mechanics for such systems. In particular, a simple computation within the quark model for nucleons (disregarding all relativistic effects which, however, could turn out to be very important at this level) gives a dissociation rate for the proton well below the experimental bound, while this constraint is not met by the standard CSL model. The advantages of taking the above position have also been discussed by A. Rimini.[14]

3.2. Mass Density Function

Let us now consider a physical system S which will constitute "our universe," and let us denote by $H^{(S)}$ the associated Hilbert space. Let

$|\Psi(t)\rangle$ be the normalized state vector describing our individual system at time t; in terms of it we define an average mass density c-number function $\mathcal{M}(\mathbf{r}, t)$ in ordinary space as

$$\mathcal{M}(\mathbf{r}, t) = \langle \Psi(t)|\, M(\mathbf{r})\, |\Psi(t)\rangle \tag{3.4}$$

If one assumes, as one can consistently do within a nonrelativistic quantum framework, that the system S contains a fixed and finite number of particles, Eq. (3.4) establishes, for a given t, a mapping of $H^{(S)}$ into the space of positive and bounded functions of $\mathbf{r}$.

Obviously this map is many to one; in particular, to better focus on this point as well as for future purposes it turns out to be useful to compare two state vectors $|\Psi^{\oplus}\rangle$ and $|\Psi^{\otimes}\rangle$ defined as follows. Let us consider a very large number N of particles and two space regions A and B with spherical shape and radius R. The state $|\Psi^{\oplus}\rangle$ is the linear superposition, with equal amplitudes, of two states $|\Psi^{\mathrm{A}}_{N}\rangle$ and $|\Psi^{\mathrm{B}}_{N}\rangle$ in which the N particles are well localized with respect to the characteristic length (10^{-5} cm) of the model and uniformly distributed in regions A and B, respectively, in such a way that the density turns out to be of the order of 1 g/cm^3. On the other hand, $|\Psi^{\otimes}\rangle$ is the tensor product of two states $|\Phi^{\mathrm{A}}_{N/2}\rangle$ and $|\Phi^{\mathrm{B}}_{N/2}\rangle$ corresponding to $N/2$ particles being distributed in region A and $N/2$ in region B, respectively:

$$|\Psi^{\oplus}\rangle = \frac{1}{\sqrt{2}}\{|\Psi^{\mathrm{A}}_{N}\rangle + |\Psi^{\mathrm{B}}_{N}\rangle\}, \qquad |\Psi^{\otimes}\rangle = |\Phi^{\mathrm{A}}_{N/2}\rangle \otimes |\Phi^{\mathrm{B}}_{N/2}\rangle \tag{3.5}$$

It is trivially seen that the two considered states give rise to the same function $\mathcal{M}(\mathbf{r})$, and it is clear that if one attempts to give some meaning to it one has to be very careful in keeping in mind from which state $\mathcal{M}(\mathbf{r})$ originates.

In particular, it is quite obvious that in the case of $|\Psi^{\oplus}\rangle$, $\mathcal{M}(\mathbf{r})$ cannot be considered as an "objective" mass density function. To see this, let us suppose that one can use quantum mechanics to describe the gravitational interaction between massive bodies, and let us consider the following *gedanken* experiment: a test mass is sent through the middle point of the line joining the centers of regions A and B with its momentum orthogonal to it (see Figs. 1a and 1b). In the case of the state $|\Psi^{\otimes}\rangle$ for the system of the N particles, quantum mechanics predicts that the test particle will not be deflected. On the other hand, if the same test is performed when the state is $|\Psi^{\mathrm{A}}_{N}\rangle$ ($|\Psi^{\mathrm{B}}_{N}\rangle$), quantum mechanics predicts an upward (downward) deviation of the test particle. Due to the linear nature of the

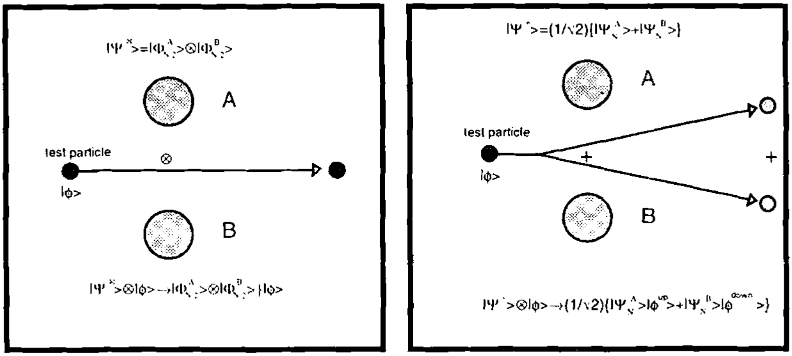

Fig 1a Fig 1b

Fig. 1. Objective and nonobjective mass density distribution $\mathcal{M}(\mathbf{r})$. The shading intensity is proportional to the value of $\mathcal{M}(\mathbf{r})$ in the shaded region. In case 1a, corresponding to the factorized state $|\Psi^{\otimes}\rangle$, the mass density in regions A and B is objective and the test particle, interacting with $|\Psi^{\otimes}\rangle$, behaves in such a way as to give rise to the appropriate density along its natural trajectory. In case 1b, corresponding to the superposition $|\Psi^{\oplus}\rangle$, the densities in A and B are nonobjective and the same holds for the density distribution generated by the interaction of the test particle with $|\Psi^{\oplus}\rangle$.

theory, this implies that if one were able to prepare the state $|\Psi^{\oplus}\rangle$, the final state would be

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left\{|\Psi_N^A\rangle \otimes |\phi^{\mathrm{UP}}\rangle + |\Psi_N^B\rangle \otimes |\phi^{\mathrm{DOWN}}\rangle\right\} \tag{3.6}$$

with obvious meaning of the symbols. If one includes the test particle into the "universe" and considers the mass density operator in regions corresponding to the wave packets $|\phi^{\mathrm{UP}}\rangle$ and $|\phi^{\mathrm{DOWN}}\rangle$, one discovers once more that nowhere in the universe is there a density corresponding to the density of the test particle. In a sense, if one would insist in giving a meaning to the density function, he would be led to conclude that the particle has been split by the interaction into two pieces of half its density.

This analysis shows that great attention should be paid in attributing an "objective" status to the function $\mathcal{M}(\mathbf{r})$. We will tackle this problem in the next subsection.

Before going on we consider also another quantity which will be useful in what follows. It is the mass density variance at $\mathbf{r}$ at time t defined by the following map from $H^{(S)}$ into $\Re^3$:

$$\mathcal{V}(\mathbf{r}, t) = \langle \Psi(t) | [M(\mathbf{r}) - \langle \Psi(t) | M(\mathbf{r}) | \Psi(t) \rangle]^2 | \Psi(t) \rangle \tag{3.7a}$$

$|\Psi(t)\rangle$ being a normalized state vector.

With these premises we have all the elements which are necessary to discuss the problems one meets when dealing with $\mathcal{M}(\mathbf{r}, t)$ and the way to overcome them. We will do this in the next subsection.

Before doing that, we consider it appropriate to simply mention the obvious fact that the states giving rise to puzzling, nonobjective density functions are those corresponding to superpositions of differently located macroscopic bodies, i.e., the infamous states which are at the center of the long debated problems about the meaning of quantum mechanics at the macrolevel.

For future purposes it is useful to introduce a mathematical criterion which allows one to make clear the different status of the mass densities in the two cases considered above (corresponding to the states $|\Psi^{\oplus}\rangle$ and $|\Psi^{\otimes}\rangle$, respectively). This is more easily expressed by resorting once more to a discretization of space in analogy with what has been done in Subsection 2.2. Obviously, in place of the space functions $\mathcal{M}(\mathbf{r}, t)$ and $\mathcal{V}(\mathbf{r}, t)$ we will consider the mean value $\mathcal{M}_i(t)$ and the variance $\mathcal{V}_i(t)$ of the mass operator in the ith cell. In correspondence to an arbitrary cell i we define the ratio:

$$\mathcal{R}_i^2 = \mathcal{V}_i / \mathcal{M}_i^2 \tag{3.7b}$$

We then state that the mass $\mathcal{M}_i$ is objective if $\mathcal{R}_i$ turns out to be much smaller than one, that is:

$$\mathcal{R}_i \ll 1 \tag{3.8}$$

This criterion is clearly reminiscent of the probabilistic interpretation of the state vector in standard quantum mechanics. Actually, within such a theory Eq. (3.8) corresponds to the fact that the spread of the mass operator M_i is much smaller than its mean value. Even though in this paper we take a completely different attitude with respect to the mean value $\mathcal{M}_i$, it turns out to be useful to adopt the above criterion also within the new context. In fact, as we will discuss in what follows, when one has a space region such that for all cells contained in it (3.8) holds, it behaves as if it would have the "classical" mass corresponding to $\mathcal{M}_i$.

With reference to the previous example we stress that in the case of $|\Psi^{\otimes}\rangle$ all cells within regions A and B are such that criterion (3.8) is very well satisfied. In the case of $|\Psi^{\oplus}\rangle$ for the same cells one has

$$\mathcal{M}_i \cong \frac{n}{2} m_0, \qquad \mathcal{V}_i \cong \frac{n^2}{4} m_0^2 \tag{3.9}$$

where n is the number of particles per cell. There follows

$$\mathcal{R}_i \cong 1 \tag{3.10}$$

3.3. The Mass Density Function in the Context of Dynamical Reduction Models

In the previous subsection we have presented a meaningful example of the difficulties one meets when one keeps the standard quantum dynamics and tries to base a description of the world and an acceptable psychophysical correspondence on the mass density function $\mathcal{M}(\mathbf{r})$. The unacceptable features find their origin in the fact that, when the macrostate is $|\Psi^{\oplus}\rangle$, while the density function takes the value of about 1/2 g/cm^3 within regions A and B, if one performs a measurement of the density in the considered regions, or if a measurement like process (such as the passage of the test particle in between A and B) occurs, things proceed in such a way that it is incompatible with the above density value. Actually one could state that no outcome emerges in the measurement. To understand fully the meaning of this statement one could identify, for example, the final position of the test particle with a pointer reading; the pointer would then not point to the middle position (corresponding to equal densities in A and B) but would be split into "two pointers of half density" pointing upward and downward, respectively (compare with Fig. 1b).

If one attempts to take an analogous attitude with reference to dynamical reduction theories, one does not meet the same difficulties because they imply that linear superpositions of states corresponding to far-apart macroscopic systems are dynamically suppressed in extremely short times and measurements have outcomes.[3,4,8,15] Therefore, we can guess that, within the context of the dynamical reduction program, the description of the world in terms of the mass density function $\mathcal{M}(\mathbf{r})$ is a *good* description; moreover, it is such as to allow one to base on it a sensible psychophysical correspondence.

Obviously, some *fuzzy* situations can occur also in this context, when the mass density is not "objective," i.e., when (in the simplified discretized version) criterion (3.8) is not satisfied. However, as we are going to show, this does not give rise to any difficulty for the program we are furthering.

In order to show this, we will examine, along the above lines, the status of the mass density function $\mathcal{M}(\mathbf{r})$ for the various possible states which are not forbidden by the reducing dynamics. We will discuss the cases of microsystems and macrosystems, and, with reference to the latter, we will identify two physically relevant classes of states which can occur. As we have done previously, we will deal with a discretized space.

3.3.1. Microscopic Systems. For the sake of simplicity, let us consider a single nucleon. As is well known, the reducing dynamics does not forbid the persistence, for extremely long times, of linear superpositions of far-away states of the particle, typically states like

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|0, 0,..., 1_i,..., 0_j,...\rangle + |0, 0,..., 0_i,... 1_j,...\rangle) \tag{3.11}$$

where i and j are two distinct and far-apart cells. Such microscopic states which are not eigenvectors of the operators M_i will be called "microscopically nondefinite," the term "nondefinite" making reference to the characteristic preferred basis of the model. As is evident from (3.11), the mean values of M_i and M_j are $(1/2)\,m_0$, and criterion (3.8) is not satisfied in correspondence of both cells. A measurement of the mass in one of these two cells would give the *definite outcome* 0 or m_0 with equal probability (corresponding to the fact that wave packets of microsystems diffuse, but, however, the reaction of a detector devised to reveal them remains spotty) and not $(1/2)\,m_0$, the value taken by the density function within the considered cells. This discrepancy, this *nonclassical* character of $\mathcal{M}_i$ and $\mathcal{M}_j$, cannot, however, be considered a difficulty for the theory with the proposed interpretation; it simply amounts to recognizing that we cannot legitimately apply our *classical* pictures to the microworld. On the contrary, we must allow[16] *microsystems to enjoy the cloudiness of waves.*

3.3.2. Macroscopic Systems. The theory allows the persistence of two general classes of states for macroscopic systems, i.e., those corresponding to a macroscopic number of microsystems in microscopically nondefinite states and states like those of almost rigid bodies with sharply defined (with respect to the characteristic length of the model) center-of-mass position. Due to the fact that the center-of-mass wavefunction has, in general, noncompact support, this second class obviously includes also the states wich are of particular interest for our discussion, i.e., the states which, being brought out by the reducing dynamics, have "tails."

Concerning states of the first class, it is of extreme relevance to make clear that they have a conceptual status which is very different from that of the superpositions of macroscopically distinguishable states like $|\Psi^{\oplus}\rangle$ of Subsection 3.2. This important difference has already been appropriately stressed by A. Leggett,[17] who, even though in a different context, has introduced the mathematically precise concept of disconnectivity to distinguish states of this type from states like $|\Psi^{\oplus}\rangle$.

To be more specific, let us consider a system of N nucleons and a discretization of space in cells of linear dimensions 10^{-8} cm. We consider again two macroscopic regions A and B, and we label with the indices k_A and k_B pairs of cells within A and B, respectively. For $k_A \neq \tilde{k}_A$, the two cells are disjoint and the union of all cells k_A (k_B) covers the region A(B). The index k runs from 1 to N, a very large number; typically if A and B have volumes of the order of 1 dm^3, N will be of the order of 10^{27}.

Let us denote by $|\varphi_{k_A}\rangle$ and $|\varphi_{k_B}\rangle$ the states of a particle whose wavefunctions in coordinate representation are well localized within k_A and k_B, respectively. As an example, we could choose

$$\langle \mathbf{r} \mid \varphi_{k_A}\rangle = \chi(k_A) \tag{3.12}$$

$\chi(k_A)$ being the characteristic function of cell k_A. We now consider the following microscopically nondefinite state for the kth particle:

$$|\varphi^k\rangle = \frac{1}{\sqrt{2}}(|\varphi_{k_A}\rangle + |\varphi_{k_B}\rangle) \tag{3.13}$$

and the factorized state of the N particles

$$|\varphi\rangle = |\varphi^1\rangle \otimes \cdots \otimes |\varphi^k\rangle \otimes \cdots \otimes |\varphi^N\rangle \tag{3.14}$$

In spite of the fact that the state $|\varphi\rangle$ is a direct product of microscopically nondefinite states, it is nevertheless "almost" an eigenstate of the operators M_i [remember that the linear dimensions of the cell to which the index i refers are of the order of 10^{-5} cm so that one such cell contains about 10^9 cells of the kind of k_A (k_B)]. In fact, denoting by $n \cong 10^9$ the number of k_A (k_B) cells contained in the ith cell, one can easily see that $|\varphi\rangle$ gives rise to "objective" mass $\mathcal{M}_i$ in regions A and B respectively:[10]

$$\langle M_{i(A,B)}\rangle \cong \tfrac{1}{2} n m_0, \qquad \langle M^2_{i(A,B)}\rangle \cong \tfrac{1}{4}(n^2+n)\, m_0^2 \tag{3.15}$$

hence

$$\mathcal{V}^2_{i(A,B)} \cong \tfrac{1}{4} n m_0^2 \qquad \text{and} \qquad \mathcal{R}_{i(A,B)} \cong \frac{1}{\sqrt{n}} \ll 1 \tag{3.16}$$

To clarify the physical implications of the state $|\varphi\rangle$, from the point of view which interests us here, we can imagine that we are performing once more the *gedanken* experiment with a test particle we have already considered in the previous subsection, assuming, for simplicity, that the interactions between the test particle and the considered N particles do not

[10] In making the computations we have identified the operators M_i with the sum of the projectors (multiplied by the nucleon mass m_0) of the various particles on the ith cell.

change the state[11] of the latter. By substituting Eq. (3.13) into Eq. (3.14), we see that $|\varphi\rangle$ is a superposition of 2^N states in which each particle is well localized. In such a superposition all states have an equal amplitude $(1/\sqrt{2^N})$ and almost all states correspond to about $N/2$ particles being in regions A and B, respectively. Therefore, in the language of dynamical reduction models, the probability of occurrence of a realization of the stochastic potential leading to the "actualization" of an almost completely undeflected trajectory for the test particle is extremely close to one.[12] This shows that the mass density function $\mathcal{M}(\mathbf{r})$ corresponding to the state $|\varphi\rangle$ behaves in a "classical way," so that no trouble arises in this case. It has to be noted that, obviously, the mass $\mathcal{M}_i$ corresponding to the state (3.14) coincides with the one corresponding to the state $|\Psi^{\otimes}\rangle$ of Subsection 3.2, in spite of the fact that both states are dynamical allowed and are quite different as physical states. However, as we have shown, the masses $\mathcal{M}_i$ in the two cases behave practically in the same way and give rise to no trouble, contrary to what happens in the case of $|\Psi^{\oplus}\rangle$.

We come now to the consideration of the other type of allowed states of interest for us, i.e., macroscopic states which are "almost" eigenstates of the mass operators M_i but which, however, have tails. Let $|\Psi\rangle$ be the normalized state

$$|\Psi\rangle = \alpha\,|\Psi_N^{\mathrm{A}}\rangle + \beta\,|\Psi_N^{\mathrm{B}}\rangle \tag{3.17}$$

where $|\Psi_N^{\mathrm{A}}\rangle$ and $|\Psi_N^{\mathrm{B}}\rangle$ are the states defined in the previous subsection and $|\beta|^2$ is extremely close to zero. In region A we have

$$\mathcal{M}_{i(\mathrm{A})} \cong |\alpha|^2\, nm_0, \quad \mathcal{V}_{i(\mathrm{A})} \cong |\alpha|^2\,|\beta|^2\, n^2 m_0^2 \qquad \text{and} \qquad \mathcal{R}_{i(\mathrm{A})} \cong |\beta|^2 \ll 1 \tag{3.18}$$

so that the masses $\mathcal{M}_{i(\mathrm{A})}$ are objective and practically equal to those corresponding to the state $|\Psi_N^{\mathrm{A}}\rangle$. In region B we have

$$\mathcal{M}_{i(\mathrm{B})} \cong |\beta|^2\, nm_0, \quad \mathcal{V}_{i(\mathrm{B})} \cong |\alpha|^2\,|\beta|^2\, n^2 m_0^2 \qquad \text{and} \qquad \mathcal{R}_{i(\mathrm{B})} \cong |\beta|^{-2} \gg 1 \tag{3.19}$$

hence the masses $\mathcal{M}_{i(\mathrm{B})}$ are not objective.

[11] At any rate, possible changes in such a state would be symmetrical with respect to the middle plane, so that the subsequent considerations would still hold true.

[12] It could be useful to remark that if one would analyze the same experiment in terms of the linear quantum dynamics, the test particle would end up in the linear superposition of an extremely large number of states. However, since such states correspond to trajectories which are very near and almost undeflected, the evaluation of the mass density associated to the final state vector would show that in the "middle" region there would practically be the total mass of the test particle. Therefore, this represents a case in which even without any reduction process the mass density referring to the test particle would correspond to a precise outcome of the measurement.

At this point it is appropriate to make a detailed analysis, within the CSL scheme, to get a quantitative estimate of $\mathscr{R}_{i(\mathrm{A})}$ and of the total mass in region B. To this purpose [as is evident from Eqs. (3.18) and (3.19)] one has to evaluate explicitly the order of magnitude of the parameter $|\beta|^2$ implies by the reducing dynamics. In order to do this, to cover also the case of nonhomogeneous bodies, we consider again two far-apart regions A and B, each containing K cells and a system of nucleons which at time $t=0$ is in a (normalized) state of the type (the overall phase factor being irrelevant)

$$|\Psi\rangle = \alpha(0)\, |n_{1(\mathrm{A})},\ldots, n_{K(\mathrm{A})},\ldots, 0,\ldots, 0\rangle + \beta(0)\, e^{i\gamma(0)}\, |0,\ldots, 0, n_{1(\mathrm{B})},\ldots, n_{K(\mathrm{B})}\rangle \tag{3.20}$$

where $\alpha(0)$ and $\beta(0)$ are comparable positive numbers and $n_{i(\mathrm{A,B})}$ represents the occupation number in the ith cell in regions A and B respectively.[13] We then study the ensemble of systems brought in by the reducing dynamics after a time interval of the order of, for example, 10^{-2} secs (the reason for this choice will become clear in what follows).

According to the CSL model of Subsection 3.1, after such a time interval the normalized state corresponding to a definite realization of the stochastic potential would be of the type

$$\begin{aligned} |\Psi_{\mathrm{B}}(t)\rangle &= \alpha_{\mathrm{B}}(t)\, |n_{1(\mathrm{A})},\ldots, n_{K(\mathrm{A})}, 0,\ldots, 0\rangle \\ &\quad + \beta_{\mathrm{B}}(t)\, e^{i\gamma(0)}\, |0,\ldots, 0, n_{1(\mathrm{B})},\ldots, n_{K(\mathrm{B})}\rangle \end{aligned} \tag{3.21}$$

with $\alpha_{\mathrm{B}}(t)$ and $\beta_{\mathrm{B}}(t)$ as positive numbers. The ensemble of systems corresponding to all possible realizations of the stochastic potential would be described by the statistical operator

$$\rho(t) = \int dB_1 \cdots dB_{2K} P_{\mathrm{cook}}[B(t)]\, |\Psi_{\mathrm{B}}(t)\rangle\langle \Psi_{\mathrm{B}}(t)| \tag{3.22}$$

satisfying[14]

$$\begin{aligned} &\langle n_{1(\mathrm{A})},\ldots, n_{K(\mathrm{A})}, 0,\ldots, 0|\, \rho(t)\, |0,\ldots, 0, n_{1(\mathrm{B})},\ldots, n_{K(\mathrm{B})}\rangle \\ &\quad = e^{-\lambda t \sum_i^K n_i^2} \langle n_{1(\mathrm{A})},\ldots, n_{K(\mathrm{A})}, 0,\ldots, 0|\, \rho(0)\, |0,\ldots, 0, n_{1(\mathrm{B})},\ldots, n_{K(\mathrm{B})}\rangle \end{aligned} \tag{3.23}$$

[13] We disregard the cells which are not contained in regions A and B since they are irrelevant for the following discussion.

[14] Even though we are using the CSL model relating decoherence to the mass, the formulas of this section coincide with the analogous ones of standard CSL. This is due to the fact that we deal only with nucleons and that we have chosen the coupling to the noise to be governed by the ratio γ/m_0^2, taking the standard CSL value.

with $\lambda t \cong 10^{-18}$. From (3.23) we see that the matrix elements of $\rho(t)$ between the considered states are exponentially damped by a factor which is proportional to $\sum_i^K n_i^2$.

In the following we consider only situations in which $\sum_i^K n_i^2$ turns out to be much greater than 10^{18}, so that in the considered time interval the linear superposition (3.20) is actually suppressed, i.e., either $\alpha_B(t)$ or $\beta_B(t)$ of Eq. (3.21) become very small. The states at time t are then typical states with "tails," i.e., states whose existence is considered as a drawback of the theory by the authors of Refs. 5 and 6. Equation (3.23) implies [taking into account Eqs. (3.21) and (3.22)] that

$$\int dB_1 \cdots dB_{2K} P_{\text{cook}}[B(t)]\, \alpha_B(t)\, \beta_B(t) = \alpha(0)\, \beta(0)\, e^{-\lambda t \sum_i^K n_i^2} \tag{3.24}$$

From (3.24), since $\alpha_B(t)$ and $\beta_B(t)$ are positive, one can easily deduce that the probability of occurrence of realizations of the stochastic potential which would lead to a value for the product $\alpha_B(t)\,\beta_B(t)$ much greater than $e^{-\lambda t \sum_i^K n_i^2}$ must be extremely small. Therefore, one can state that in practically all cases

$$\alpha_B(t)\, \beta_B(t) \cong e^{-\lambda t \sum_i^K n_i^2} \tag{3.25}$$

If we assume that $\alpha_B(t) \cong 1$, so that we consider an individual case for which the reduction leads to the state corresponding to the nucleons being in region A, $|\beta_B(t)|^2$ must be of the order of $e^{-\lambda t \sum_i^K n_i^2}$. On the basis of this fact, we can then estimate the value of $|\beta|^2$, e.g., for a homogeneous cube of normal density (so that $n_i = n \cong 10^9$ is the number of particles per cell) and of size 1 dm^3 (so that $K \cong 10^{18}$ is the number of cells in regions A and B), getting a figure of the order of $e^{-10^{18}}$. Correspondingly, we have

$$\mathscr{R}_{i(A)} \cong e^{-10^{18}} \tag{3.26}$$

while for the total mass in region B we get the value

$$\mathscr{M}_B \cong e^{-10^{18}}\, 10^{27}\, m_0 \tag{3.27}$$

Equation (3.26) shows that the mass in region A is "objective" to an extremely high degree of accuracy, and Eq. (3.27) shows that the total mass in region B is much smaller than the mass of a nucleon. If we consider a situation in which K or n are greater than those of the example we have discussed now, we find values for $\mathscr{R}_{i(A)}$ and $\mathscr{M}_B$ which are even smaller[15] than those of Eqs. (3.26) and (3.27). This fact by itself (see also the analysis

[15] Note that this holds also for objects like a galaxy or a neutron star.

of the following subsection) shows that the states with "tails" allowed by CSL cannot give rise to difficulties for the proposed interpretation of the theory. If we would perform the usual *gedanken* experiment with the test particle, it would be deflected just as if in region A there would be the "classical" mass Knm_0.

Concluding, we have made plausible that in the context of the dynamical reduction program one can consistently describe the macro-world, at a given time, in terms of the mass density function $\mathcal{M}(\mathbf{r})$. Obviously, since with the elapse of time the state of the world changes, a complete description requires the consideration of the motion picture of the density, i.e., of $\mathcal{M}(\mathbf{r}, t)$ defined in Eq. (3.4). We will discuss in greater detail this crucial point in Section 4.

3.4. Defining an Appropriate Topology for the CSL Model

Let us consider a system S of finite mass which will constitute our "universe" and its associated Hilbert space $H^{(S)}$. We denote by $\mathcal{U}^{(S)}$ the unit sphere in $H^{(S)}$, and we consider the nonlinear map[16] $\mathcal{M}$ associating to the element $|\varphi\rangle$ of $\mathcal{U}^{(S)}$ the element $\mathbf{m} = \{\mathcal{M}_i(|\varphi\rangle)\}$ of l_2, $\mathcal{M}_i(|\varphi\rangle)$ being the quantity $\langle\varphi| M_i |\varphi\rangle$.

On $\mathcal{U}^{(S)}$ we define a topology by introducing a mapping Δ: $\mathcal{U}^{(S)} \otimes \mathcal{U}^{(S)} \rightarrow \Re^+$ according to

$$\Delta(|\varphi\rangle, |\psi\rangle) = d(\mathbf{m}, \mathbf{n}) = \sqrt{\sum_i (m_i - n_i)^2} \tag{3.28}$$

where $\mathbf{m} = \{\mathcal{M}_i(|\varphi\rangle)\}$, $\mathbf{n} = \{\mathcal{M}_i(|\psi\rangle)\}$. Such a mapping is not a distance since, as it emerges clearly from the analysis of the previous subsection, it may happen that $\Delta(|\varphi\rangle, |\psi\rangle) = 0$ even though $|\varphi\rangle \neq |\psi\rangle$. However, Δ meets all other properties of a distance:

$$\Delta(|\varphi\rangle, |\psi\rangle) = \Delta(|\psi\rangle, |\varphi\rangle) \geqslant 0 \tag{3.29}$$

and

$$\Delta(|\varphi\rangle, |\psi\rangle) \leqslant \Delta(|\varphi\rangle, |\chi\rangle) + \Delta(|\chi\rangle, |\psi\rangle) \tag{3.30}$$

as one easily proves by taking into account the fact that d is a distance in l_2.

[16] To be rigorous, one should consider the map $\mathcal{M}$ from the unit sphere of $H^{(S)}$ into the space L^2 of the square integrable functions of r. However, we can deal, without any loss of generality, with the discretized version of the model.

From now on we will limit our considerations to the proper subset $\mathscr{A}^{(S)}$ of $\mathscr{U}^{(S)}$ of those states which are allowed by the CSL dynamics. In the previous subsection we have already identified, even though in a rough way, the set $\mathscr{A}^{(S)}$. One could obviously be very precise about such a set by adopting, for example, the following criterion: let $|\varphi\rangle \in \mathscr{U}^{(S)}$, and let us consider the ensemble $\mathscr{A}^{(S)}(|\varphi\rangle)$ of states which have a nonnegligible (this obviously requires the definition of a threshold) probability of being brought in by the reducing dynamics after a time interval of the order of 10^{-2} sec for the given initial condition $|\varphi\rangle$. The union of all subsets $\mathscr{A}^{(S)}(|\varphi\rangle)$ for $|\varphi\rangle$ running over $\mathscr{U}^{(S)}$ is then $\mathscr{A}^{(S)}$. For our purposes, however, it is not necessary to go through the cumbersome management of a very precise definition of the set $\mathscr{A}^{(S)}$, the consideration of the cases we have discussed in the previous subsection being sufficient to lead to the interesting conclusions.

For any element $|\varphi\rangle$ of $\mathscr{A}^{(S)}$ we consider the set of states of $\mathscr{A}^{(S)}$ for which $\Delta(|\varphi\rangle, |\psi\rangle) \lesssim \varepsilon$. Here the quantity ε has the dimensions of a mass and is chosen of the order of $10^{18} m_0$, with m_0 the nucleon mass. From the properties of the map Δ it follows that:

i. $\{\Delta(|\varphi\rangle, |\psi\rangle) \lesssim \varepsilon$ and $\Delta(|\varphi\rangle, |\chi\rangle) \lesssim \varepsilon\}$ implies $\Delta(|\chi\rangle, |\psi\rangle) \lesssim \varepsilon$

ii. $\{\Delta(|\varphi\rangle, |\psi\rangle) \gg \varepsilon$ and $\Delta(|\varphi\rangle, |\chi\rangle) \lesssim \varepsilon\}$ implies $\Delta(|\chi\rangle, |\psi\rangle) \gg \varepsilon$

We have introduced the parameter ε in such a way that it turns out to be sensible to consider states similar to each other whose "distance" Δ is smaller than (or of the order of) ε. More specifically, when

$$\Delta(|\varphi\rangle, |\psi\rangle) \lesssim \varepsilon \tag{3.31}$$

we will say that $|\varphi\rangle$ and $|\psi\rangle$ are "physically equivalent."

To understand the meaning of this choice, it is useful to compare it with the natural topology of $H^{(S)}$. We begin by pointing out the inappropriateness of the Hilbert space topology to describe the concept of similarity or difference of two macroscopic states. In fact, suppose our system S is an almost rigid body, and let us consider the following three states: $|\varphi^{A}\rangle$, $|\varphi^{B}\rangle$, and $|\tilde{\varphi}^{A}\rangle$. The state $|\varphi^{A}\rangle$ corresponds to a definite internal state of S and to its center of mass being well localized around A, the state $|\varphi^{B}\rangle$ is simply the translated of $|\varphi^{A}\rangle$ so that it is well localized in a far region B, and the state $|\tilde{\varphi}^{A}\rangle$ differs from $|\varphi^{A}\rangle$ simply by the fact that one or a microscopic number of its "constituents" are in states which are orthogonal to the corresponding ones in $|\varphi^{A}\rangle$.

It is obvious that, on any reasonable assumption about similarity or difference of the states of the universe, $|\tilde{\varphi}^{A}\rangle$ must be considered very similar (identical) to $|\varphi^{\mathrm{A}}\rangle$ while $|\varphi^{\mathrm{B}}\rangle$ must be considered very different from $|\varphi^{\mathrm{A}}\rangle$. On the other hand, according to the Hilbert space topology

$$\|(|\varphi^{\mathrm{A}}\rangle - |\tilde{\varphi}^{\mathrm{A}}\rangle\| = \|(|\varphi^{\mathrm{A}}\rangle - |\varphi^{\mathrm{B}}\rangle\| = \sqrt{2} \tag{3.32}$$

This shows with striking evidence that the Hilbert space topology is totally inadequate for the description of the macroscopic world. As a consequence, such topology is also quite inadequate to base on it any reasonable psychophysical correspondence.

We now discuss the "distorted" (with respect to the Hilbert space one) topology associated to the "distance" Δ. First of all, we stress that the two states $|\varphi^{\mathrm{A}}\rangle$ and $|\tilde{\varphi}^{\mathrm{A}}\rangle$, which are maximally distant in the Hilbert space toplogy, turn out to be equivalent, i.e., to satisfy condition (3.31) in the new topology. This represents an example showing how such a topology takes more appropriately into account the fact that, under any sensible assumption, the "universes" associated to the considered states are very similar.

Obviously, one problem arises. Criterion (3.31) leads to consider equivalent states which are quite different from a physical point of view, even at the macroscopic level. To clarify this statement we take into account two states $|\varphi\rangle$ and $|\psi\rangle$ corresponding to an almost rigid body located, at $t=0$, in the same position but with macroscopically different momenta, let us say $P=0$ and P, respectively. Even though the two states are physically quite different, their distance at $t=0$ is equal to zero. However, if one waits up to the time in which the state $|\psi\rangle$ has moved away from $|\varphi\rangle$, the "distance" $\Delta(|\varphi(t)\rangle, |\psi(t)\rangle)$ becomes large and the two states are no longer equivalent. We will discuss the now outlined problem in great detail in the next section.

Before concluding this part, it is important to analyze the case of two states $|\psi\rangle$ and $|\psi_T\rangle$ such that $|\psi\rangle$ corresponds to an almost rigid body with a center-of-mass wavefunction perfectly localized and $|\psi_T\rangle$ corresponds to the same body with a "tail" in a distant region. As we have already discussed, the CSL dynamics allows the existence of this latter type of states; however, it tends to depress more and more the tail in such a way as to make the mass in the distant region extremely close to zero (much less than one nucleon mass) in very short times. As a consequence, according to the topology that we propose, the two states $|\psi\rangle$ and $|\psi_T\rangle$ turn out to be identical. This is quite natural. In fact, in the same way in which taking away a single particle from a macroscopic system would be accepted as being totally irrelevant from a macroscopic point of view, when one chooses, as we do, to describe reality in terms of mass density, one must consider equivalent situations in which their difference derives entirely from the location of a small fraction of the mass of a nucleon in the whole universe. We remark that $|\psi\rangle$ and $|\psi_T\rangle$ are extremely close to each other also in the standard Hilbert space topology.

4. DEEPENING THE PROPOSED INTERPRETATION

We consider it appropriate to devote this section to discuss in great generality the problem of giving an acceptable description of the world within a given theory. Usually one tries to do so by resorting to the notion of observable. As repeatedly remarked, such an approach meets, within standard quantum mechanics, serious difficulties since the formal structure of the theory allows only probabilistic statements about outcomes conditional under the measurement being performed. In brief, the theory deals with *what we find* not with *what is*. This is why J. S. Bell has suggested[18] to replace the notion of observable with the one of "beable," from the verb *to be*, to exist. Obviously, the identification of the beables, of what is real, requires the selection of some formal ingredients of the theory we are dealing with.

4.1. The Case of the Pilot-Wave Theory

To clarify our point, it turns out to be useful to analyze the de Broglie–Bohm pilot-wave theory. It describes the world in terms of the wave function and of the actual positions of the particles of our "universe," each of which follows a definite trajectory. Therefore, in such a theory it is quite natural to consider as the beables the positions (which are the local elements accounting for reality at a given instant) and the wave function (which is nonlocal and determines uniquely the evolution of the positions). It is important to stress that, within the theory under discussion, all other "observables" (in particular, e.g., the spin variables) turn out, in general, to be contextual. This simply means that *the truth value* of a statement about the outcome of the measurement of one such observable (which in turn is simply a statement about the future positions of some particles) may in general depend (even nonlocally) on the *overall* context. This obviously implies that the attribution of a value to the considered observable cannot be thought of as corresponding, in general, to an "intrinsic property" of the system.

Before coming to discuss the problem of the beables within CSL we would like to call attention to the fact that[2] within the pilot-wave theory, one can construct, from the microscopic variables $\mathbf{r}$, macroscopic variables

R including pointer positions, images on photographic plate, etc. Obviously this requires some fuzziness, but such a limitation is not relevant for a consistent account of reality. Thus, in this theory we are led to suppose that it is from the **r**, rather than from the wave function, that the observables we use to describe reality are constructed. The positions are also the natural candidates to be used in defining a psychophysical parallelism, if we want to go so far. An appropriate way to express the now discussed features of the theory derives from denoting, as J. S. Bell proposed, as "exposed variables" the positions of particles and as a "hidden variable" the wave function Ψ.

4.2. The Case of CSL

Let us now perform a corresponding analysis for the model theory considered in Section 3. Since, as it should be clear from the discussion given there, the most relevant feature of the modified dynamics is that of suppressing linear superpositions corresponding to different mass distributions, one is actually led to identify as the local beables of the theory the mass density function $M(\mathbf{r}, t)$ at a given time. Obviously, also within CSL just as for the pilot-wave, the wave function plays a fundamental role for the evolution so that it too acquires the status of a nonlocal beable.

It has to be remarked that in the interpretation we are proposing, even though the wave function is considered as one of the beables of the theory, the "exposed variables" are the values of the mass density function at different points. It is then natural to relate to them, as we have done in the previous section, the concept of similarity or difference between universes.

In doing so one is led to consider equivalent, at a fixed time t, two "universes" which are almost identical in the exposed beables [i.e., they satisfy the condition (3.31)]. Obviously the fact that the above condition holds at t does by no mean imply that the two universes will remain equivalent as time elapses.

It has to be stressed that the above-mentioned feature is not specific of the model and of the interpretation we are proposing, but it is quite general and occurs whenever one tries to make precise the idea of "similarity" of physical situations. In fact, within all theories we know, and independently of the variables we choose to use to define nearness, situations can occur for which *nearby states* at a given time can evolve in extremely short times in *distant states.*

To focus on this important fact, we can consider even classical mechanics with the assumption that both positions and momenta are the beables of the theory.[17] As is obvious, even if such an attitude is taken, there are at least two reasons for which nearby points in phase space can rapidly evolve into distant ones. First of all, one must take into account one of the most important conceptual achievements of recent times, i.e., the discovery that many systems exhibit dynamical instability so that the distance between "trajectories" grows exponentially with time. Secondly, even for a "dynamically standard" situation, one can consider cases in which just the present conditions can give rise to completely different evolutions depending on some extremely small difference in the whole universe. Suppose in fact you consider two universes A, $\tilde{A}$ differing only in the *direction* of propagation of a single particle (such universes have to be considered as very close in any sensible objective interpretation). If the trajectory of the particle in $\tilde{A}$ is such that in a very small time it triggers, for example, the discharge of a Geiger counter, wich in turn gives rise to some relevant macroscopic effect, while in A it does not, the evolved universes soon become quite different. An analogous argument obviously holds for standard quantum mechanics, the pilot-wave theory and, as previously remarked, for CSL too.

It is appropriate to stress that, in a sense, the above considerations favor taking a position about reality which can be described in the following terms. One chooses the sensible beables for its theory at a fixed time and one distinguishes similar or different universes on the basis of such a snapshot. Obviously, one must then also pay attention to the way in which the beables evolve, i.e., to compare snapshots at different times.[18]

4.3. The Role of Mass Density

The previous analysis has shown that the proposed interpretation can be consistently taken. Obviously it gives an absolutely prominent role to the mass in accordance with the fact that mass is the handle by which the reduction mechanism induces macro-objectification.

Other features of natural phenomena, such as the effects related to the charge, are, in a sense, less fundamental since to become objective they need mass as a support. To clarify this point, we remark that one could consider, for example, a condenser with two plates of about 1 cm^2, at a distance of 1 cm. The plates are supposed to be perfectly rigid and in perfectly

[17] Obviously, within classical mechanics any function of these variables can be considered as a beable, but since all information about the system can be derived from the positions and the momenta, consideration of such variables is sufficient.

[18] From this point of view, one could state that also the classical world would be most appropriately described in terms of positions at fixed time.

defined positions.[19] Let us also consider the following *gedanken* situation: the condenser can be prepared in the superposition of two states, $|C_0\rangle$ and $|C_c\rangle$, the first corresponding to its plates being neutral, the second to its plates having been charged by displacing 10^{12} *electrons* from one plate to the other. We remark that for the two states the decoupling rate (recall that electrons, being very light, are quite ineffective in suppressing superpositions) is about 10^{-8} $\sec^{-1}$, i.e., that the superposition can persist for more than 10 years. The electric field within the plates is zero or about 10^8 V/m in the two states, respectively. Suppose now we consider a small sphere of radius 10^{-5} cm and density 10^{-2} g/cm^3 carrying a charge corresponding to 10^4 electrons. We send such a test particle through the plates of the condenser. What happens? The final state is the entangled state

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[|C_0\rangle\,|\text{undeflected}\rangle + |C_c\rangle\,|\text{deflected}\rangle\right] \tag{4.1}$$

the location of the particle in the state $|\text{undeflected}\rangle$ and $|\text{deflected}\rangle$ differing by macroscopic amounts. According to the CSL model of Subsection 3.1, one can easily evaluate the rate of suppression of the superposition. As already remarked, the contribution of the electrons on the plates is totally negligible, so that the decoherence is governed mainly by the mass of the particle. Then, with the above choices for the radius and the density of the test particle, the superposition will persist for more than 1 min. In spite of the fact that macroscopically relevant forces enter into the game, no reduction takes place for such a time interval. On the contrary, if we put the same charge on a particle of normal density and of radius 10^{-3} cm, we see that the macroscopic force acting on it when the condenser is in the state $|C_c\rangle$ leads to a displacement of the order of its radius in about 10^{-5} sec and that within the same time the reducing effect of the dynamics suppresses one of the two terms of the superposition.

This example is quite enlightening since it shows that superpositions of charge distributions generating different forces which are relevant at the macroscopic level, are not suppressed unless they induce displacement of masses.

It goes without saying that any attempt to relate reduction to charge is doomed to fail since it will not suppress superpositions of macroscopically different but electrically neutral mass distributions.

[19] This assumption must be made because we are just discussing the role of the charge with respect to the one of the mass within the model. If one would allow deformations and/or displacements of the plates, once more the ensuing reduction would be due to the mass and not directly to the charge density difference in states $|C_0\rangle$ and $|C_c\rangle$.

We hope to have made clear, with this perhaps tedious analysis, the real significance of treating the mass function as the "exposed beables" allowing one to describe reality.

4.4. The Stochastic Nature of the Evolution

So far we have discussed the description of the world allowed by the CSL theory in terms of the values taken by the mass density function $\mathcal{M}(\mathbf{r})$ which have been recognized to constitute the exposed beables of the theory. According to Eq. (3.4) it is the wave function associated to the system which determines $\mathcal{M}(\mathbf{r})$. It is useful to analyze the evolution of the beables. As we have discussed in Section 2, the dynamical evolution equation for the wave function is *fundamentally* stochastic, being governed by the stochastic processes $w(\mathbf{r}, t)$. The "cooked" probability of occurrence of such processes, based on the analogs of Eq. (2.3), depends on the wavefunction which describes the system, and this fact is of crucial importance for getting the "right" (i.e., the quantum) probabilities of measurement outcomes. Therefore, in the CSL theory, the wave function has both a descriptive [since it determines $\mathcal{M}(\mathbf{r})$] and a probabilistic (since it enters in the prescription for the cooking of the probability of occurrence of the stochastic processes) role.

As we have seen in Section 2, also the "tails" of the wavefunction have a precise role. In fact, suppose our "universe" is described at $t=0$ by a normalized state

$$|\Psi(0)\rangle = \alpha(0)\, |a\rangle + \beta(0)\, |b\rangle \tag{4.2}$$

with $|\beta(0)|^2$ being extremely small. The "reality" of the universe at $t=0$ is "determined" by the state $|a\rangle$, as we have explicitly shown in Section 3. However, one cannot ignore the (extremely small) probability $|\beta(0)|^2$ that a realization of the stochastic potential occurs which, after a sufficiently long time, leads to a normalized state

$$|\Psi(t)\rangle = \alpha(t)\, |\tilde{a}\rangle + \beta(t)\, |\tilde{b}\rangle \tag{4.3}$$

with $|\alpha(t)|^2$ being extremely small and with $|\tilde{a}\rangle$ and $|\tilde{b}\rangle$ two of the most probable states at time t for the initial conditions $|a\rangle$ and $|b\rangle$, respectively. Then, the "reality" at time t is that associated to the state $|\tilde{b}\rangle$ which has its origin in the negligible component $|b\rangle$ at time $t=0$. Thus, some "memory" of a situation which at time zero did not correspond to the "reality" of the world remains at time t. Obviously, if such an extremely improbable case would occur, one would be tempted (wrongly) to retrodict that "reality" at $t=0$ was the one associated to $|b\rangle$ and not the one associated to $|a\rangle$. However, we stress that such peculiar events, which we could denote as the "reversal of the universe," have absolutely negligible probabilities. As made plausible by the estimate for the values of $\beta(t)$ given in Section 3, the "risk to be wrong" in retrodicting from the present to the past "status of the world" is comparable with the probability that observing now a table standing on the floor, and knowing that it has been kept isolated, we can infer that it was standing there even one hour ago, in spite of the fact that thermodynamically a very peculiar situation corresponding to its "levitation" at that time could in principle have occurred.

5. THE PSYCHOPHYSICAL PARALLELISM WITHIN CSL

The most characteristic and appealing feature of CSL and of its interpretation we have proposed in this paper consists in the fact that it allows one to give a satisfactory account of reality, to take a realistic view about the world, to talk about it as if it is really there even when it is not observed. However, one cannot avoid raising the problem of also including conscious observers into the picture, for[2]: *what is interesting if not experienced*? Thus, one is led to consider the problem of the psychophysical parallelism within the considered theory.

The previous analysis has already given clear indications about the way to follow to reach this goal. In this section we will first of all clarify how to relate the "different states of the universe" as characterized by the formalism according to the lines of the previous sections to the specificity of conscious perceptions. Secondly, we will show how the theory itself supports the proposed correspondence. To get this, we will perform a very sketchy analysis, from the point of view of CSL, of the current ideas about the physical processes leading to perceptions.

5.1. External World and Internal Perceptions

As we have seen, the CSL dynamics lead naturally to consider as the exposed beables accounting for "reality," the values taken by the mass density function $\mathcal{M}(\mathbf{r}, t)$ at different points and at different times. We have also shown that in the case of macroscopic objects, the dynamical evolution forces the mass density to be "objective" at almost all times in the regions where such objects are. According to the above picture "reality," "what is out there," is identified with a precise mass distribution in real space. The reality of a massive macro-object in front of us corresponds to the fact that in the region it occupies there is the objective mass density which characterizes it.

On the other hand, it is a fundamental feature of our perceptions that they correspond to objects having precise locations and extensions. The problem of establishing a map between reality and perceptions is then naturally solved by correlating our perceptions to mass density distributions.[20]

After these simple remarks we can come to discuss how one can account, within CSL, for the emergence of perceptions.

5.2. Describing the Perceptive Process

To clarify how CSL is able to "describe" the occurrence of definite perceptions of conscious beings, it turns out to be quite appropriate to start by discussing a criticism which has been put forward in some recent papers.[5,6,19] The idea is quite simple but raises a problem which deserves a detailed investigation. One considers the following process: a neutral microsystem with spin is sent through a Stern–Gerlach apparatus. The spin state is such that the system ends up in the superposition of being deflected, with equal probabilities, upwards or downwards, respectively. The two "potential trajectories" cross a fluorescent screen in two macroscopically far-apart regions A and B, respectively. The particle–screen interaction is such as to lead to the excitation of a small number, e.g., of about ten atoms, which subsequently undergo a transition to the ground state accompanied by the emission of photons.

The argument of Refs. 19 goes as follows. Since only a few atoms of the screen are excited during the process, since their excitations involve displacements which are much smaller than the characteristic localization length of the model, and since photons are not spontaneously localized, there is no way for the CSL mechanism to yield decoherence of the two superposed states. Thus, the superposition of states corresponding to ten photons emerging from the different space regions A and B will persist for extremely long times. On the other hand, since the visual perception threshold is quite low (about 7 photons) there is no doubt that the naked eye of a human observer is sufficient to detect whether the luminous spot on the screen is at A or at B. This raises an interesting question: in the considered situation how can it happen that a definite perception about the location of the spot on the screen emerges? Are we compelled to accept that, at least in some circumstances, also within CSL the conscious

[20] Obviously, our perceptions are much richer than those (corresponding to position and shape) we have listed here. In the next subsection we will make clear how also more complex perceptions (such as color, etc.) can be naturally included in the picture we are presenting.

observer and his perceptions play a peculiar role analogous to the one they have in Wigner's views?

That this is not the case has been discussed in great detail in a recent paper.[20] If one takes, as one must, the above remark seriously one is compelled to consider the actual systems which enter into play and to analyze the implications of the CSL dynamics for them. We simply sketch here the argument of Ref. 20. One takes into account what we know about the transmission of nervous signals from, let us say, the retina and the higher visual cortex. Such a transmission requires, among other changes, the displacement of ions along the axons involved in the process. A very rough estimate of the mass associated to these ions and of the displacements they have to perform to flow through the ion channels which open at Ranvier's nodes to transmit the electric pulse can be given. This estimate makes perfectly plausible that the conditions which are sufficient, according to CSL, for the suppression of one of the two superposed states (nervous signals) within the visual perception time (which is of the order of 10^{-2} sec), are satisfied.

We do not want to be misunderstood; this analysis does by no means amount to attributing a special role to the conscious observer or to his perception. The observer's brain is simply the only system among those which are present in which a superposition of two states involving different locations of a large number of particles occurs. As such it is the only place where the reduction can and actually must take place according to the theory. If, in place of the eye of a human observer, one puts in front of the photon beams a spark chamber of any device leading to the displacement of a macroscopic pointer or producing ink spots on a computer output, reduction will take place. In the considered example, the human nervous system is simply a physical system, a specific assembly of particles, which has the same function as any one of these devices, if no such device is interacting with the photons before the human being does.[21] In Section 3, in order to study the states allowed by the CSL theory and the "size of tails," we have considered a time interval of the order of 10^{-2} sec. The reason for such a choice should now be clear: it is the time interval corresponding to the perception time, the time in which our brain, acting as a physical system, must (and actually does) suppress the linear superpositions of states corresponding to different stimuli in order that the observer has a

[21] We consider appropriate a specification. The above analysis could be taken as indicating that we adopt a very naive and oversimplified attitude about the deep problem of the brain–mind correspondence. We do not claim and we do not pretend that CSL yields a physicalistic explanation of consciousness. We simply point out that, for what we know about the purely physical aspects of the perceptual process, the conditions guaranteeing that superpositions of different perceptions cannot occur are satisfied.

definite perception. Analogous considerations have been taken into account in choosing the parameter ε characterizing the similarity or difference of physical situations.

The above analysis should be sufficient by itself to clarify our problem. Our perceptions are triggered by our sensory apparatuses. In many cases, such as for the auditory or tactile perceptions, the stimulus itself cannot be ambiguous, i.e., it cannot correspond to superpositions of different perceptions, since it requires macroscopic displacements (of the hearing membrane or of the skin). In other cases, as in the one discussed above, the nervous signal can be triggered by a microscopic system, which can very well be in a superposition of states capable of inducing different perceptions. But in all cases the nervous transmission of the signal involves a "macroscopic" (on the appropriate scale) displacement of mass in the brain. And, as repeatedly stressed, the CSL dynamics does not tolerate nonobjective macroscopic mass distributions lasting for a time of the order of the perception time.

In a sense the analysis shows that the problem of the psychophysical correspondence admits a simple solution quite similar to the one which is usually assumed to hold for classical theories. Reality and perceptions involve the same and fundamental type of "exposed beables": the values of the mass density function. On one hand "reality," as previously discussed, is related to mass density, and macroscopic situations to "objective" mass density distributions. On the other, definite perceptions are related to objective macroscopic mass density distributions within the brain.

We do not want to spend further time in elaborating on this point. Our aim here is not that of performing a detailed technical analysis of any conceivable situation but simply that of making plausible that the problem of the psychophysical parallelism admits, within CSL a solution which is quite analogous to the one of the unproblematic classical case. Thus CSL can be claimed, according to J. S. Bell's definition,[2] to be an "exact theory" in the precise and limited sense that *it neither needs nor is embarrassed by an observer*.

6. CONCLUDING REMARKS

We consider the previous analysis to be sufficient to give a clear idea of the reasons and the formal aspects which allow one to close the circle within the dynamical reduction program. We cannot, however, conclude our analysis without stressing the crucial role of some stimulating remarks by J. S. Bell for the elaboration of the ideas of this paper. He was the first to call attention[21] to the fact that spontaneous reduction models, unlike standard quantum mechanics, allow one to take a density rather than a probabilistic interpretation of the modulus square of the wavefunction. Therefore, the line of thought of this paper represents, in a sense, an implementation of his suggestions. But there are important differences between the present attitude and the one he was inclined to take. He repeatedly insisted that the density he was referring to was not a mass or a charge density but the density of stuff of which the world is made. Moreover, he stressed strongly (probably since he was worried about the consequences of adopting an interpretation à la Schrödinger) that the density function had to be taken seriously only in the $3N$-dimensional configuration space and not in the real 3-dimensional space.[22] We hope to have shown that, within CSL, a quite satisfactory interpretation can be obtained along the (more traditional) lines we have presented in this paper.

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REFERENCES

1. A. Shimony, in *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*, J. T. Cushing and E. McMullin, eds. (University of Notre Dame Press, Notre Dame, Indiana, 1989).
2. J. S. Bell, in *Quantum Gravity 2*, C. Isham, R. Penrose, and D. Sciama eds. (Claredon Press, Oxford, 1981).
3. G. C. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D* **34**, 470 (1986).
4. P. Pearle, *Phys. Rev. A* **39**, 2277 (1989); G. C. Ghirardi, P. Pearle, and A. Rimini, *Phys. Rev. A* **42**, 78 (1990); G. C. Ghirardi and A. Rimini, in *Sixty-Two Years of Uncertainty*, A. Miller ed. (Plenum, New York, 1990).
5. A. Shimony, in *PSA 1990*, Vol. 2, A. Fine, M. Forbes, and L. Wessels, eds. (Philosphy of Science Association, East Lansing, Michigan, 1991).
6. D. Z. Albert and B. Loewer, in *PSA 1990*, Vol. 1, A. Fine, M. Forbes, and L. Wessels eds. (Philosophy of Science Association, East Lansing, Michigan, 1990).

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[22] Of course, the interpretation we have adopted is not à la Schrödinger; while, for example, for a particle, the wavefunction, and consequently the mass density, is extended in space, the detection of the particle remains spotty. This fact, as we have already discussed, simply implies that the mass density function associated to the particle is not "objective."

7. G. C. Ghirardi, R. Grassi, and P. Pearle, *Found. Phys.* **20**, 1271 (1990); in *Symposium on the Foundations of Modern Physics 1990*, P. Lahti and P. Mittelstaedt eds. (World Scientific, Singapore, 1991); G. C. Ghirardi, and P. Pearle in *PSA 1990*, Vol. 2, A. Fine, M. Forbes, and L. Wessels eds. (Philosophy of Science Association, East Lansing, Michigan, 1991).
8. G. C. Ghirardi, R. Grassi, J. Butterfield, and G. N. Fleming, *Found. Phys.* **23**, 341 (1993); J. Butterfield, G. N. Fleming, G. C. Ghirardi, and R. Grassi, *Int. J. Theor. Phys.* **32**, 2287 (1993).
9. J. S. Bell, in *Schrödinger-Centenary Celebration of a Polymath*, C. W. Kilmister ed. (Cambridge University Press, Cambridge, 1987).
10. F. Karolyhazy, *Nuovo Cimento A* **42**, 390 (1966) and references therein; A. B. Komar, *Int. J. Theor. Phys.* **2**, 157, (1969); R. Penrose, in *Quantum Concepts in Space and Time*, R. Penrose and C. J. Isham, eds. (Clarendon, Oxford, 1986); J. Ellis, S. Mohanty, and D. V. Nanopoulos, *Phys. Lett. B* **221**, 113 (1989); L. Diosi, *Phys. Rev. A* **40**, 1165 (1989); A. Frenkel, *Found. Phys.* **20**, 159 (1990).
11. G. C. Ghirardi, R. Grassi, and A. Rimini, *Phys. Rev. A* **42**, 1057 (1990).
12. P. Pearle and E. Squires, to be published in *Phys. Rev. Lett.*
13. F. Benatti, G. C. Ghirardi, and R. Grassi, in *Proceedings of the International Course on Advances in Quantum Phenomena*, E. Beltrametti and J. M. Levy-Leblond eds. (Plenum, New York, to appear).
14. A. Rimini, in *Proceedings of the International Course on Advances in Quantum Phenomena*, E. Beltrametti and J. M. Levy-Leblond, eds. (Plenum, New York, to appear).
15. F. Benatti, G. C. Ghirardi, A. Rimini, and T. Weber, *Nuovo Cimento B* **100**, 27 (1987).
16. J. S. Bell, in *Proceedings of the Nobel Symposium 65: Possible Worlds in Arts and Sciences* (Stockholm, 1986).
17. A. J. Legett, *SSuppl. Prog. Theor. Phys.* **69**, 80 (1980).
18. J. S. Bell, CERN-TH. 4035/84, 1984.
19. D. Z. Albert and L. Vaidman, *Phys. Lett. A* **139**, 1 (1989); D. Z. Albert in *Sixty-Two Years of Uncertainty*, A. Miller ed. (Plenum, New York, 1990); D. Z. Albert, *Quantum Mechanics and Experience* (Harvard University Press, Cambridge, Massachusetts, 1992).
20. F. Aicardi, A. Borsellino, G. C. Ghirardi, and R. Grassi, *Found. Phys. Lett.* **4**, 109 (1991).
21. J. S. Bell, in *Sixty-Two Years of Uncertainty*, A. Miller, ed. (Plenum, New York, 1990).