THE HEAT FLUX DENSITY IN A NON-HOMOGENEOUS BARE LOESSIAL SOIL

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Abstract. This work describes the relationship between the diurnal patterns of the radiant energy exchange in the atmospheric surface layer and the soil heat flux density of a bare irrigated soil in an arid environment. The measurements show that the soil heat flux density is a large fraction of the net radiation. The soil moisture content has little effect on this fraction but modifies the phase relationship between the net radiation and the soil heat flux density waves. Differences between the thermal regimes of the 'wet' and 'dry' soil appear to be caused by latent heat exchanges rather than changes of the soil thermal properties. The data also show that the variation with depth of the soil thermal properties strongly influences the propagation of the temperature and heat flux density waves in the soil. A heat diffusion theory for non-homogeneous conductors (Lettau, 1962) which enables the thermal properties of the soil to be predicted is tested by comparison with experimental determinations in the field.

1. Introduction

The undisturbed loessial soil of the northern Negev is normally bare of vegetation during the summer as a result of drought and heavy pasturing by nomadic herds. The energy exchange processes at the surface of such a soil determine the microclimate near the ground.

The typical soil surface of the area has uniform properties. Furthermore, as the soil moisture is uniformly low, the homogeneous top soil can be regarded as an isotropic porous medium for which the temperature regime and heat flow pattern are readily determined. Measurements carried out by Stearns (1969), in a homogeneous desert sand, support this view. During the summer months, the evaporation from the dry homogeneous, loessial soil is practically nil, and therefore the relationship between absorbed radiant energy, soil heat flow and soil temperature can be predicted (Lettau, 1951).

If the soil is cultivated, tillage and irrigation modify the soil porosity, the state of aggregation and the water content of the soil profile. These modifications also affect the thermal properties of the soil (de Vries, 1963) and, consequently transform the heat flow and the temperature regime inside the soil profile. As the changes with depth in thermal properties of the soil occur gradually, the heterogeneity is a continuous function of depth. The mathematical description of the heat flow in homogeneous soils is inadequate for such a condition. A model by Lettau (1954, 1962), based on a Fourier analysis of harmonic oscillations as induced by diurnal radiant energy waves, generalizes the mathematical treatment of the problem to non-homogeneous soils, and includes the depth dependency of the thermal diffusivity in its formulation. Direct experimental verification of this theory has been obtained only in homogeneous soils (Stearns, 1969), for which case the classical theory is also satisfactory.

In this paper we investigate the relationship between the radiation balance, the soil heat flux density and the soil surface heating of a tilled and irrigated loessial soil as a function of its thermal properties. The experimental data are used to check the assumption of two models of thermal diffusion in non-homogeneous soils. We show that the model proposed by Lettau accounts best for the observed soil temperature regime and that the resulting prediction of the soil thermal properties is qualitatively correct.

2. Experimental Procedure

Profiles of soil temperature and soil heat flux were measured in a bare plot at the Gilat Experimental Farm (lat. $31^{\circ}20'$ N, long. $34^{\circ}40'$ E, elevation 150 m above m.s.l.). The soil profile is texturally homogeneous to a depth of about 2 m. Its mechanical composition is 18% clay, 27% silt and 55% fine sand.

The chosen plot and its surroundings were plowed to a depth of about 25 cm, then irrigated, at first by flooding and then by sprinkling with a total amount of water equivalent to a 130-mm rainfall.

Three days after irrigation, heat flux plates and thermocouples were installed at depths of 0.1, 2, 4, 8 and 16 cm below the soil surface. The heat flux plates were constructed and calibrated in the laboratory according to a procedure described by Fuchs and Tanner (1968). The average sensitivity of the heat flux plates was 4.20 mV/cal cm⁻² min⁻¹. The temperature sensors consisted of five copper-constantan junctions connected in series, with common reference junctions buried at 32-cm depth. The temperature measurements were taken as differences between consecutive depths. The absolute temperature at 32 cm was measured to the nearest 0.2 °C by a calibrated germanium diode. The accuracy of the differential temperature measurements was ± 0.05 °C.

The heat flux plates were placed singly at 0.1 cm, in pairs in series at 2 and 4 cm. At depths of 8 and 16 cm, four heat flux plates were connected in series to increase the measuring sensitivity.

The incoming radiation was measured with an Eppley precision spectral pyranometer, the reflected radiation with a Kipp solarimeter and the net radiation with a Funk net pyrradiometer. All the measured parameters were scanned within three minutes and recorded every 15 min.

The long-wave radiation terms were computed from the temperature measured at 0.1 cm assuming that the soil emissivity is 0.95 (Hovis, 1966).

Soil bulk densities and water contents were determined by sampling, weighing, oven drying for 24 hr at 105 °C and reweighing. Soil bulk density profiles are given in Figure 1.

The first set of measurements was taken on 7–8 June, 1970, two weeks after irrigation. The soil profile was still wet and the water content showed marked diurnal fluctuations (Figure 2). A second set of measurements was collected on 21–22 July, 1970, when the soil moisture content had decreased considerably but exhibited an accentuated profile (Figure 3). Diurnal water content fluctuations could not be detected.

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Fig. 1. Bulk density profiles of Gilat loess.



Fig. 2. Soil water content fluctuations in Gilat loess two weeks after irrigation.



Fig. 3. Stationary soil water content profile in Gilat loess 8 weeks after irrigation.

The dates of measurements were chosen around the summer solstice when the solar radiation input varies little, to isolate the soil condition as our only variable. The average air temperatures were 22.5 °C and 24.0 °C for the first and second run, respectively, and weather conditions were similar as well.

3. Results and Discussion

As our measurements were carried out in a soil which contained water, a preliminary requirement was to assess the respective contribution of sensible and latent heat in the energy transport within the soil. This problem pertains mainly to the first set of measurements, for which significant diurnal fluctuations of soil water content were observed. As we cannot discriminate directly between liquid and vapor flow of water in the soil, we checked whether the soil heat flux divergence $\partial G/\partial z$, measured by the heat flux plates satisfied the one-dimensional continuity equation for heat without a source term, i.e.,

$$\partial G/\partial z = C \ \partial T/\partial t \,, \tag{1}$$

where $\partial T/\partial t$ is the measured time rate of change of the soil temperature and C is the volumetric heat capacity given as (de Vries, 1963):

$$C = (\theta + 0.46 \chi) \text{ cal cm}^{-3} \text{ C}^{-1}.$$
 (2)

In Equation (2) θ is the volumetric soil water content and χ is the volumetric fraction of solids in the soil. Figure 4 shows the results of this test in the soil layer between 2- and 16-cm depth, on an hourly basis. In both runs the points are randomly scat-



Fig. 4. Comparison between directly measured soil heat flux density divergence and calorimetrically determined rate of change of soil heat content in Gilat loess for the layer between 2- and 16-cm depth.

tered along a straight line. This indicates that experimental errors are larger than possible latent heat exchanges. A noteworthy conclusion derives from the unity slope of the best fitted lines in Figure 4 as it proves that the laboratory calibration of the heat flux plates is correct and that their sensitivity does not depend appreciably upon the conditions of the surrounding medium.

The diurnal variations in the radiation balance terms for the two sets of measurements are shown in Figure 5. The radiant energy input terms are similar, but the sur-



Fig.5. Diurnal waves of the radiation balance terms and the soil heat flux density of the bare Gilat loess.

face heating, expressed by the emitted long-wave radiation is much stronger in the second run.

Linear regressions were fitted to the relation between emitted radiation and total absorbed radiation to provide a quantitative estimate of the surface heating. The slope of the regression lines expresses the average change of emitted radiation per unit absorbed total radiation for a given day:

slope =
$$\partial (\varepsilon \sigma T^4) / \partial R = 4\varepsilon \sigma T^3 (\partial T / \partial R),$$
 (3)

where ε is the surface emissivity, σ is the Stefan-Boltzmann constant, T is the absolute surface temperature, and R is the total absorbed radiation. The coefficient of $4\varepsilon\sigma T^3$ is a specific radiant heating rate for the soil surface, which defines the radiant energy flux density required to increase the temperature of the soil surface by 1 °C. This coefficient is a property of the soil-atmosphere interface and depends upon the energy transfer processes in both the atmosphere and the soil. Its values for the wet and dry soil are 0.0497 and 0.0388 cal cm⁻² min⁻¹ C⁻¹, respectively.

The soil moisture content data in Figure 2 indicate that during the daylight hours of 8 June, 1970, the average latent heat flux density from the wet soil was 0.3 cal cm⁻² min⁻¹. In the dry soil, no evaporation could be detected but the surface heating was much stronger. Over the daylight hours, the measured surface temperature of the dry soil was 8.5°C higher than the temperature of the wet soil.

The energy flux density equivalent to this different surface temperature is made up of the difference in long-wave radiation emitted by the surface, i.e., approximately $0.08 \text{ cal cm}^{-2} \text{ min}^{-1}$, and the reduction of surface heating energy flux density given by the product $0.0388 \times 8.5 = 0.33 \text{ cal cm}^{-2} \text{ min}^{-1}$, or a total of 0.41 cal cm⁻² min⁻¹. This energy flux density compensates largely for the lack of latent heat flux density from the dry soil, and shows that the difference between the thermal regimes in the wet and dry soils is essentially due to the difference between the rates of evaporation.

The soil heat flux density wave in Figure 5 follows closely the net radiation wave. The relative magnitude of the ordinates of the two curves indicates that the soil heat flux density accounts for a large fraction of the energy budget of the bare soil surface.

Figure 6, where the soil heat flux density is plotted against the net radiation, further illustrates the relationship between these two parameters. The phase shifts between the waves generate the diurnal loops, but for the purpose of comparing the two sets of measurements on a daily average basis, these can be ignored. Linear regressions fitted through the points indicate that approximately 30% of the net radiation is dissipated as soil heat flux density. We observe also that the fraction of the net radiation dissipated as soil heat flux density is nearly identical for the wet and dry soil. Lettau (1951) has shown that if evaporation is assumed to be small or nearly constant, the partition of sensible heat flux between the air and the soil depends exclusively upon the thermal admittance of the soil $(\lambda C)^{1/2}$, the turbulent transfer characteristics of the atmospheric surface layer, and the period of the net radiation wave considered. Since the wind and air temperature conditions above the wet and the dry surfaces were similar, the fact that the fractional soil heat flux densities were nearly identical indicates that the effect



Fig. 6. Relationship between soil heat flux density and net radiation of the bare Gilat loess.

of the larger latent heat dissipation is compensated by the larger thermal admittance of the wet soil. However, this observation is not readily generalized because the quantitative prediction of the fraction of net radiation dissipated as soil heat flux density can be made only if vertical homogeneity of the soil is assumed. The data presented in Table I and the subsequent discussion invalidate this assumption. The occurrence of phase differences of the type illustrated in Figure 6 stresses the necessity for direct measurements of the soil heat flux density in energy balance studies of non-homogeneous soils.

A model frequently used to describe the heat flow in cultivated soils, approximates the vertical heterogeneity of the medium by a series of homogeneous layers stacked together with sharp discontinuities between boundaries (van Wijk and Derksen, 1963). In such a model, for each of the homogeneous layers, the equation of continuity (1) transforms into:

$$\partial T/\partial t = (\lambda/C) \,\partial^2 T/\partial z^2,\tag{4}$$

where λ , the thermal conductivity is constant and defined as:

$$\lambda = G/(\partial T/\partial z). \tag{5}$$

Equation (4) predicts that a plot of $\partial T/\partial t$ against $\partial^2 T/\partial z^2$ for a full diurnal cycle should yield a straight line with a slope equal to λ/C . Instead, the points in Figure 7 loop along a quasi-ellipse and show that the soil temperature field does not satisfy the basic differential equation used in the model.

A more general approach to heat propagation in heterogeneous media (Lettau, 1954), constructed on the assumption that (1) holds through the soil profile, that both the fluctuations of the soil temperature and the soil heat flux density wave at any depth can be described by the first n harmonics of a Fourier series, and that the thermal properties of the soil are depth dependent only, predicts that the plot of the partial deriva-



Fig. 7. Relationship between partial derivatives of soil temperaturecalculated from the measured soil temperature waves at 4-cm depth in the soil profile.

tives for each harmonic of the temperature waves yields an ellipse. This result is well supported by the data in Figure 7. Secondary loops and irregularities of the actual trace result mainly from measurement errors, but could also be due to the diurnal variation of the thermal properties of the soil.

An extension of the theory enables us to calculate the soil thermal conductivity as a function of depth (Lettau, 1962; Stearns, 1969):

$$\lambda(z) = -\frac{G_n^*(z)\cos\delta_n(z)}{\partial/\partial z [T_n^*(z)]} = \frac{G_n^*(z)\sin\delta_n(z)}{T_n^*(z)\,\partial/\partial z [\tau_n(z)]},\tag{6}$$

where $G_n^*(z)$ and $T_n^*(z)$ are the amplitudes of the *n*th harmonic of the soil heat flux density wave and the temperature wave, respectively, $\tau_n(z)$ is the angular phase shift of the temperature wave, $\delta_n(z)$ is the angular dephasing between the temperature and the heat flux waves.

Similarly the volumetric heat capacity of the soil is given as:

$$C(z) = -\frac{\partial/\partial z \left[G_n^*(z)\right]}{n\omega T_n^*(z)\sin\delta_n(z)} = \frac{G_n^*(z) \,\partial/\partial z \left[\gamma_n(z)\right]}{n\omega T_n^*(z)\cos\delta_n(z)},\tag{7}$$

where $\gamma_n(z)$ is the heat flux wave phase shift and ω is the angular frequency.

The angular phase difference $\delta_n(z)$ for n=1, was computed according to (6) from the slope of the curve relating $\ln[T_n^*(z)/T_n^*(0)]$ to τ_n . This determination, based upon soil temperature measurements only, agrees fairly well with $\delta_n(z) = \tau_n(z) - \gamma_n(z)$, which was obtained from the Fourier analysis of the soil temperature and heat flux density waves (Table I). Both determinations indicate that for the soil layer above 8 cm, $\delta_n(z)$ is smaller than the value of 0.78, $(\pi/4)$, predicted by the classical solution of the heat flow equation in uniform conductors (Carslaw and Jaeger, 1959). This is in agreement with

the expectation based on Figures 1, 2 and 3, that the greatest variation of the soil thermal properties occurs near the surface.

Equation (7) shows that the value of $\delta_n(z)$ can also be derived from the relationship between the logarithmic amplitude attenuation of the soil heat flux density waves and their angular phase shift with depth. The resulting values are usually smaller than those obtained from the temperature data. The discrepancy could be due to positioning errors of the heat flux plates with depth as suggested by Stearns (1969).

Comparison between theoretical and observed (first harmonic of Fourier series) angular phase relationships (radians) for the temperature and the heat flux density in the soil.											
z cm	'wet' soil			'dry' soil							
	Fourier $n=1$	Eq. (6)	Eq. (7)	Fourier $n=1$	Eq. (6)	Eq. (7)					
0.1	0.43	0.46	0.28	0.38	0.43	0.39					
2	0.51	0.57	0.41	0.50	0.75	0.62					
4	0.67	0.72	0.55	0.61	0.78	0.64					
8	0.77	0.78	0.79	0.89	0.78	0.64					

0.98

0.82

TABLE I

The thermal conductivity and heat capacity computed for n=1 from (6) and (7), respectively, are in fair agreement with diurnal average values experimentally obtained from (5) and (2) (Table II). Since the coefficient of variation of the average experimental values is of the order of 15%, the accuracy of the theoretical derivation of the soil thermal properties is quite acceptable.

0.67

16

0.58

It is noteworthy that the theoretical values corroborate the known relationships between soil moisture content and thermal properties, i.e., that the thermal conduc-

TABLE II

Comparison between direct determinations of the soil thermal properties and those predicted from Lettau's theory.

z cm	'wet' soil				'dry' soil				
	$\frac{\lambda \times 10^3}{\text{cal cm}^{-1} \text{ s}^{-1} C^{-1}}$		$\frac{C}{\operatorname{cal}\operatorname{cm}^{-3}C^{-1}}$		$\frac{\lambda \times 10^3}{\text{cal cm}^{-1}\text{s}^{-1} C^{-1}}$		$\frac{C}{\operatorname{cal}\operatorname{cm}^{-3}C^{-1}}$		
									Eq. (5)
	0.1	0.83	0.91	0.230	0.234	0.80	0.62	0.230	0.220
2	1.65	1.91	0.315	0.316	1.17	1.20	0.283	0.265	
4	1.86	2.13	0.390	0.345	1.40	1.43	0.302	0.272	
8	2.18	1.97	0.426	0.342	2.20	2.05	0.344	0.304	
16	3.07	3.05	0.459	-	2.80	3.10	0.388	-	

tivity varies over a wider range than the heat capacity (de Vries, 1963). Thus soil temture and heat flux density data convey valuable information regarding the moisture distribution in the soil profile.

4. Conclusions

The soil heat flux density is often treated as a minor term in energy budget investigations. The data presented in this paper show that in the bare soils which occupy very large areas in steppe and desert climates, nearly one third of the net radiant energy available at the soil surface can be dissipated in this flux. As the exact relationship between net radiation and soil heat flux density cannot be predicted if evaporation is unknown, and if the soil thermal properties vary with depth, direct measurement of the soil heat flux density is essential for the definition of the physical environment of non-homogeneous bare soils.

Increased soil water content reduces the surface heating. The more moderate microclimate near the ground which occurs when the soil is wet, is a consequence of latent heat exchanges and of the larger thermal conductivity and the volumetric heat capacity of the soil.

Finally, the measurements show that Lettau's theory of heat diffusion in heterogeneous soils, predicts to a good approximation the thermal behavior of cultivated soils in which moisture content and physical properties vary with depth.

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