Reynolds numbers, however, the falling speed of a sphere diminishes as its size approaches that of the suspended particles. The significance of these results to the theory of the viscosity of suspensions is discussed.

References

1) *Whitmore, R. L.*, Brit. J. Appl. Phys. **6**, 239 (1955).
2) *Ward, S. G.* and *R. L. Whitmore*, Brit. J. Appl. Phys. **1**, 286 (1950).
3) *Fidleris, V.*, Ph. D. Thesis, University of Nottingham (1958).
4) *Fidleris, V.* and *R. L. Whitmore*, Institute of Fuel Conference Science in the Use of Coal, Paper 10 (Sheffield 1958).
5) *Lunnon, R. G.*, Proc. Roy. Soc. A **118**, 680 (1928).
6) *Ward, S. G.*, J. Oil Col. Chemists Assoc. **38**, 9 (1955).
7) *Faxén, H.*, Dissertation (Uppsala 1921).
8) *Francis, A. W.*, Physics **4**, 403 (1933).
9) *Fidleris, V.* and *R. L. Whitmore* (to be published).
10) *Allen, H. S.*, Phil. Mag. **50**, 323 (1900).
11) *Liebster, H.*, Ann. Physik **82**, 541 (1927).
12) *Schmiedel, J.*, Phys. Z. **29**, 593 (1928).
13) *Möller, W.*, Phys. Z. **39**, 57 (1938).
14) *Wieselsberger, C.*, Phys. Z. **23**, 219 (1922).
15) *Davies, C.*, Proc. Phys. Soc. **57**, 259 (1945).
16) *Eirich, F. R.*, *M. Bunzl* and *H. Margaretha*, Kolloid-Z. **74**, 276 (1936).
17) *Maude, A. D.* and *R. L. Whitmore*, Brit. J. Appl. Phys. **7**, 98 (1956).
18) *Whitmore, R. L.*, Brit. Soc. of Rheology Conference (Swansea 1957).
19) *Eveson, G. F.*, *S. G. Ward* and *R. L. Whitmore*, Disc. Faraday Soc. **11**, 11 (1951).
20) *Roscoe, R.*, Brit. J. Appl. Phys. **3**, 267 (1952).
21) *Brinkman, H. C.*, J. Chem. Phys. **20**, 571 (1952).

Discussion

G. W. Scott Blair (Reading):

Has Dr. *Whitmore* any experimental evidence for, or any theoretical comments on a phenomenon which was, I believe, predicted or found by Prof. *Faxén*, namely that spherical particles falling near the wall of a container are deflected from their vertical motion?

F. R. Eirich (New York):

I agree fully with all your results but wish to point out that your projected experiments at a variety of concentrations were carried out by us some time ago (Kolloid-Z. **74**, 276, 1936) and led to the same findings as just described by you. In addition, we carried out experiments with non-spherical particles.

We did not anticipitate your elegant method of varying the sphere sizes, but concluded the prevalence of *Newton*ian flow from the absence of shear-rate dependance, except in the case of capillary flow. As for the latter, as you have also indicated, we found anomalies and explained them by inertia effects (Kolloid-Z. **85**, 260, 1938; Sitz.ber. Akad. Wiss. Wien **1938**). That the latter are absent in your falling sphere experiments is proven by the identical fall times observed by you, whether your falling spheres followed a straight or a round-about path; it shows that there is no relative translational motion between the resting spheres and the liquid passed by the falling sphere. The curvilinear path, incidentally, of two spheres passing at close distance, was postulated sometimes ago by myself and others, has been recently calculated by *Mooney* (J. Colloid Sci. **12**, 575, 1957) and experimentally shown by *Mason*.

J. E. Roberts (Seven Oaks/Kent):

Dr. *Fidleris* and Dr. *Whitmore* have used *Reynolds* number as a criteria. May I ask how they calculated this for the falling sphere? A sphere has different rates of shear at various points, so may I ask how they evaluated their *Reynolds* number.

R. L. Whitmore (Nottingham) Schlußwort:

In reply to Dr. *Scott Blair* we have not made any measurements of the motion of a sphere moving parallel to a plane-boundary but, besides the work of *Faxén*, at least one other author (*F. C. Karal*, J. App. Phys. **24**, 1947, 1953) has shown that a force will be developed which will push the sphere away from the wall.

We are of course aware of Professor *Eirich*'s pioneer work on the fall of particles in suspensions and we refer to it in our paper. In his experiments the falling sphere was very much larger than the suspended bodies and the flow was, in general, streamline or nearly so. This work we have extended to turbulent flow and shown that suspensions of spheres still behave as simple fluids. Our special interest, however, has been in the fall of spheres comparable in size with the suspended spheres. It might well be anticipated from the work of others that the rate of energy dissipation from shearing the general mass of suspension at a distance from the falling sphere would depend only on the concentration of spheres present and not on their size, but what did surprise us was that the rate of energy dissipation by the sphere, falling under streamline-flow conditions, while interacting with particles in its track (which involved the movement of falling and suspended particles along curvilinear paths, rotation about their own axes and probably the vertical displacement of the suspended particles) exactly equalled the rate at which energy would haye been dissipated if the sphere had fallen straight and unrotating through a *Newton*ian fluid of the same viscosity as the suspension. It is, of course, possible that the energy dissipated in particle interactions by the falling sphere is small compared with that dissipated in the remainder of the suspension but we did not investigate this point.

In reply to Mr. *Roberts* the *Reynolds* numbers for the falling spheres were calculated in the conventional way as used by many other workers and as described in the test of our paper. (Eg. see *Lamb, H.*, Hydrodynamics, 6th Ed. Cambridge 1932).

From the Koninklijke Shell Laboratorium Amsterdam (The Netherlands)

B 17

On the Rheology of Concentrated Dispersions

By C. van der Poel

(Published in Rheol. Acta 1, 198, 1958)

Discussion

J. G. Oldroyd (Swansea):

I am interested to know if Dr. *van der Poel* has a simple argument to justify in principle the use of the perturbation method of *Fröhlich* and *Sack* for values of the concentration which are not infinitesimal. It would be of interest to compare Dr. *van der Poel*'s extension of *Einstein*'s formula to large concentrations with that of *G. J. Kynch* (Proc. Roy. Soc. London A **237**, 90, 1956), which was derived theoretically by making use of an electromagnetic analogue of this problem.

W. Fritz (Braunschweig):

Bei der Strömung einer Flüssigkeit mit Fremdpartikeln anderer Dichte als die reine Flüssigkeit sollte man erwarten, daß die Partikel sich relativ zur Flüssigkeit bewegen. Die Versuche scheinen zu zeigen, daß der Einfluß dieser verschiedenen Bewegung von Partikel und Flüssigkeit gering ist oder nicht beobachtet werden kann. Welche Ansicht hat der Vortragende zu dieser Frage?

From the Department of Physics, Indian Institute of Science, Bangalore (India)

The Viscosity of Polydisperse Emulsions

B 18

By E. S. Rajagopal

(Received July 17, 1958)

1. Introduction

The classical problem of the *Newton*ian flow of disperse systems has attracted considerable attention in recent times owing to its importance in the study of such systems. The elegant investigations of *Einstein* (1) on the intrinsic viscosity of suspensions have been the fountain head of all the subsequent theoretical developments. His theory for the suspensions of solid incompressible spheres was extended to the dispersions of liquid spheres by *Taylor* (2). These investigations showed that the viscosity was proportional to the concentration of the dispersed phase, in the limit of low concentrations. The theoretical and experimental studies are summarised by *Hermans* (3) in his book.

But recent experiments have shown that the size of the particles affects the viscosity slightly [*Sherman* (4); *Orr* and *Blocker* (5)], a fact not understood in the usual theory. The effect can be explained if a more realistic model of the emulsion is considered, in which partial slipping can occur at the interface between the dispersed particle and the dispersion medium. Such a slipping will occur due to the presence of emulsifiers and other surface adsorbent materials. In order to explain the effects in detail, a very simple quantitative theory has been developed using the methods of *Fröhlich* and *Sack* (6) and of *Oldroyd* (7).

Instead of computing the viscous resistance experienced by the medium containing spherical particles by the usual hydrodynamic methods, as was done by *Einstein* and by *Taylor*, *Fröhlich* and *Sack* gave a new procedure of calculating the equivalent viscosity directly. Their method consists in identifying the macroscopic flow of the composite medium with the flow of an equivalent homogeneous medium. This method has been successfully applied by *Oldroyd* to elucidate the complex visco-elastic behaviour of emulsions. The present work is essentially similar but a simpler treatment which is applied to polydisperse emulsions.

2. Formulation of the Hydrodynamic Problem

Consider a dilute emulsion of concentration c and viscosity $\bar{\eta}$. The dispersed particles (not necessarily monodisperse) will be so far apart that mutual interactions can be neglected. The dispersed liquid is of viscosity η', while the dispersion medium is of viscosity η. One need consider only the uniform slow motion of the spheres and so the motion can be taken to be axisymmetric. For the small shears, the particles will remain spherical during the motion.

Let a be the radius of any dispersed particle. One forms a composite element by surrounding it from $r = a$ to $r = b$ (such that $a^3/b^3 = c \ll 1$) with the dispersion medium of viscosity η, and from $r = b$ to $r = R$ ($R \gg b$) with the equivalent medium of viscosity $\bar{\eta}$. The macroscopic flow of this composite element is identified with the flow of the homogeneous element which consists of a sphere of the equivalent medium of viscosity $\bar{\eta}$, filling the whole space $0 \leq r \leq R$. The two elements are identical except for the small part within $r = b$ and the influence of the part within $r = b$ upon the flow at $r = R$ must be of the order of the ratio of the two volumes, i. e., $\sim (b/R)^3$.

Fröhlich and *Sack* therefore demand that the ratio of the flow at $r = R$ of the two elements be

$$1 + \sum_{n>3} \text{const.}\,(1/R)^n \qquad [1]$$

that is, unity up to the third power of (b/R).

One can write the axisymmetric equations of continuity and of slow motions in polar coordinates as [e. g.. *Lamb* (8); *Milne-Thomson* (9)].

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 U_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, U_\theta) = 0 \qquad [2]$$