Fluid Dynamics, Vol. 29, No. 5, 1994

EXPERIMENTAL AND THEORETICAL INVESTIGATION OF THERMAL CONVECTION IN A TERRESTRIAL MODEL OF A CONVECTION DETECTOR

G. P. Bogatyrev, M. K. Ermakov, A. I. Ivanov, S. A. Nikitin,
D. S. Pavlovskii, V. I. Polezhaev, G. F. Putin, and S. F. Savin UDC 532.5.013.4:536.25

The effect of small inclinations and low-amplitude rocking on thermal convection in an air-filled cube with sides 3 cm long has been investigated experimentally and numerically for steady and periodic heating from above. Expressions have been obtained for the dependence of the temperature difference between two points inside the cube on the angle of inclination, the applied vertical temperature difference, the rocking frequencies and the oscillations of the heating power. The possibility of estimating the magnitude and direction of the microacceleration vector on the basis of temperature field measurements made under orbital flight conditions is discussed.

A feature of processes carried out under orbital flight conditions is the nonconstancy of the acceleration vector, which varies widely with respect to both magnitude and direction. Even when full information about these fluctuations is available, the numerical or ground laboratory simulation of unsteady three-dimensional flows in a true state of weightlessness is a difficult task. As a result of the low value of the residual accelerations gravitational convection on board spacecraft should be relatively weak. At present, its existence can be judged only from indirect indicators, the results of the various field experiments carried out so far being contradictory. Accordingly, it is desirable to formulate a direct experiment to detect and investigate gravitational convection directly in orbit.

On the other hand, the gravitational and inertial sensitivity of flows in nonisothermal liquid and gaseous media makes it possible to propose a method of measuring the microaccelerations on board a spacecraft based on recording the temperature stratification due to free convection [1]. In [2, 3] a method of recording weak convection based on observing the propagation of a thermal front between coaxial cylinders was considered. Especially promising is the use of a convection detector to measure the magnitude and direction of the low-frequency components of the force field inaccessible to most other instruments currently in use.

Below, we present the results of a laboratory simulation of the process of measuring microaccelerations from observations of the convective motion of air in a cubical enclosure, together with the results of numerically testing the corresponding two-dimensional model.

1. JUSTIFICATION OF THE METHOD OF MEASURING MICROACCELERATIONS BY MEANS OF A CONVECTION DETECTOR

The intensity of the convective motion of a liquid or gas in a gravity field is determined by the Rayleigh number $\mathrm{Ra}=g\beta\nabla TH^4/\nu\chi$. Here, β, ν, and χ are the volume expansion coefficient, the kinematic viscosity coefficient and the thermal diffusivity, ∇T is the temperature gradient, and H is the characteristic dimension of the enclosure. Under surface conditions $g=g_0$, where g_0 is the acceleration of gravity. For small values of Ra there is a one-to-one relation between the Rayleigh number and the intensity of the convection and, if it can be found, then from this relation it is possible to determine the quantity g. If the acceleration gravity is close to zero (conditions approximating weightlessness), but there are small inertial accelerations of another nature, then the intensity of the convection will be determined by the resultant of these accelerations. Usually the magnitude of such accelerations is characterized by the ratio g/g_0.

Under laboratory conditions, when the temperature gradient is uniform and directed strictly upwards, there is no convective motion. However, if ∇T is rotated about the vertical through some angle α, convective motion, whose intensity depends on the horizontal component of the gradient, develops in the liquid (gas). Thus, a small horizontal component of ∇T will cause the same convective motion as a weak force field, and the quantity g/g_0 is equivalent to the ratio $(\nabla T\sin\alpha)/\nabla T$.

On board an orbital station the magnitude and direction of the microaccelerations vary with time. Therefore, under

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Moscow. Translated from Izvestiya Rossiiskoi Akademii Nauk, Mekhanika Zhidkosti i Gaza, No. 5, pp. 67-75, September-October, 1994. Original article submitted February 3, 1994.

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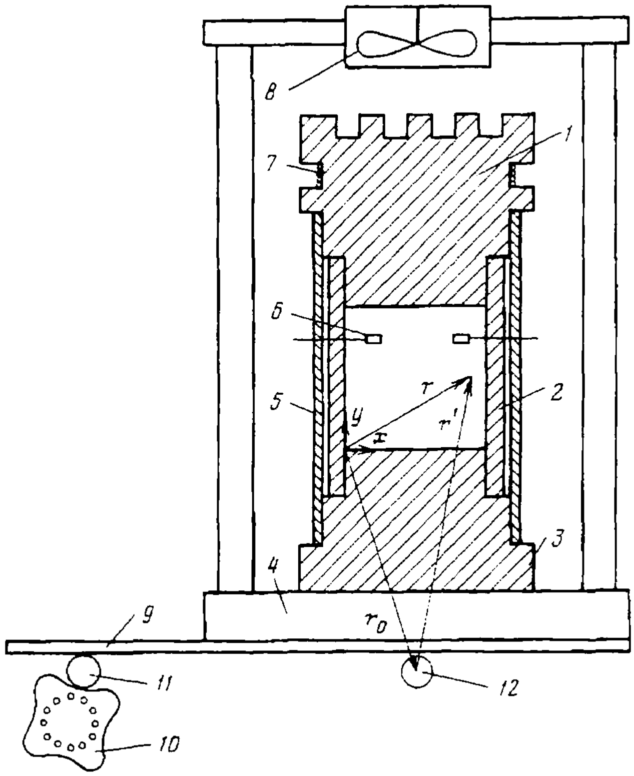

Fig. 1

surface conditions it is important to carry out laboratory simulation of the variable component of the microaccelerations and investigate its effect on the convective motion. In the experiments described below only the low-frequency (frequency not greater than 1 Hz) component of the microaccelerations was simulated.

Laboratory simulation was carried out by rocking the cell. As the working fluid we used air, for which the Prandtl number $Pr = \nu/\chi$ is close to unity. Symmetrical rocking of the model about the vertical corresponds to the situation in which the time-average value of the microaccelerations is equal to zero. Otherwise microaccelerations having both a variable and a constant component are simulated.

2. DESIGN OF THE DETECTOR, EXPERIMENTAL AND MEASURING PROCEDURES

The detector was a cubical enclosure with sides 3 cm long bounded above and below by aluminum heat exchangers 1 and 3 (Fig. 1). The side walls 2 were made of acrylic plastic 4 mm thick. A jacket 5 of copper foil-clad paper-based laminate was used to provide thermal protection from external temperature oscillations. The air gap between the jacket and the side walls was about 1 mm and served as an insulating layer. The leads of a differential copper-constantan thermocouple 6 were built into the side walls. The thermocouple junctions were located near one of the faces 7 mm from the upper heat exchanger and the side walls. The junctions of a second differential thermocouple were located in the lower and upper heat exchangers. The thermocouples were made of copper wire 0.1 mm in diameter and 0.06 mm constantan wire. The upper heat exchanger was equipped with an electric heater 7. An electric fan 8 was used to cool the upper heat exchanger, while the lower heat exchanger was attached to a massive metal slab 4.

In order to rock the model the slab 4 was mounted on a plate 9, which was made to rock about the axis of the shaft 12 by means of a disk cam 10 and bearing 11. The disk cam was shaped to produce harmonic oscillations of the plate with an angular oscillation amplitude of $\pm 1.5°$. The distance from the axis of the shaft 12 to the center of the enclosure was 97 mm. The disk cam was driven by a synchronous motor, whose speed was regulated by varying the frequency of the supply voltage.

After amplification by a factor of 1000 the thermo-emf of the detector thermocouples was measured by a digital voltmeter, and the voltmeter data were fed into the computer via a matching system. The heater was supplied from a stabilized source. In the periodic heating regime the latter was connected to the heater by means of an electronic relay, controlled by the computer through the matching system. The heater switching frequency was programmed. The

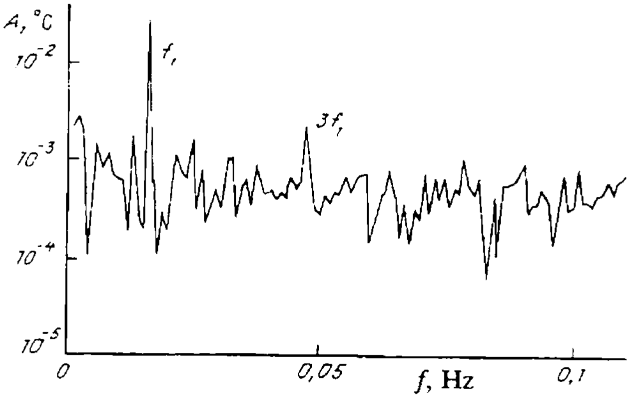

Fig. 2

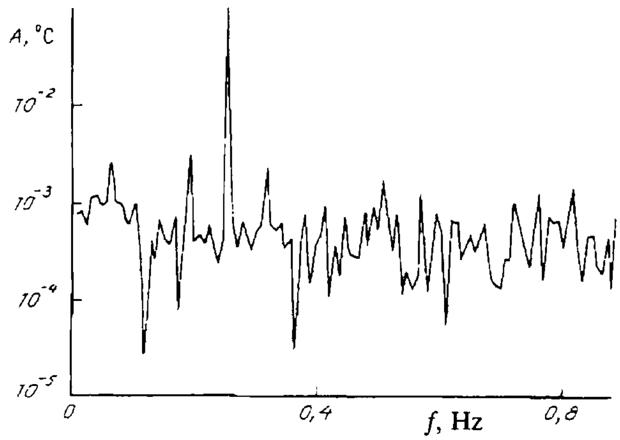

Fig. 3

voltmeter was actuated by the computer through the matching system or a trigger controlled by the disk cam.

The experiments can be divided into three groups: at constant heating power; with pulsating heating power; and with rocking of the model and constant or periodic heating. In all cases the fan was used to blow air over the upper heat exchanger. This will be necessary to ensure a pulsating temperature difference between the heat exchangers in microgravity when the natural convective heat transfer is very weak. In the experiments the model was inclined and rocked so that the two side faces of the enclosure remained in the vertical plane and the junctions of the differential thermocouple 6 (Fig. 1) on a line parallel to those faces. The angle of inclination of the model to the vertical was varied from 0 to 0.05 rad. The temperature difference between the heat exchangers ΔT was maintained on the interval 4—60°C with an accuracy of 0.1°C. The steady-state regime was established after 30—40 min. As a measure of the intensity of the convective motion we took the temperature difference ΔT, which was determined by means of the differential thermocouple 6 (Fig. 1).

In carrying out the experiments at a constant heating power we averaged the voltmeter readings over 200 measurements with a time interval between measurements of 0.04 sec. In the experiments with pulsating power the time step between measurements was usually 1 sec, and the number of points of realization was 256. In all the experiments with pulsating heating the heater was switched on for 32 sec and switched off for the same time interval, so that the periodicity was 64 sec. In these experiments the timing was controlled by the computer.

When the model was being rocked, the voltmeter was triggered by means of the disk cam, a photodiode and a light source. The disk cam had 32 holes (Fig. 1), through which light fell on the photodiode. After amplification and shaping, the signal from the photodiode was fed to the voltmeter trigger. Thus, the time step of the measurements was determined by the angular velocity of the cam. In the experiments we used two cams. The first caused the model to rock once and the second four times per revolution. Both cams produced angular oscillations of the model with an amplitude of 0.026 rad. The rocking frequency was varied from 0.05 to 1 Hz and determined from the number of triggerings of the voltmeter and the duration of the measurements.

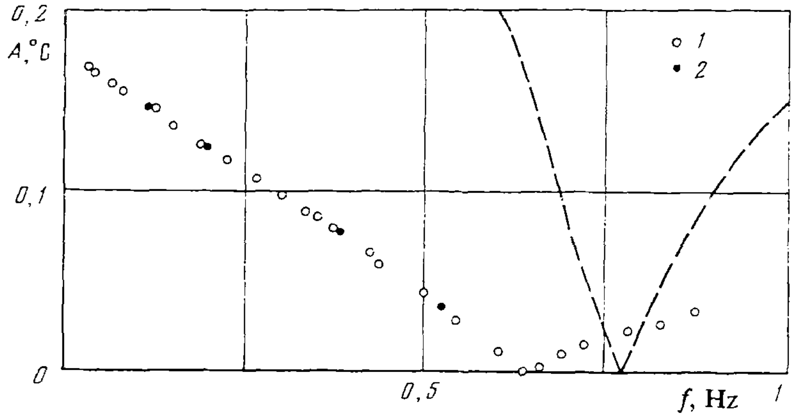

Fig. 4

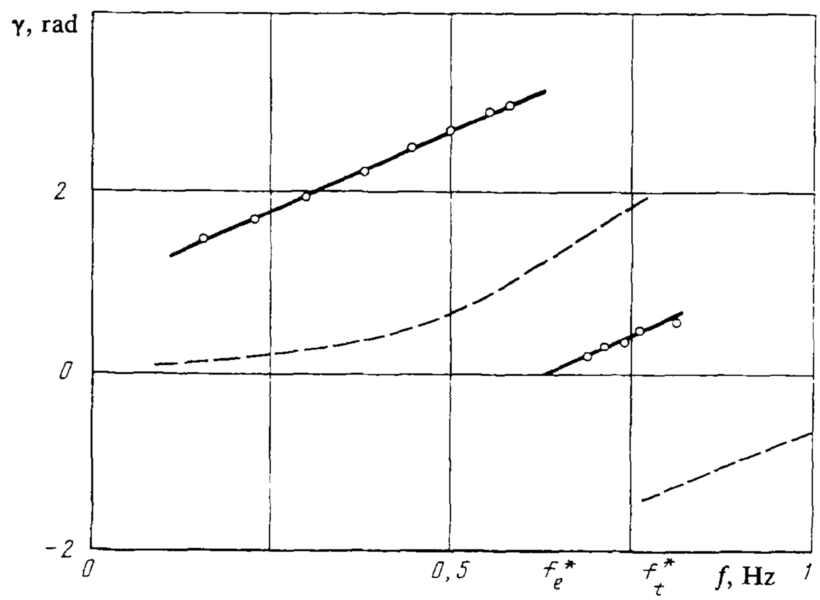

Fig. 5

3. EXPERIMENTAL RESULTS AND DISCUSSION

When the enclosure is heated from above, the deviation of the temperature gradient form the vertical can be used to model convection at low values of the ratio g/g_0. However, the angular deflections α must be small enough to simulate weak convective motion. In the experiments described α did not exceed 0.05 rad. For these small inclinations the transverse temperature difference Δt and the angle α are linearly related: $\Delta t = (8.52 \pm 0.04)\alpha$ (for constant heating power) and $\Delta t = (8.57 \pm 0.04)\alpha$ (for periodic heating). In both cases the dependence $\Delta t(\alpha)$ is the same within the limits of the random error of the measurements. These results were obtained in experiments with $\Delta T = 30.2°$ C. In the case of pulsating heating, the time-average values were used as Δt and ΔT.

On board an orbital station it is difficult to maintain a constant value of ΔT. Therefore we investigated the dependence Δt (ΔT) for a fixed angle of inclination of the model $\alpha = 0.0174$ rad. As a result of processing the experimental data we obtained: $\Delta t = (49 \pm 1)0.0001\Delta T$. Combining $\Delta t(\alpha)$ and $\Delta t(\Delta T)$ gives $\Delta t = 0.28\alpha\Delta T$. If the accuracy of the Δt measurements is 0.01° C, then when $\Delta T = 50°$ C the sensitivity threshold of the detector described will be equal to $0.0007g_0$.

The direction and magnitude of the constant component of the microaccelerations can be determined from the readings of several, for example, two detectors, in which the temperature gradients are mutually orthogonal, by geometric summation of the values of Δt measured by differential thermocouples whose junctions also lie in mutually perpendicular planes.

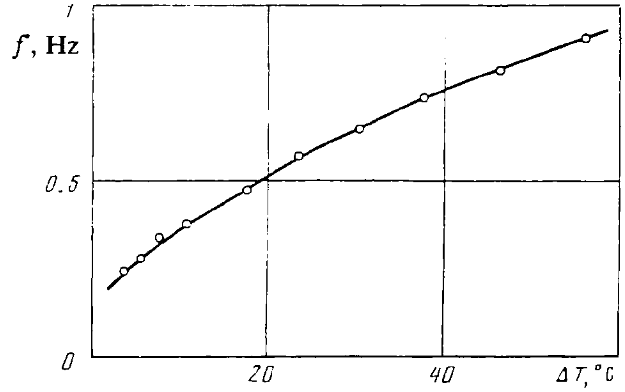

Fig. 6

Under orbital station conditions parasitic signals, associated with nonuniform contact heating, amplifier drift, voltage surges, etc., represent the greatest threat to the measuring capabilities of the instrument. We accordingly carried out experiments with pulsating heating. In this case not only the average value of Δt but also the amplitude A of the fundamental harmonic in the Δt pulsation spectrum provide a measure of the intensity of the convective motion. This method has the advantage of enabling frequency filtration of the useful signal. However, it is not possible to determine the direction of the constant component of the microaccelerations in terms of the amplitude A. The pulsation spectrum for the transverse temperature difference under periodic heating conditions is presented in Fig. 2 for an experiment in which the average value of $\Delta T = 30.2°$C and $\alpha = 2°$.

In this case the random noise level was 0.001°C (Fig. 2), which makes it possible to measure small values of the pulsation amplitudes. The dependence of the pulsation amplitude A on the angle of inclination, obtained in experiments with an average temperature different $\Delta T = 30.2°$C, is described by the expression $A = (80 \pm 2)0.01\alpha$. Thus, the sensitivity threshold of the detector with respect to the amplitude of the pulsations of the temperature difference Δt converted to $\Delta T = 50°$C is $0.0008 g_0$.

As indicated above, the variable component of the microaccelerations was simulated by rocking the enclosure. Figure 3 shows the pulsation spectrum for the transverse temperature difference obtained in an experiment with a model rocking frequency of 0.25 Hz. The frequency of the fundamental harmonic coincided with the model rocking frequency, and there were no multiple harmonics. The experiments established that under constant heating conditions the amplitude of the fundamental harmonic A decreases with increase in the rocking frequency f (Fig. 4) and at a certain critical frequency $f = f^*$ it vanishes. With further increase in f the amplitude of the temperature pulsations begins slowly to increase. In the phase plane as f increases the oscillations of the transverse temperature difference begin to lag behind the rocking. The points in Fig. 5 represent the experimental dependence of the phase difference γ on the model rocking frequency. On the phase dependence curve $\gamma(f)$ a discontinuity is observed at the point $f = f_e$ where the phase value jumps from π to 0. The value of the critical frequency f_e increases with increase in the temperature difference between the heat exchangers ΔT (Fig. 6).

If the model is rocked relative to some nonzero angle α, then convection in the presence of both a variable and a constant microacceleration component is simulated. In this case the convective oscillations will be about a steady value corresponding to the particular angle, and the time-average value of Δt will satisfy the dependence $\Delta t(\alpha)$ found in the experiments without rocking.

In these experiments the pulsation amplitude coincides with the dependence $A(f)$ obtained for $\alpha = 0$. In Fig. 4 points 1 correspond to a time dependence of the angle of inclination $\alpha = 0.026\cos(2\pi f\tau)$ and points 2 to $\alpha = 0.026 + 0.026\cos(2\pi f\tau)$. If the rocking is combined with pulsating heating (a typical time dependence of the transverse temperature difference is shown in Fig. 7), then multiple harmonics appear in the pulsation spectrum (Fig. 8), which indicates interaction of the modes corresponding to periodic heating and rocking. The amplitudes of the fundamental harmonics f_1 and f_2 differ somewhat from the amplitudes determined from the functions $A(\alpha)$ and $A(f)$. The interaction may be associated with the multiplicity of the frequencies f_1 and f_2 and the given phase shift between them.

5. MATHEMATICAL MODEL

For modeling these experiments we used the Navier—Stokes equations in the Boussinesq approximation in the right-handed coordinate system xyz related to the cubic enclosure, which moves with angular velocity $\Omega(\tau)$ about the fixed

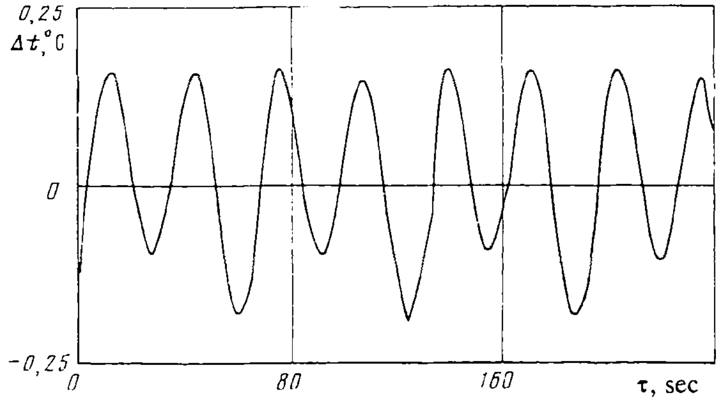

Fig. 7

axis (Fig. 1). In this coordinate system the axis of rotation is parallel to the z axis and is given by the equations $x=x_0$ and $y=y_0$:

$$\operatorname{div} \mathbf{V}=0 \tag{5.1}$$

$$\rho_0\left(\frac{\partial \mathbf{V}}{\partial \tau}+(\mathbf{V}\nabla)\mathbf{V}-2(\mathbf{V}\times\boldsymbol{\Omega})\right)=-\nabla P+\mu\Delta\mathbf{V}+\rho(\mathbf{g}(\tau)-\boldsymbol{\Omega}\times(\boldsymbol{\Omega}\times\mathbf{r}')-\dot{\boldsymbol{\Omega}}\times\mathbf{r}') \tag{5.2}$$

$$\frac{\partial T}{\partial \tau}+(\mathbf{V}\nabla)T=\chi\Delta T \tag{5.3}$$

where $\mathbf{V}$ is the velocity, P is the pressure, $\mathbf{r}'$ is the radius vector connecting the axis of rotation with a certain point inside the enclosure (Fig. 1), $\mathbf{r}'=(x', y')=\mathbf{r}-\mathbf{r}_0$, $\mathbf{r}=(x, y)$, $\mathbf{r}_0=(x_0, y_0)$, $\dot{\boldsymbol{\Omega}}$ is the angular acceleration, and $\mathbf{g}(\tau)$ is the acceleration of gravity vector.

Assuming that the variables do not depend on the z coordinate, we reduce the two-dimensional equations of motion to the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

$$\rho_0\left(\frac{\partial u}{\partial \tau}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-2v\Omega\right)=-\frac{\partial P}{\partial x}+\mu\Delta u+\rho(|\Omega|^2x'+\dot{\Omega}y'+g_x)$$

$$\rho_0\left(\frac{\partial v}{\partial \tau}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+2u\Omega\right)=-\frac{\partial P}{\partial y}+\mu\Delta v+\rho(|\Omega|^2y'-\dot{\Omega}x'+g_y)$$

introducing the vorticity $\omega=\partial v/\partial x-\partial u/\partial y$ and the stream function $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$, we obtain

$$\rho_0\left(\frac{\partial \omega}{\partial \tau}+u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}+2\Omega\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right)=\mu\Delta\omega+\frac{\partial \rho}{\partial x}(|\Omega|^2y'-\dot{\Omega}x'+g_y)+$$

$$\rho\frac{\partial}{\partial x}(|\Omega|^2y'-\dot{\Omega}x'+g_y)-\frac{\partial \rho}{\partial y}(|\Omega|^2x'+\dot{\Omega}y'+g_x)-\rho\frac{\partial}{\partial y}(|\Omega|^2x'+\dot{\Omega}y'+g_x)$$

$$\Delta\psi=-\omega$$

If $\rho=\rho_0(1-\beta(T-T_0))$, where T_0 is the average temperature, then

$$\begin{gathered}\frac{\partial \omega}{\partial \tau}+u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}=\nu\Delta\omega-\beta\frac{\partial T}{\partial x}(|\Omega|^2y'-\dot{\Omega}x'+g_y)+\\ \beta\frac{\partial T}{\partial y}(|\Omega|^2x'+\dot{\Omega}y'+g_x)-2\dot{\Omega}\end{gathered} \tag{5.4}$$

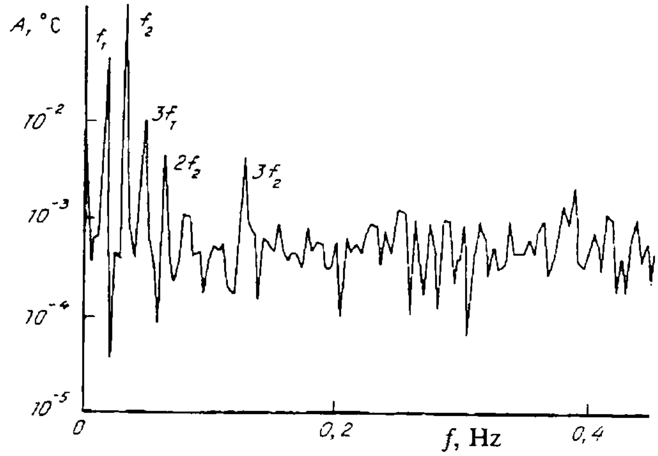

Fig. 8

$$\Delta\psi = -\omega \tag{5.5}$$

$$\frac{\partial T}{\partial \tau} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \chi \Delta T \tag{5.6}$$

On all the boundaries the boundary conditions for the velocity are the no-slip conditions. The temperature on the lower boundary of the enclosure was assumed to be equal to 20°C, and the temperature on the upper boundary was made constant on the interval 24—80°C. On the side faces of the enclosure for the temperature we consider either adiabatic conditions or a linear profile.

The rotation (rocking) of the chamber was specified as follows:

$$\alpha = \alpha_0 \cos(2\pi f\tau), \qquad \Omega = \dot{\alpha} = -\alpha_0 2\pi f \sin(2pf\tau), \qquad \dot{\Omega} = \alpha_0 (2\pi f)^2 \cos(2\pi f\tau)$$

where α is the angle of inclination of the chamber, α_0 is the inclination amplitude, equal to 1.5°, and f is the rocking frequency, which was varied between 0.1 and 1 Hz. Moreover, we made calculations for a constant angle of inclination on the interval from 0 to 3°.

The results presented below were obtained for the system of equations (5.4)—(5.6) by the finite-difference method using 17×17 and 33×33 grids. Some control calculations were also made by the finite-difference method for the system of equations (5.1)—(5.3) in the two-dimensional and three-dimensional cases for a constant angle of inclination of the chamber using 33×33 and 15×15 grids. Test calculations on various grids using different methods showed that acceptable accuracy (within 5% with respect to temperature and 10% with respect to velocity) can be achieved using a 17×17 grid, and this grid was adopted for the main calculations.

6. RESULTS OF THE CALCULATIONS

Calculations were made only for the cases of steady heating at constant and variable angles of inclination. As in the experiments described above, the principle characteristic investigated was the temperature difference between points with coordinates $x = 0.7$, $y = 2.3$, and $z = 0.7$ cm and $x = 2.3$, $y = 2.3$, and $z = 0.7$ cm (Fig. 1). The properties of the air were calculated for a temperature midway between the temperatures of the hot and cold faces of the chamber using the data of [4]. On the temperature interval in question the Prandtl number varied from 0.7 to 0.72, while the Rayleigh number, defined above, did not exceed $2 \cdot 10^5$.

In the calculations based on Eqs. (5.4)—(5.6) we determined the dependence of the transverse temperature difference Δt on the angle α and found that on the angular interval up to 3° this dependence, as in the experiments, is strictly linear; however, the proportionality factor differs considerably (by more than 1.5 times) from that obtained experimentally: $\Delta t = 13.5\alpha$. These calculations were made for $\Delta T = 30.2$°C with linear boundary conditions for the temperature on the vertical walls. Calculations made for the same parameters with adiabatic boundary conditions gave a value of the proportionality factor equal to 13.6. In this case the flow pattern in the chamber changed, especially in the core; for example, the horizontal velocity at the site of the thermocouples decreased from 0.3 to 0.05 mm/sec. In order to check the influence of three-dimensional effects on the flow we calculated the three-dimensional problem from Eqs. (5.1)—(5.3) for linear boundary conditions on the side faces using a 15×15×15 grid and obtained a value of 13.5 for the proportionality factor (13.1 on a 10×10×10 grid). Thus, three-dimensional effects and the type of boundary conditions do not explain the discrepancy between the experimental and numerical results, which may be attributable to the local temperature dependence of the physical properties, disregarded in the model in question, the nonvalidity of the Boussinesq approximation, the effect of the thermocouples on the slow flow and heat transfer in the neighborhood of the points at which they are located, and the leakage of heat through the thermoelectrodes. A similar discrepancy with respect to the experimental data was obtained in calculating the dependence of Δt on ΔT and also in the chamber rocking regime. The temperature difference between the junctions of the thermocouple (6) can be estimated by assuming that when the chamber rotates through an angle α the flow leads to strictly vertical stratification with a temperature gradient $\nabla T = \Delta T/H$ and $\Delta t = \nabla T L \alpha$, where L is the distance between the thermocouple junctions ($L = 1.6$ cm). Then $\Delta t = \Delta T L \alpha / H = 0.53 \Delta T \alpha$, which approximately corresponds to the calculations.

As a result of calculating variants in which the chamber was rocked we found, as in the experiments, that the transverse temperature difference Δt performs strictly sinusoidal oscillations with the same frequency as the rocking frequency. If in constructing the spectrum a time interval which is a multiple of the rocking period is used, then the amplitudes of all the other harmonics will be negligibly small. In Fig. 4 the broken curve represents the dependence of the amplitude of the transverse temperature difference pulsations on the model rocking frequency. As in the experiments, with increase in frequency the amplitude at first gradually decreases, until the oscillations are completely damped at a frequency $f_r^* = 0.77$ Hz, after which the amplitude begins to increase. As in the stationary case, the calculations give too high a value of the amplitude and the critical frequency f^* is displaced. At low frequencies the amplitude stabilizes and tends to the steady value of 0.35 obtained in the calculations for a constant angle of inclination of 1.5°. The broken curve in Fig. 5 represents the calculated phase difference between the oscillations of the transverse temperature difference and the angle of inclination of the chamber.

The causes of the quantitative discrepancy require further investigation.

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