ALGEBRAICALLY CLOSED METABELIAN LIE ALGEBRAS

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Bokut' [1] introduced the concept of a Lie algebraic equation over a Lie algebra and proved that any Lie algebra can be embedded in an algebraically closed Lie algebra, i.e., in an algebra in which any Lie algebraic equation with coefficients in this algebra is solvable. He also raised questions about the validity of a similar assertion in other varieties of algebras, in particular, solvable Lie algebras.

In the present paper we prove (Theorem 2) that any metabelian Lie algebra can be embedded in an algebraically closed metabelian Lie algebra. We make some natural changes in the concept of a Lie algebraic equation over a metabelian Lie algebra, which make the equation conform to the metabelian structure of the algebra.

The spirit of the present paper is close to that of [1] and the principal method is the method of compositions, due to A. I. Shirshov [2], applied to metabelian Lie algebras.

We also prove that if each of two elements generates the same ideal of a free metabelian Lie algebra, then they are conjugate, to within a factor from the ground field, by an inner automorphism (Theorem 1). This assertion is connected with a well-known theorem of Magnus to the effect that if in a free group each of two elements g and h generates the same normal subgroup, then $g^{\pm 1}=c^{-1}hc$, and also with the paper of A. L. Shmel'kin [3], in which it was shown that the Magnus theorem is not true for free solvable groups.

1. A COMPOSITION FOR METABELIAN LIE ALGEBRAS

Suppose L is a free metabelian lie algebra over a field k and has a set $X=\{x_\alpha\}$ of free generators, ordered by their subscripts. We can take a basis of L consisting of the regular R_0- and R_1-words on X, i.e., words of the form

$$x_\alpha\,,\ x_{\alpha_0}x_{\alpha_1}\ \ldots x_{\alpha_s}\,,$$

where $\alpha_0>\alpha_1\leqslant\ldots\leqslant\alpha_s$, the arrangement of parentheses being right-normed [4]. Regular words of greater length are larger than words of smaller length, and regular words of the same length are ordered lexicographically from left to right.

Any element $v\in L$ can be represented in the form $v=v^{(0)}+v^{(1)}$, where $v^{(0)}\in L\setminus L^{(1)}$, $v^{(1)}\in L^{(1)}$. It is clear that $v^{(0)}$ is a linear combination of R_0-words, $v^{(1)}$ a linear combination of regular R_1-words.

The largest regular word occurring in v with nonzero coefficient is called the *leading word* of v and is denoted by $\overline{v}$.

If $u=x_\alpha$, then by a *subword* of u we mean u itself. Suppose $v=x_{\alpha_0}x_{\alpha_1}\ldots x_{\alpha_s}$ is a regular R_1-word. By the *subwords* of v we mean v itself, the R_0-words $x_{\alpha_0},\ldots,x_{\alpha_s}$,

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and also the regular R_1-words $x_{\alpha_0}x_{\beta_1}\dots x_{\beta_t}$ where $(\beta_1,\dots,\beta_t)$ is a subsequence of $(\alpha_1,\dots,\alpha_s)$. The subwords $x_{\alpha_2},\dots,x_{\alpha_s}$, and also x_{α_1} if $x_{\alpha_0} > x_{\alpha_2}$, will be called *strict sudwords* of the regular word v.

Suppose f and g are nonzero elements of the algebra L. We define their composition (f,g). The following cases, which are not mutually exclusive, are possible.

1) $f^{(1)} \neq 0$, $g^{(1)} \neq 0$. Suppose there exists a smallest regular R_1-word u that contains $\bar f$ and $\bar g$ as subwords, i.e., u is the leading word of elements $\bar f x_{\alpha_1}\dots x_{\alpha_s}$ and $\bar g x_{\beta_1}\dots x_{\beta_t}$. If $f=\alpha\bar f+\dots;\ g=\beta\bar g+\dots,\alpha,\beta\in k$, then the composition of f and g with respect to u is the element $(f,g)=\alpha^{-1}f x_{\alpha_1}\dots x_{\alpha_s} - \beta^{-1}g x_{\beta_1}\dots x_{\beta_t}$.

2) $f^{(1)}\neq 0$, $g^{(1)}\neq 0$, $g^{(0)}\neq 0$. Suppose $v=\bar f$ contains $\bar g^{(0)}$ as a strict subword, i.e., v is the leading word of $x_{\alpha_0}x_{\alpha_1}\dots x_{\alpha_s} g^{(0)}$, $s\geq 1$. If $g=\gamma\bar g^{(0)}+\dots,\gamma\in k$, then the composition of f and g with respect to v is the element $(f,g)=\alpha^{-1}f-\gamma^{-1}x_{\alpha_0}x_{\alpha_1}\dots x_{\alpha_s}g$.

3) $g^{(1)}=0$. Suppose $\bar f$ contains $\bar g^{(0)}$ as a subword, i.e., $\bar f=x_{\alpha_0}x_{\alpha_1}\dots x_{\alpha_s}$, $x_{\alpha_i}=\bar g^{(0)}=\bar g$ for some $0\leq i\leq s$. Then the composition of f and g with respect to $\bar f$ is the element $(f,g)=\alpha^{-1}f-\gamma^{-1}f_1$, where f_1 is obtained from $\bar f$ by replacing x_{α_i} by g.

In all other cases, f and g do not form compositions.

It follows at once from the definition of composition that the leading word of a composition is always strictly smaller than the word with respect to which this composition is formed.

Suppose $P=\{r_i\}$ is a set of elements of the algebra L; R is the ideal generated by this set; $S(P)=\{\bar s_i\}$ is the set of leading words of the elements r_i and of the elements obtained from the r_i by means of all possible composition (in any number); $S^{(0)}(P)$ is the set of leading words of the linear combinations of the elements $s_i^{(0)}$, $\bar s_i\in S(P)$.

<u>LEMMA 1.</u> If $f\in R$, then $\bar f$ either contains a subword that is an element of $S(P)$ or contains a strict subword that is an element of $S^{(0)}(P)$.

The proof of this lemma is similar to the proof of a lemma of A. I. Shirshov [2, Lemma 3], and so we omit it.

A set P is called *closed under compositions* if the composition of any elements $r_i, r_j \in P$ can be represented in the form

$$(r_i,r_j)=\sum \alpha_k r_k R_{v_{k,1}}\dots R_{v_{k,s}},$$

where the $v_{k,i}$ are regular words, R_v is the adjoint multiplication operator, $\alpha_k\in k$, and the leading words of all summands in the right-hand side are distinct (hence each of these leading words is smaller than the word with respect to which the composition is formed).

<u>COROLLARY.</u> If an ideal R is generated by a set P that is closed under compositions, then $\bar f$ either contains a subword that is the leading word of some element r_i or contains a strict subword of the form $\bar s^{(0)}$, where s is a linear combination of elements of P.

2. IDEALS GENERATED BY A SINGLE ELEMENT

Let us consider the case where the ideal R is generated by a single element r. If $r=r^{(0)}$ or $r=r^{(1)}$, then r cannot form compositions with itself. Now suppose $r^{(0)}\neq 0$,

$\tau^{(1)} \neq 0$. In this case, τ can form with itself only the composition of case 2), and $\bar{\tau}$ must contain $\bar{\tau}^{(0)}$ as a strict subword. If this condition is satisfied, consider the indicated composition $\tau_1 = (\tau, \tau) = \alpha^{-1}\tau - \alpha^{-1} a\tau$, $a \in L^{(1)}$. It is obvious that τ_1 also generates the ideal R, and $\bar{\tau}_1 < \bar{\tau}$, $\tau_1^{(0)} = \tau^{(0)}$. If $\bar{\tau}_1$ contains $\bar{\tau}^{(0)}$ as a strict subword, we again consider the composition $\tau_2 = (\tau_1, \tau_1)$, and so on. By induction on the leading word, we obtain an element τ', that generates the ideal R and is such that $\tau'^{(0)} = \tau^{(0)}$, $\bar{\tau}' < \bar{\tau}$, and also either $\tau' = \tau^{(0)}$ or else $\bar{\tau}'$ does not contain $\bar{\tau}^{(0)}$ as a strict subword.

Arguments analogous to the one given above for the leading word also apply to the other regular R_1-words occurring in the expansion of τ and containing $\bar{\tau}^{(0)}$ as a strict subword. Thus, we have the following

LEMMA 2. Suppose R is an ideal of L that is generated by an element τ . Then there exists an element $\tau^* \in L$ which also generates R and is such that $\bar{\tau}^* \leqslant \bar{\tau}$, $\tau^{*(0)} = \tau^{(0)}$, and no regular R_1-word occurring in τ^* contains $\bar{\tau}^{(0)}$ as a strict subword.

An automorphism φ of an algebra L is called the inner automorphism corresponding to $a \in L^{(1)}$ if for any $x \in L$ we have $\varphi(x) = x + xR_a$. It is known that the inner automorphisms form a normal subgroup of the group of all automorphisms of L.

It follows immediately from the definition of composition in case 2) that the composition $\tau_1 = (\tau, \tau)$ of an element τ with itself is obtained by the action on the element $\alpha^{-1}\tau$ of some inner automorphism. Therefore, the element τ^* mentioned in Lemma 2 also has the form $\tau^* = \beta\varphi(\tau)$, where φ is an inner automorphism.

THEOREM 1. Suppose each of the elements τ_1 and τ_2 generates the same ideal R of the algebra L. Then there exist an element $\alpha \in k$ and an inner automorphism φ such that $\tau_1 = \alpha\varphi(\tau_2)$.

Proof. Obviously, the elements $\tau_1 = \tau_1^{(0)}$ and $\tau_2 = \tau_2^{(1)}$ cannot generate the same ideal. Assume that $\tau_1 = \tau_1^{(0)}$, $\tau_2 = \tau_2^{(0)}$ or $\tau_1 = \tau_1^{(1)}$, $\tau_2 = \tau_2^{(1)}$. In these cases the elements τ_1 and τ_2 do not form compositions with themselves. By Lemma 1, $\bar{\tau}_1$ contains the subword $\bar{\tau}_2$, and $\bar{\tau}_2$ the subword $\bar{\tau}_1$, i.e., $\bar{\tau}_1 = \bar{\tau}_2$. Since the composition (τ_1, τ_2), which in this case has the form $\alpha_1^{-1}\tau_1 - \alpha_2^{-1}\tau_2$, lies in R and has a smaller leading word, it follows that $(\tau_1, \tau_2) = 0$, i.e, $\tau_1 = \alpha\tau_2$.

Now assume that $\tau_2^{(0)} \neq 0$, $\tau_2^{(1)} \neq 0$. Since each of τ_1 and τ_2 generates the same ideal, we have $\tau_1^{(0)} = \beta\tau_2^{(0)}$. Consider the elements $\tau_1^* = \beta_1\varphi_1(\tau_1)$, $\tau_2^* = \beta_2\varphi_2(\tau_2)$, defined by Lemma 2, where the φ_i are inner automorphisms. Since τ_1^* and τ_2^* do not form compositions with themselves, we again have $\bar{\tau}_1^* = \bar{\tau}_2^*$. Since the leading word of the composition $(\tau_1^*, \tau_2^*) = \alpha_1^{-1}\tau_1^* - \alpha_2^{-1}\tau_2^*$ is strictly smaller than $\bar{\tau}_2^*$, it follows that either $(\tau_1^*, \tau_2^*) = 0$ or else $\overline{(\tau_1^*, \tau_2^*)}$ contains, by Lemma 1, the strict subword $\bar{\tau}_2^{(0)}$. But the latter alternative is impossible, since the regular R_1-words occurring in τ_1^* and τ_2^* do not contain the indicated strict subword. Thus, $\tau_1^* = \beta\tau_2^*$, i.e., $\tau_1 = \beta_1^{-1}\beta\beta_2\varphi_1^{-1}\varphi_2(\tau_2)$.

The theorem is proved.

3. ALGEBRAICALLY CLOSED METABELIAN LIE ALGEBRAS

Suppose A is a metabelian Lie algebra over the field k and $\{v_i\} \cup \{w_j\}$ is a basis of this algebra, where the v_i are linearly independent modulo the commutant and where each $w_j \in$

$A^{(1)}$. Let S_n denote the free metabelian Lie product $S_n = A * k\langle x_1\rangle * \ldots * k\langle x_n\rangle$ of A and the free Lie algebras $k\langle x_i\rangle$ with generators x_i.

We associate with the elements w_i the symbols a_i, and with the elements v_j the symbols b_j and consider the free metabelian Lie algebra F_n with free generators $\{a_i\}\cup\{b_j\}\cup\{x_1,\ldots,x_n\}$, where a_i, b_j are ordered by their subscripts and $x_n > \ldots > x_1$, $x_i > a_j > b_k$ for any i, j, k. Let Q be the ideal of F_n generated by the set M consisting of the elements

$$a_i b_j - \sum \gamma_{i,j}^{k} a_k, \quad \text{if} \quad w_i v_j = \sum \gamma_{i,j}^{k} w_k \quad \text{in the algebra } A; \tag{1}$$

$$b_i b_j - \sum \delta_{i,j}^{k} a_k, \quad \text{if} \quad v_i v_j = \sum \delta_{i,j}^{k} w_k, \ i > j; \tag{2}$$

$$a_i a_j \quad \text{for all} \quad i > j. \tag{3}$$

Then S_n is isomorphic to F_n/Q, and this isomorphism extends the mappings $a_i \longmapsto w_i$, $b_j \longmapsto v_j$, $x_k \longmapsto x_k$.

<u>LEMMA 3.</u> The set M is closed under compositions.

<u>Proof.</u> Denote the free generators of F_n different from $x_1,\ldots,x_n$, by c_k. Since $c_j < x_i$, the elements of the set M can form compositions in case 1) only under multiplication by c_i. For $c_i > c_k \geqslant c_j$ consider the composition

$$\begin{aligned}&(c_i c_j - \sum \alpha_{i,j}^{\ell} c_\ell)c_k - (c_i c_k - \sum \alpha_{i,k}^{\ell} c_\ell)c_j = \\ &= (c_i c_j - \sum \alpha_{i,j}^{\ell} c_\ell)c_k - (c_i c_j - \sum \alpha_{i,j}^{\ell} c_\ell)c_k - (c_k c_j - \sum \alpha_{k,j}^{\ell} c_\ell)c_i = -(c_k c_j - \sum \alpha_{k,j}^{\ell} c_\ell)c_i,\end{aligned}$$

where the $\alpha_{i,j}^{k}$ are structure constants of the Lie algebra A.

Compositions in case 2) can be formed only by the elements (3) among themselves. For these elements and $i > k \geqslant j$ we also obtain

$$(a_i a_j)a_k - (a_i a_k)a_j = -(a_k a_j)a_i.$$

The elements of M do not form compositions of type 3).

The lemma is proved.

Following [5], a regular word u of the algebra F_n will be called *special* if u contains no subwords of the form $a_i a_j$, $b_i b_j$, $a_i b_j$, or strict subwords a_i.

It follows directly from Lemma 1 that if $q \in Q$, then $\overline{q}$ is not special. Therefore, the images of the special words in the algebra $S_n \cong F_n/Q$ are linearly independent and can be taken as a basis of this algebra, which we do.

From the algorithm for reducing words to regular form and Lemma 1 it is easy to obtain.

<u>LEMMA 4.</u> Any element f of the algebra can be uniquely represented in the form $f = f^{o} + q$, where f^{o} is a linear combination of special words and $q \in Q$.

By a Lie algebraic equation (in n unknowns) over a metabelian Lie algebra A we mean an expression of the form $p(x_1,\ldots,x_n) = 0$, where $p \in S_n$, $p \notin A$, and if the linear part $p^{(0)}$ of p lies in A, then it is contained in the annihilator of the commutant of A.

A metabelian Lie algebra B is called algebraically closed if every Lie algebraic equation over B has a solution in B.

The last condition in the definition of a Lie algebraic equation over a metabelian Lie algebra is necessary because if the linear part of an equation contains no unknowns and does not lie in the annihilator of the commutant of the algebra, then such an equation cannot have a solution in any metabelian extension of the original algebra. Therefore, ignoring similar cases, we consider only those Lie algebraic equations that are compatible with the metabelian structure of the algebra.

If $v=\sum \alpha_i v_i$ is an element of the annihilator of the commutant $A^{(1)}$, then A can be embedded in a metabelian algebra $\hat{A}$ in which v lies in the commutant. Indeed, consider a free metabelian Lie algebra F with free generators $\{c_1, c_2\} \cup \{a_i\} \cup \{b_j\}$, where $c_1 < c_2 < b_i < a_j$ and the ideal R of this algebra generated by the set M and the element $c_2c_1 - \sum \alpha_i b_i$. This element, being multiplied by certain regular words, can form only compositions of type 2) with the elements (1) and (2) of M. We have $(c_2c_1 - \sum \alpha_i b_i)a_j = c_2c_1a_j$ modulo the ideal generated by the set M in F, since v lies in the annihilator of the commutant $A^{(1)}$ of A. Since, by Lemma 3, the set M is closed under compositions, we obtain, by applying Lemma 1, that the leading word of any element of R either is the leading word of some element of the ideal generated by M or else contains c_1 or c_2. Therefore, the quotient algebra $\hat{A} = F/R$ contains a subalgebra isomorphic to A, and the image of the element $v=\sum \alpha_i v_i$ lies in the commutant of $\hat{A}$.

By embedding, if necessary, the algebra A in $\hat{A}$ we will assume that any Lie algebraic equation p = 0 over A satisfies the following condition: if $p^{(0)} \in A$, then $p^{(0)} \in A^{(1)}$.

Suppose p = 0 is a Lie algebraic equation over A, f is one of the preimages of p in the algebra F_n, and N is the ideal of F_n, generated by the ideal Q and element f. By the observation made above and the definition of a Lie algebraic equation over A we have for two possibilities:

$$(i)\quad f^{(0)} = \sum \lambda_i a_i ,$$

$$(ii)\quad f^{(0)} = \sum \lambda_i x_i + \sum \mu_i a_i + \sum \nu_i b_i .$$

In case (ii), we renumber the generators $x_1, \dots, x_n$ so that $\overline{f^{(0)}} = x_n$. Since each of f and f^*, defined in Lemma 2, generate the same ideal, we may assume without loss of generality that $f = f^*$. In view of Lemma 4, we may also assume that $f = f^0$.

<u>LEMMA 5.</u> Any element $u \in N$ can be represented in the form

$$u = \sum \alpha_i f R_{a_i} + \sum \beta_i f R_{c_i} + \sum \gamma_i f R b_{i_1} \dots R_{b_{i_s}} R_{x_{j_1}} \dots R_{x_{j_t}} + q, \tag{4}$$

where the c_i are special R_1-words, $i_1 \leq \dots \leq i_s$, $j_1 \leq \dots \leq j_t$, $q \in Q$.

<u>Proof.</u> Suppose $u \in N$. Using, if necessary, the relation $R_u R_v = R_v R_u + R_{[u,v]}$, we may assume that

$$u=\sum \alpha_i f R_{b_{i_1}} \dots R_{b_{i_s}} R_{x_{j_1}} \dots R_{x_{j_t}} R_{a_{k_1}} \dots R_{a_{k_\ell}} R_{c_i} + q', \tag{5}$$

where $i_1 \leq \dots \leq i_s$, $j_1 \leq \dots \leq j_t$, $k_1 \leq \dots \leq k_\ell$, the c_i are regular R_1-words, $q' \in Q$.

Since for any $y \in F_n$ we always have $fR_y \in F_n^{(1)}$, it follows that $fR_y R_{c_i} = 0$, hence the R_{c_i} are present only in those summands of (5) in which all other adjoint multiplication operators are absent. Since the w_i form a basis of the commutant $A^{(1)}$, each a_i is the linear part of some element q_i of the ideal Q, so that $fR_y R_{a_i} = fR_y R_{q_i} \in Q$. Therefore, we may also assume that the operator R_{a_i} is present only in those summands of (5) in which all other adjoint multiplication operators, including the R_{a_j} are absent.

If some c_i is not special, then $c_i = \overline{q}_i$, $q_i \in Q$. Using the relation $R_{c_i} = R_{\alpha q_i} - R_{(\alpha q_i - c_i)}$, where $q_i = \alpha^{-1}\overline{q}_i + \dots$, we can insure that all of the c_i be special R_1-words.

The lemma is proved.

<u>LEMMA 6.</u> If an element u belongs to the ideal N and $u \notin Q$, then the word $\overline{u}^o$ contains one of the elements x_i as a subword.

The *proof* of this lemma is basically the same as the proof of a lemma of A. I. Shirshov [2, Lemma 3]. Therefore, we will only make a few comments. We represent u in the form (4).

1) Suppose f satisfies condition (i). Then fR_{a_i} and fR_{c_i} lie in Q. Since $\rho \notin A$, we have $f^{(1)} \neq 0$. Also, since any special R_1-word must begin with some x_i, the leading word of the element $fR_{b_{i_1}} \dots R_{b_{i_s}} R_{x_{j_1}} \dots R_{x_{j_t}}$ must be special. Inasmuch as such elements have distinct leading words, $\overline{u}^o$ contains some x_i.

2) Suppose f satisfies condition (ii).

a) If $f^{(0)} = 0$, then $fR_{a_i} \in Q$ and $fR_{c_i} = 0$, hence we can again apply the argument of 1).

b) If $f^{(0)} \neq 0$, then $\overline{(fa_i)}^o = x_n a_i$, $\overline{(fc_i)}^o = c_i x_n$. Elements of the form $(fR_{a_i})^o$ and $(fR_{c_i})^o$ cannot have equal leading words. There remain two possibilities.

First, the leading words of $fR_{b_{i_1}} \dots R_{b_{i_s}} R_{x_{j_1}} \dots R_{x_{j_t}}$ and fR_{c_i} can be the same. Then $x_{j_t} = x_n$, and c_i is the leading word of $h = fR_{b_{i_1}} \dots R_{b_{i_s}} R_{x_{j_1}} \dots R_{x_{j_{t-1}}}$. Using the equalities

$$fR_{c_i} = fh^{(1)} - f(h^{(1)} - c_i),$$

$$fR_{x_n} = \alpha^{-1} h f^{(0)} - h(\alpha^{-1} f^{(0)} - x_n), \text{ where } f^{(0)} = \alpha x_n + \dots,$$

$$fh^{(1)} = (f^{(0)} + f^{(1)})h^{(1)} = f^{(0)}h^{(1)} = f^{(0)}h - f^{(0)}h^{(0)},$$

we obtain

$$fR_{c_i} = -\alpha h R_{x_n} - h(f^{(0)} - \alpha x_n) - f(h^{(1)} - c_i) - f^{(0)}h^{(0)}. \tag{6}$$

If $h = \beta f$, then $f^{(0)}h^{(0)} = 0$. If h is obtained by the action on f of at least one adjoint multiplication operator, then $h^{(0)} = 0$. Therefore, in the right-hand side of (6) there appear elements formed analogously to fR_{c_i} and h whose leading words are no larger than

The second possibility is that $\overline{f} = x_n a_i$. But then, as is easily seen, $(\overline{f - fa_i})^0$ contains some x_i.

The lemma is proved.

LEMMA 7. Suppose $p(x_1, \ldots, x_n) = 0$ is any Lie algebraic equation over a metabelian algebra A. Then A can be embedded in a metabelian algebra B over the same field in which this equation has a solution.

Proof. As B we can take F_n/N. The given equation is solvable in B, and it follows directly from Lemma 6 that B contains a subalgebra isomorphic to A.

THEOREM 2. Any metabelian Lie algebra over a field k can be embedded in an algebraically closed metabelian Lie algebra over the same field.

Proof. The set Λ of all Lie algebraic equations over the metabelian algebra A can be well ordered. Suppose $\Lambda = \{p_{\varkappa} = 0\}$, $1 \leqslant \varkappa \leqslant \tau$, τ an ordinal number. By Lemma 7, the algebra A can be embedded in a metabelian algebra B in which the equation $p_1 = 0$ has a solution. Therefore, by a simple transfinite induction we can establish the existence of a metabelian algebra A_1 in which any equation in Λ is solvable. In the same way we embed A_1 in , and so on. As a result, we obtain an ascending chain of metabelian Lie algebras

$$A \subseteq A_1 \subseteq \ldots \subseteq A_n \subseteq \ldots .$$

The union D of the algebras in this chain satisfies the requirement of the theorem, since any Lie algebraic equation over D is an equation over some algebra A_κ.

The theorem is proved.

It is known that any submodule of a Lie algebra containing the commutant of the algebra is an ideal of the algebra. It follows from the solvability of the Lie algebraic equations $ax = b$ that an algebraically closed metabelian Lie algebra contains no other ideals. Therefore, we have the following.

COROLLARY. Any metabelian Lie algebra can be embedded in a metabelian Lie algebra in which each ideal contains the commutant.

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