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GROUPS WITH ABELIAN SUBGROUPS OF FINITE RANKS

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One of the basic finiteness conditions in groups is the condition of finiteness of special rank in the sense of Mal'tsev [1]. The fact that this condition is fundamental was brilliantly displayed in the investigations on locally solvable groups. The achievements in this direction are well reflected in the familiar survey of Robinson [2]. In what follows, the special rank in the sense of A. I. Mal'tsev will be called simply the *rank* of the group. In the study of groups of finite rank the following question turned out to be an exceptionally fruitful direction: for which groups does the finiteness of the rank of the group group follow from the finiteness of the ranks of Abelian subgroups? Precisely in solving this question the most profound results in the theory of locally solvable groups were obtained. Thus, in the class of solvable groups A. I. Mal'tsev characterized the polycyclic groups as groups in which all Abelian subgroups are finitely generated [3], M. I. Kargapolov characterized groups of finite rank as groups in which Abelian subgroups have finite ranks [4], and Yu. I. Merzlyakov characterized groups of finite rank in the class of locally solvable groups as groups for which the ranks of Abelian subgroups are bounded in aggregate [5]. Merzlyakov [6] also showed that in his theorem the boundedness of ranks of Abelian subgroups is essential.

For periodic groups the theorem of Kargapolov formulated above was successfully generalized to locally solvable groups (Gorchakov [7]). In the present paper Kargapolov's theorem [4] is generalized to periodic binary solvable groups and thus a new characterization of periodic locally solvable groups of finite rank is obtained (Theorem 2). From Theorem 2, in particular, there follows the positive solution of a question from [8] for the case of periodic groups. Moreover, new characterizations are also given of locally solvable finitely layered groups and groups with the primary minimality condition (Theorems 3 and 4). Theorem 5 gives necessary and sufficient conditions for the local finiteness of a periodic F^*-group with Abelian subgroups of finite ranks and with finite Sylow p-subgroups, and its corollary allows us to single out finite groups from an arbitrary class of groups.

1. Preliminary Information

1. Definition. The group G has finite rank r, if r is the smallest number with the property that any finite set of elements of G generates a subgroup with no more than r generators.

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2. Kargapolov's Theorem [3]. A periodic almost locally solvable group with Chernikov Sylow p-subgroups for all p has a complete part.

3. Kargapolov's Theorem [4]. Let $A \triangleleft G$ and A be Abelian. If all Abelian subgroups of the group G have finite rank, then the Abelian subgroups of the group G/A also have finite rank.

4. Gorchakov's Theorem [7]. A periodic locally solvable group has finite rank if and only if the ranks of Abelian subgroups in it are finite.

5. Hall's Theorem [9]. Let the automorphism φ of the finite p-group P induce the identity automorphism on $P/\Phi(P)$. Then φ is the identity automorphism.

6. Jordan–Brauer–Feit Theorem [10]. Let G be a finite linear p'-group of degree n over a field of characteristic p. Then G has an Abelian normal subgroup A such that $|G:A|<f(n)$, where $f(n)$ is a number depending only on n.

7. Definition. The group G is said to be q-biprimitively finite, if for any finite subgroup H of the group G in $N_G(H)/H$ any two elements of order q generate a finite subgroup (q is a prime number). If G is q-biprimitively finite for all $q \in \pi(G)$, then G is said to be biprimitively finite. The group G is said to be a π-biprimitively finite group ($\pi \subset \pi(G)$), if it is q-biprimitively finite for any $q \in \pi$.

8. Let G be a q-biprimitively finite group; N be a Chernikov normal subgroup. Then G/N is a q-biprimitively finite group [11].

9. Let G be a q-biprimitively finite group; N be a normal q'-subgroup. Then G/N is a q-biprimitively finite group.

Proof. We consider in G/N the subgroup generated by elements $\bar{a}$ and $\bar{b}$ of order q. Some preimages of $\bar{a}$ and $\bar{b}$ in G we denote respectively by a and b. Obviously $a^q \in N$ and $b^q \in N$. Without loss of generality one can assume that a and b are elements of order q. By the definition of q-biprimitive finiteness, gr (a, b) is finite and hence gr $(a, b)=$ gr (aN, bN) is finite. The assertion is proved.

10. Let G be a q-biprimitively finite group and let some Sylow q-subgroup of it be finite. Then all Sylow q-subgroups in G are finite and conjugate [12].

11. A biprimitively finite p-group is a Chernikov group if and only if it has at least one finite maximal elementary Abelian subgroup [13].

12. Frattini's Lemma. Let G be a group; N be a normal subgroup; S be a Sylow q-subgroup of N. If Sylow q-subgroups are conjugate in N, then $G = N_G(S)N$.

13. Let G be a q-biprimitively finite group with Chernikov Sylow q-subgroups; N be a normal subgroup. If Sylow q-subgroups are conjugate in N, then G/N is a q-biprimitively finite group with Chernikov Sylow q-subgroups.

Proof. By hypothesis, Sylow q-subgroups in N are conjugate. Let Q be one of them. By Frattini's lemma, $G=N_G(Q)N$, and by the isomorphism theorem [14, Theorem 4.2.2], $G/N \simeq N_G(Q)/T$, where $T=N_G(Q)\cap N$. By Proposition 8, $N_G(Q)/Q$ is q-biprimitively finite. Since T/Q does not contain q-elements, by Proposition 9 the group $G/N \simeq N_G(Q)/Q/T/Q$ is q-biprimitively finite.

Now we shall show that Sylow q-subgroups of G/N are Chernikov. For this, using Propositions 8, 12, the already proved q-biprimitive finiteness of G/N, and the isomorphism theorem, it suffices for us to consider the case when N is a q'-group. Let us assume that our assertion is false. In this case by Propositions 9, 11, G/N has an infinite elementary Abelian q-subgroup. Without loss of generality we shall assume that G/N itself is an infinite elementary Abelian q-subgroup. From this, Propositions 10, 12, and the isomorphism theorem, it follows that in G one can construct a strictly increasing chain of finite q-subgroups:

$$S_1 < S_2 < \dots < S_n < \dots \quad . \tag{1}$$

Obviously S_n are elementary Abelian q-subgroups and the union of the chain (1) is not a Chernikov group, contrary to the hypothesis of the proposition. Consequently, the Sylow q-subgroups of G/N are Chernikov, and the proposition is proved.

14. Let G be a q-biprimitively finite group with a finite Sylow q-subgroup S; N be a normal subgroup in G. Then SN/N is Sylow in G/N and Sylow q-subgroups are conjugate in G/N.

Proof. By Proposition 13, $\bar{S} = SN/N < B$, where B is a Chernikov Sylow q-subgroup of G/N. Let us assume that $\bar{S} \neq B$. Since B satisfies the normalizer condition [14, Theorem 16.2.1], one has $\bar{P} = N_B(\bar{S}) \neq \bar{S}$. Let P be the complete preimage of $\bar{P}$ in G. Obviously $SN = K \lhd P$. By hypothesis, S is a finite Sylow q-subgroup in P and P is a q-biprimitively finite group. But then in view of Propositions 10, 12, $P = N_p(S)K$, and by the isomorphism theorem, $P/K \simeq N_p(S)/V = M$, $V = K \cap N_p(S)$, where $S \leqslant K \cap N_p(S)$. Obviously M contains nontrivial q-elements of S. But this is impossible, since M is a quotient group of $N_p(S)$, and S is a subgroup of $N_p(S)$. Consequently, $\bar{S} = B$ is a Sylow q-subgroup in G/N, and on the basis of Proposition 10 we conclude that Sylow q-subgroups are conjugate in G/N.

The proposition is proved.

15. The group G is said to be binary solvable if any two elements of it generate a solvable subgroup.

16. Thompson's Theorem [16]. A finite binary solvable group is solvable.

17. Definition. One says that the group G satisfies the p-min condition, if any decreasing chain of subgroups $H_1 > H_2 > \dots > H_n > \dots$ such that there are p-elements in $H_n \setminus H_{n-1}$, stops at a finite index. The group G satisfies the primary minimality condition, if it satisfies the p-min condition for any $p \in \pi(G)$.

18. Polovitskii's Theorem [19]. A periodic locally solvable group G satisfies the primary minimality condition if and only if it is an extension of a complete Abelian subgroup A with Chernikov Sylow subgroups by a locally normal group with finite Sylow p-subgroups for all p, while each element of G is elementwise noncommutative with only a finite number of Sylow subgroups of A.

19. Shunkov's Theorem [11]. An infinite biprimitively finite group has an infinite Abelian subgroup.

20. Myagkova's Theorem [18]. A locally finite p-group has finite rank if and only if it is a Chernikov group.

21. Definition. Let G be a group, S be any finite p-subgroup for some $p \in \pi(G)$. We consider

$$\left| N_G(S) \;/\; SC_G(Q) \right| = p^{\alpha} n_S,\ (n_S, p) = 1.$$

We denote by $u_p(G)$ the number equal to $\max n_S$, if the numbers n_S are bounded in aggregate. If not we set $u_p(G) = \infty$. We call $u_p(G)$ the exponent of p-inclusion of a Sylow p-subgroup in G. (This concept was introduced by V. P. Shunkov in connection with the present paper.) If $H < G$, then obviously $u_p(H) \leqslant u_p(G)$.

22. Let G be a finite group, $N \triangleleft G$. Then $u_p(G/N) \leqslant u_p(G)$.

Proof. First we consider the case when N is either a p-group or a p'-group.

Let N be a p-group. We denote by B, P, C, respectively, the complete preimages of $N_{\bar{G}}(\bar{P})$, $\bar{P}$, $C_{\bar{G}}(\bar{P})\bar{P}$ in G. Obviously $PC_G(P) \leqslant C$, and if $|B:C| = p^{\alpha} m$, $(p, m) = 1$, then from $|B:PC_G(P)| = |B:C||C:C_G(P)|$ we get $m \leqslant u_p(G)$. This means that $u_p(G/N) \leqslant u_p(G)$.

Now let N be a p'-group. By Proposition 24, we get $u_p(G/N) \leqslant u_p(G)$.

Now we consider the arbitrary case. Let S be some Sylow p-subgroup of N. By Frattini's Lemma, we have $G = N_G(S)N$. From the preceding, in view of Theorem 4.2.3 and 4.2.4 of [14],

$$G/N \simeq N_G(S)N/N \simeq N_G(S)/N \cap N_G(S) \simeq N_G(S)/S/T/S,$$

where $T = N \cap N_G(S)$, we get $u_p(G/N) \leqslant u_p(G)$.

The assertion is proved.

23. Let G be a finite solvable group and a be some q-element, $q > u_p(G)$, $q \neq p$. Then $a \in O_{p'}(G)$.

We shall give a *proof* by induction on the order of the group. Let K be a minimal normal subgroup in G. In view of [14, Theorem 19.1.7], it is an elementary Abelian r-subgroup. If $r \neq p$, then by Proposition 22, $G/K = \bar{G}$ satisfies all the hypotheses of the assertion and by the inductive hypothesis, $aK \in O_{p'}(\bar{G})$. Now taking the complete preimage of the subgroup indicated, it is easy to get $a \in O_{p'}(G)$, which is what was required.

Let r = p. We shall show that $a \in C_G(K)$. In fact, if $a \notin C_G(K)$, then from the hypotheses of the proposition and Definition 21 it would follow that q divides n_K, where $n_K p^{\alpha} = |G : C_G(K)|$ and $q > n_K$, but this is impossible. Thus, $a \in C_G(K) \triangleleft G$. Further, $O_{p'}(C_G(K))$ is a characteristic subgroup in $C_G(K)$, and hence $O_{p'}(C_G(K)) \triangleleft G$. If $G \neq C_G(K)$, then by the inductive assumption $a \in O_{p'}(C_G(K))$ and hence $a \in O_{p'}(G)$ also. Now let $G = C_G(K)$, i.e., $K \leqslant Z(G)$. In view of Proposition 22, $aK \in O_{p'}(\bar{G})$. We take B, the complete preimage of $O_{p'}(\bar{G})$. Since $K \leqslant Z(B)$ and K is a Sylow p-subgroup of B, B has the form $B = K \times T$, where T is a p'-subgroup and $T \triangleleft G$. Obviously $a \in T$ and hence $a \in O_{p'}(G)$.

The proposition is proved.

24. Let G be a q-biprimitively finite group; T be a normal q'-subgroup and P be a finite q-subgroup, $\bar{G} = G/T$, $\bar{P} = PT/T$. Then

$$N_{\bar{G}}(\bar{P}) = N_G(P)T/T,$$

$$C_{\bar G}(\bar P) = C_G(P)T/T.$$

Proof. The validity of this assertion is easily gotten, using Propositions 10, 12.

25. Let G be a group, B be a π-subgroup, N be a normal π'-subgroup. If $G/N = BN/N \times S$ and gr $(B, N) = B \times N$, then $G = B \times L$.

The proof is obvious.

26. Definition. We shall say that two groups have isomorphic Sylow series (the same Sylow series), if the lengths of these series coincide and for corresponding indices the quotients are isomorphic.

2. Biprimitively Finite Groups with Finite Sylow p-Subgroups

LEMMA 1. Let G be a periodic binary solvable group with Chernikov Sylow p-subgroups for all p. If $\pi(G)$ is finite, then G is a Chernikov group.

Proof. As is known from [14, Theorem 10.1.2], any periodic Abelian group splits into the direct product of its Sylow p-subgroups. From this and the hypotheses of the lemma and the finiteness of $\pi(G)$ it follows that G satisfies the minimality condition for Abelian subgroups. But then G is a Chernikov group [17].

The lemma is proved.

LEMMA 2. Let G be a periodic binary solvable group with Chernikov Sylow p-subgroups for all p. Then G has a complete part.

Proof. First we shall show that all quasicyclic subgroups of G generate an Abelian subgroup. Let P and Q be arbitrary quasicyclic subgroups of G. Using the same method as in [17], we show that T = gr(P, Q) is a periodic locally solvable group. In fact, we represent P and Q as unions of chains of subgroups:

$$\underset{p}{(a_1)} < \underset{p^2}{(a_2)} < \ldots < \underset{p^n}{(a_n)} < \ldots < P,$$

$$\underset{q}{(b_1)} < \underset{q^2}{(b_2)} < \ldots < \underset{q^n}{(b_n)} < \ldots < Q,$$

where $|a_n| = p^n$, $|b_n| = q^n$, $n = 1, 2, \ldots$, p and q are prime numbers. We consider the series of subgroups:

$$\mathrm{gr}\,(a_1, b_1) < \mathrm{gr}\,(a_2, b_2) < \ldots < \mathrm{gr}\,(a_n, b_n) < \ldots < T.$$

In view of the definition of binary solvability, gr (a_n, b_n) $(n = 1, 2, \ldots)$ are solvable, and consequently, their union T will be a periodic locally solvable group with Chernikov Sylow p-subgroups for all $p \in \pi(T)$. In view of Kargapolov's Theorem (Proposition 2), T is an Abelian group. Thus, it is proved that all quasicyclic subgroups of G generate an Abelian subgroup R. Obviously $R \triangleleft G$ and in view of Proposition 13, 14, the quotient group G/R is a binary solvable group with Chernikov Sylow p-subgroups.

The lemma is proved.

LEMMA 3. A periodic binary solvable group with finite Sylow p-subgroups for all $p \in \pi(G)$ is finitely approximable.

Proof. Let the group G satisfy the hypotheses of the lemma. If $\pi(G)$ is finite, then in view of Lemma 1 and the hypotheses of the lemma being proved, the group G is finite. Let $\pi(G)$ be infinite. In view of Proposition 10, the Sylow p-subgroups in G are conjugate. But since they are finite, $U_p(G)$ is a finite number, independent of the choice of Sylow p-subgroup. We fix some prime number p. Let $\mathcal{L}$ be a subset of elements of G such that for all $q \in \pi(\mathcal{L})$ one has $q \neq p$ and $q > U_p(G)$.

Remark. The set $\mathcal{L}$ generates a normal p'-subgroup T.

Proof. We consider the set of words written is terms of elements of $\mathcal{L}$. Let us assume that they do not all give p'-elements. Then among them one can find an element whose order is divisible by p. We choose among such elements a word of least length. Let this be $a = s_1(s_2 \cdot s_3 \cdot \ldots \cdot s_n), |a| = pk$. We write $b = s_2 s_3 \ldots s_n$ so $a = s_1 b$, the word b has length less than n and by the inductive hypothesis is a p'-element. We consider $L = \operatorname{gr}(b, s_1)$. Since it is a periodic solvable group, by Theorem 22.3.1 of [14] it is finite. Since $s_1 \in \mathcal{L}$, in view of the definition of $\mathcal{L}$, for a prime divisor q of the order of the element s_1 we have $q > U_p(G)$ but then by Proposition 15, $s_1 \in O_{p'}(L)$, and obviously in this case $L = O_{p'}(L)(b)$ and L is a p'-group. But since $a \in L$, we have arrived at a contradiction with the assumption that a is not a p'-element. Consequently, the set $\mathcal{L}$ generates a p'-subgroup T, which is obviously normal in G. The remark is proved.

We proceed directly to the proof of Lemma 3. The quotient group G/T is again a binary solvable group with finite Sylow p-subgroups (Proposition 14), while $\pi(G/T)$ is finite. By Lemma 1, G/T is finite. In view of the arbitrariness of the choice of $p \in \pi(G)$, one obviously gets from this the finite approximability of G.

The lemma is proved.

LEMMA 4. In a π-biprimitively finite group $G(\pi \subseteq \pi(G))$ any two finite Hall π-subgroups having the same Sylow series are conjugate.

Proof. We consider two finite Hall π-subgroups T and M, having the same Sylow series. The proof will be by induction on the length of the Sylow series. Let $A_1 \lhd A_2 \lhd \ldots \lhd A_n = T$ and $B_1 \lhd B_2 \lhd \ldots \lhd B_n = M$ be Sylow series of the subgroups T and M. For n = 1 the subgroups $T = A_1$ and $M = B_1$ are Sylow p_1-subgroups in G and in this case T and M are conjugate (Proposition 10). Suppose for series of length $k < n$ the assertion of the lemma is valid. We shall prove it for n = k. By definition, A_1 and B_1 are Sylow p_1-subgroups in G, and in view of Proposition 10, there exists an $x_1 \in G$ such that $B_1^{x_1} = A_1$. Then $M^{x_1} = B_n^{x_1} \rhd \ldots \rhd B_1^{x_1} = A_1$ and $T = A_n \rhd \ldots \rhd A_1$. It is clear that $T, M < N_G(A_1)$.

We consider $D = N_G(A_1) / A_1$. The group D is π-biprimitively finite (cf. definition), and in it T/A_1 and M^{x_1}/A_1 are finite Hall π-subgroups with Sylow series of lower length (Proposition 14). By the inductive hypothesis, they are conjugate. But then by the homomorphism theorem T and M are conjugate.

The lemma is proved.

LEMMA 5. Let G be a π-biprimitively finite group with finite Sylow p-subgroups for any $p \in \pi \subseteq \pi(G)$, and suppose given in G a system of finite Hall π-subgroups $B_1, B_2, \ldots, B_n, \ldots$ having Sylow series, and $C_1, C_2, \ldots, C_n, \ldots$ are respectively the first terms of their Sylow series, where B_i and B_{i+1}/C_{i+1} have isomorphic Sylow series (i = 1, 2, ..., n). Then there exists a strictly increasing chain of subgroups $Q_1 < Q_2 < \ldots < Q_n < \ldots$ such that $Q_n \simeq B_n$.

Proof. By Theorem 20.2.6 of [14], $B_2 = C_2 \leftthreetimes S_2$, and by the hypothesis of the lemma, B_2/C_2 and B_1 have isomorphic Sylow series. In view of the isomorphism theorem and the hypotheses of the lemma being proved, B_1 and S_2 have isomorphic Sylow series. But then, by Lemma 4, B_1 and S_2 are conjugate in G, i.e., one can find an element x_1, such that $B_1 = S_2^{x_1} < B_2^{x_1}$. We write $Q_1 = B_1$, $Q_2 = B_2^{x_1}$. Then $Q_1 < Q_2$. Further, $B_3 = C_3 \leftthreetimes S_3$. As in the preceding case we show that $B_2^{x_1} = S_3^{x_2} < B_3^{x_2}$, where x_2 is some element of G. We write $B_3^{x_2} = Q_3$. Now we have a chain $Q_1 < Q_2 < Q_3$. The construction of this chain does not stop at a finite index $Q_1 < Q_2 < Q_3 < \ldots$.

The lemma is proved.

LEMMA 6. Let G be a periodic finitely approximable biprimitively finite group with finite Sylow p-subgroups for all p; $\mathfrak{M}$ be some infinite set of nonisomorphic Sylow subgroups and P_1 be some distinguished subgroup in it. Then there exists an infinite subgroup T, which is the union of finite Hall subgroups $Q_1 < Q_2 < \ldots < Q_n < \ldots$ having Sylow series, while $Q_1 \simeq P_1$ and $\pi(T) \leqslant \pi(\mathfrak{M})$.

Proof. Since G is finitely approximable, in it there exists a normal subgroup N_1, not containing p_1-elements, and $n_1 = |G : N_1| < \infty$. Then the set indicated can be represented as $\mathfrak{M} = \mathfrak{A} \cup \mathfrak{B}$ where $\mathfrak{A}$ contains all Sylow q-subgroups such that $(n_1, q) = 1$, and $\mathfrak{B}$ all others. We shall show that $\mathfrak{A} \subset N_1$.

In fact, let us assume that this is not so, i.e., there exists a q-subgroup S of $\mathfrak{A}$ and $S \not\subset N_1$. Then, by Theorem 4.24 (cf. [14]) on isomorphisms, $S/N_1 \cap S \simeq SN_1/N_1$ is a nontrivial q-subgroup in G/N_1, and hence n_1 is divisible by q. Contradiction. Consequently, $\mathfrak{A} \subset N_1$. We take $P_2 \in \mathfrak{A}$. By what was proved above, $P_2 < N_1$, and in G the Sylow p_2-subgroups are conjugate (Proposition 10). By Frattini's Lemma $G = N_G(P_2)N_1(1)$. We shall show that in $N_G(P_2)$ there is contained a subgroup conjugate with P_1. In view of the choice of P_1, it is a Sylow p_1-subgroup in G, and by the isomorphism theorem we have

$$\bar{P}_1 = P_1N_1/N_1 \simeq P_1/P_1 \cap N = P_1,$$

$N_1 \cap P_1 = 1$ while $\bar{P}_1$ is a Sylow p_1-subgroup in G/N_1 (Proposition 14). From the isomorphism theorem and (1) we get

$$G/N = N_G(P_2)N_1/N_1 \simeq N_G(P_2)/N_1 \cap N_G(P_2)$$

and

$$P_1 \simeq R \subset \bar{M} = N_G(P_2)/T_1,$$

where $N_G(P_2) \cap N_1 = T_1$ is a p_1'-subgroup. We shall show that in $N_G(P_2)$ a Sylow p_1-subgroup is nontrivial.

In fact, for us R is a nontrivial p_1-subgroup in the image M, and hence in the preimage $N_G(P_2)$ there exists a nontrivial p_1-subgroup. But then the Sylow p_1-subgroup S_1

of $N_G(P_2)$ is nontrival, and by Proposition 14, S_1T_1/T_1 is Sylow in $\overline{M}$. Further, $\overline{S}_1 = S_1T_1/T_1 \simeq S_1/T_1 \cap S_1 = S_1(T_1 \cap S_1 = (1))$. But earlier we found in $\overline{M}$ another Sylow P_1-subgroup R. The group $\overline{M}$ is P_1-biprimitively finite (Proposition 14), and by Proposition 10, R and $\overline{S}_1$ are conjugate in $\overline{M}$. But then $P_1 \simeq R \simeq \overline{S}_1 \simeq S$. Consequently, P_1 is isomorphic with the P_1-subgroup S_1 of $N_G(P_2)$, and by Proposition 10, P_1 and S_1 are conjugate in G, and hence S_1 is Sylow in G, while $S_1 < N_G(P_2)$.

We write $B_1 = S_1 \simeq P_1$ and $B_2 = P_2 \leftthreetimes S_1$. Since G is finitely approximable, there exists in it a normal subgroup N_2 such that $n_2 = |G : N_2|$ and $N_2 \cap B_2 = (1)$. Then $\mathfrak{M} = \mathfrak{A}_1 \cup \mathfrak{B}_1$, where $\mathfrak{A}_1$ is the set of Sylow q-subgroups such that $(q, n_2) = 1$, and $\mathfrak{B}_1$ contains all other subgroups of $\mathfrak{M}$. Obviously $\mathfrak{B}_1$ is a finite set. Just as for $\mathfrak{A}$, we show that $\mathfrak{A}_1 \subset N_2$. We take a Sylow P_3-subgroup $P_3 \subset \mathfrak{A}_1$. By the conjugacy of Sylow P_3-subgroups (Proposition 10) and Frattini's Lemma, we have $G = N_G(P_3)N_2$, $B_2 < G$ and

$$B_2N_2/N_2 \simeq B_2/B_2 \cap N_2 = B_2;$$

$$B_2 \simeq B_2N_2/N_2 < G/N_2 = N_G(P_3)N_2/N_2 \simeq N_G(P_3)/N_2 \cap N_G(P_3) > \overline{X},$$

where $\overline{X} \simeq B_2$.

We shall show that a subgroup conjugate with B_2 can be found in $M_2 = N_G(P_3)$ also. We write $T_2 = N_2 \cap N_G(P_3)$, $\overline{M}_2 = M_2/T > \overline{X} \simeq B_2$. Since $B_2 = P_2 \leftthreetimes S_1$, one has $\overline{X} = \overline{C} \leftthreetimes \overline{Z}$, where $\overline{C} \simeq P_2$, $\overline{Z} \simeq S_1$ (X, C are the complete preimages of $\overline{X}$, $\overline{C}$ in G). Using Proposition 14, it is easy to show that $C = T_2 \leftthreetimes Q$, Q and P_2 are conjugate in G. Since $C \lhd X$ and Sylow q_2-subgroups are conjugate in G, one has $X = N_X(Q)\,C$. By what was proved earlier one can find in $N_X(Q)$ a subgroup $F \simeq \overline{Z} \simeq P_1$. By Lemma 4, the subgroups $Q \leftthreetimes F$ and B_2 are conjugate in G. We write $B_3 = P_3(Q \leftthreetimes F)$. Thus, we have constructed subgroups B_1, B_2, B_3. Arguing analogously, we construct a subgroup B_4, etc. The constructed sequence $B_1, B_2, \ldots, B_4$ does not stop at a finite index. Now referring to Lemma 5, we complete the proof of the lemma.

<u>LEMMA 7.</u> Let G be a periodic biprimitively finite group with finite Sylow p-subgroups for all p of the form $G = M \leftthreetimes (a)$, where $|a| = p$, and M is a p'-group, and let the following conditions hold in G:

1) $\pi(G) = \pi(C_G(a))$;

2) for each $q \in \pi(G)$ a Sylow q-subgroup of $C_G(a)$ is a Sylow q-subgroup in G.

Then $a \in Z(G)$.

<u>Proof.</u> First we note that if G is finite the lemma follows obviously from Sylow's Theorem 11.1.1 of [14]. Let us assume that $a \notin Z(G)$, and let $\mathfrak{B}$ be the class of elements conjugate with a. Then we have two cases:

1) The element a commutes with any element of $\mathfrak{B}$. Since A = gr $(\mathfrak{B}) \lhd G$ and $a \in A$, in this case obviously $a \in Z(A)$. Now let R be a Sylow p-subgroup of Z(A). Since $a \in R$, $R \lhd G$ and R is a finite group, one has $|G : C_G(R)| < \infty$. We write $B = M \cap C_G(R)$. It is clear that $B \lhd G$ and $|G : B| < \infty$. We consider $\overline{G} = G/B$. From the hypotheses of the lemma and Proposition 24 it follows that $C_{\overline{G}}(aB) = C_G(a)B/B$, where B is a p'-group. From which, using Proposition 14 it is easy to see that $\overline{G}$ satisfies conditions 1 and 2 of the lemma, and since $\overline{G}$

is finite, one has (cf. the beginning of the proof of the lemma) $\bar{a} \in Z(\bar{G})$. But then, using Proposition 25, we show that $a \in Z(G)$. The consideration of case 1 is finished.

2) In $\mathfrak{L}$ there exists an element c such that L = gr (a, c) is a finite noncommutative group, having a representation $L = F \leftthreetimes (a), F \subset M$. As already noted, the assertion of the lemma is valid for finite groups, and L is finite, so F has a Sylow q-subgroup Z_1 such that $Z_1 \leftthreetimes (a)$ and $Z_1 \not\subset C_L(a)$. If Z_1 is not a Sylow q-subgroup of G, then in view of the conjugacy of Sylow q-subgroups (cf. Proposition 10) and the fact that the normalizer condition holds in finite q-groups [14, Theorem 16.2.1], Z_1 can be imbedded in a large Sylow q-subgroup Z_2 of $H_1 = N_G(Z_1)$. Obviously $H_1 = M_1 \leftthreetimes (a)$, $M_1 < M$. Since H_1 is a biprimitively finite group and Sylow q-subgroups in it are conjugate, we can take Z_2 so that $a \in N_G(Z_2)$. If Z_2 is not Sylow in G, then arguing analogously to the preceding, we get a chain $Z_1 < Z_2 < \ldots < Z_n = Z$. The construction of this chain stops at a finite index n, in view of the finiteness of the Sylow q-subgroup, i.e., Z is a Sylow q-subgroup and $a \in N_G(Z)$. Thus, in G there exists a Hall subgroup of the form $W = Z \leftthreetimes (a)$, where $Z \not\subset C_G(a)$. On the other hand, $C_G(a)$ has a q-subgroup Y, which is Sylow in G, and hence in G one can find a Hall subgroup of the form $V = Y \times (a)$. However, by Lemma 4, V and W are conjugate, which is impossible. The contradiction obtained proves the lemma.

LEMMA 8. Let G be a periodic biprimitively finite group with finite Sylow q-subgroups for any q, a be a p-element of G, satisfying the conditions:

1) $\pi(G) \setminus \pi(C_G(a))$ is finite;

2) for almost every $q \in \pi(C_G(a))$ a Sylow q-subgroup of $C_G(a)$ is Sylow in G.

If G is finitely approximable, then the element a is contained in a finite normal subgroup.

Proof. In view of the hypotheses of the lemma, in G there exists a p'-normal subgroup N, such that $\pi(N) \subset \pi(C_G(a))$ and $\pi(N)$ does not contain q for which the Sylow q-subgroups of $C_G(a)$ are not Sylow q-subgroups in G. We shall show that in the subgroup $H = N \leftthreetimes (a)$ conditions 1 and 2 of Lemma 7 hold. Let q be an arbitrary number from $\pi(N)$ and S be a Sylow q-subgroup of $C_G(a)$. In view of the choice of N, the subgroup S is a Sylow q-subgroup in G. Since Sylow q-subgroups are conjugate in G, one has $P = N \cap S$ is a Sylow q-subgroup in N, and also $P \subset C_N(a)$. In view of the arbitrariness in the choice of $q \in \pi(N)$, it follows from this that H satisfies the conditions of Lemma 7. By this lemma $H < C_G(a)$, and $|G : C_G(a)| < \infty$, and by Ditsman's lemma [15, p. 48], the element a is contained in a finite normal subgroup. The lemma is proved.

LEMMA 9. Let G be a periodic biprimitively finite group with finite Sylow p-subgroups. If G is finitely approximable, then any quotient group of it is finitely approximable.

Proof. Let K be a normal subgroup. If we show that $\bar{G} = G/K$ is finitely approximable with respect to any p-element for all p, then obviously the assertion of the lemma will follow from this. Let $\bar{c}$ be a p-element of G, c be some preimage of it, which is a p-element, $c \notin K$. Since G is finitely approximable and Sylow p-subgroups are conjugate, there exists in it a normal subgroup N, not containing p-elements, and $|G : N| < \infty$. From this and the isomorphism theorem [14] $NK/K \simeq N/K \cap N$ it follows that $c \notin NK$. But then obviously in $\bar{G} = G/K$

the subgroup NK/K has finite index and does not contain the element $\bar{c} = cK$. But as already noted at the beginning of the proof, the validity of the lemma follows from this.

LEMMA 10. Let G be a periodic biprimitively finite group with finite Sylow p-subgroups. If one of its quotient groups G/N is a periodic group of finite period S, then G/N is finite.

Proof. By Proposition 13, G/N is a biprimitively finite group with finite Sylow p-subgroups. Let us assume that $\bar{G} = G/N$ is infinite. Then by Proposition 19, it has an infinite Abelian subgroup with finite Sylow p-subgroups. However this contradicts the finiteness of the period of G/N.

The lemma is proved.

LEMMA 11. Let G be a periodic biprimitively finite group with finite Sylow p-subgroups for all p. If it is finitely approximable and the ranks of its Abelian subgroups are finite, then the ranks of its Sylow p-subgroups are bounded in aggregate.

Proof. Let us assume that this is not so. Then in G one can find an infinite sequence of Sylow p-subgroups $\mathfrak{M} = \{P_1, P_2, \ldots, P_n, \ldots\}$ such that the rank of the subgroup P_n increases with the index n. By Lemma 6, there exists a locally solvable subgroup T, containing an infinite set of nonisomorphic Sylow subgroups, each of which is conjugate with some subgroup from $\mathfrak{M}$. Obviously T has infinite rank. However, by the hypothesis of the lemma, the ranks of Abelian subgroups of T are finite, which contradicts Gorchakov's theorem [7]. The contradiction obtained proves the lemma.

LEMMA 12. Any locally finite binary solvable group is locally solvable.

Proof. The local solvability of G follows from the local finiteness and Thompson's theorem [16].

3. Basic Results

THEOREM 1. Let G be a periodic biprimitively finite group with finite Sylow p-subgroups for all p. If G is finitely approximable and in it the ranks of Abelian subgroups are finite, then G is a locally finite group.

Proof. In view of Lemma 11 we conclude that there exists a natural number K [more generally for all $p \in \pi(G)$], such that for any Sylow p-subgroup P of the group G, we have $r(P/\Phi(P)) \leqslant K$. In view of the hypotheses of the lemma, the group G has a normal p'-subgroup N such that G/N is finite. We write $H_p = G/N$ and $n_p = |H_p|$. In N we choose a Sylow q-subgroup Q such that $(q, n_p) = 1$. Sylow q-subgroups in N are conjugate (Proposition 10), and by Frattini's Lemma, $G = N_G(Q)N = MN$, where $M = N_G(Q)$. By Theorem 4.2.4 of [14] on isomorphisms, we have:

$$H_p = G/N = MN/N \simeq M/M \cap N = M/T = \bar{M},$$

where $M \cap N = T$. In Q we take the subgroup $\Phi(Q)$. It is characteristic and hence $\Phi(Q) \triangleleft M$. The subgroup $\bar{Q} = Q/\Phi(Q)$ is an elementary Abelian q-subgroup in $B = M/\Phi(Q)$ and since $\bar{Q} \triangleleft B$, one has $\bar{C} = C_B(\bar{Q}) \triangleleft B$ and by the homomorphism theorem, $C \triangleleft M$, where C is the complete pre-image of $C_B(\bar{Q})$ in M. Further, $H_p \simeq \bar{M}$. Obviously, $V_p = M/C$ is a linear group over a field of characteristic q, not dividing $|V_p|$. But then by the Jordan—Brauer—Feit Theorem (Proposition 6), V_p has an Abelian normal subgroup L such that $|V_p : L| \leqslant f(k)$, where f(k) is a function depending only on k. By the homomorphism theorem we have

$$A_q = CN/N \simeq CT/T \triangleleft M/T = \overline{M}, \quad A_q \triangleleft H_p$$

and

$$\overline{H}_p = H_p/A_q \simeq M/CT \simeq MC/CT/T = V_p/F,$$

where $F = CT/T$. In view of the structure of V_p, we get that $\overline{H}_p = V_p/F$ has an Abelian normal subgroup of index bounded by the number f(k).

The proof of the theorem splits into two cases:

1) $\bigcap_{q \in \pi(N) \setminus \pi(H_p)} A_q = D_p \neq (1)$;

2) $D_p = (1)$.

Let K(G) be the locally finite radical in G. If $K(G) = G$, then the theorem is proved. Let us assume that $K(G) \neq G$. We consider $\bar{G} = G/K(G)$. In view of Propositions 13, 14, $\bar{G}$ is a biprimitively finite group with finite Sylow q-subgroups for all q. By Lemma 9, it is finitely approximable and consequently satisfies all the hypotheses of the theorem while its locally finite radical is trivial. Hence, without loss of generality we shall assume that $K(G) = (1)$. We consider case 1). In view of the definition of the subgroup C and Proposition 5, any q'-element of C centralizes the subgroup Q. Whence, obviously if $D_p \neq (1)$, then in G one can find a nonidentity element, whose centralizer satisfies all the conditions of Lemma 8. By this lemma, G has a nontrivial normal finite subgroup, which contradicts the assumption $K(G) = (1)$. Consequently, case 2) holds. But then, by Remak's Theorem 4.3.9 of [14], H_p can be imbedded in the complete direct product of groups of type H_p/A_q and any such group, as shown above, has an Abelian normal subgroup of index no higher than f(k). Thus we have proved that H_p has an Abelian normal subgroup Y_p such that the period of the group H_p/Y_p is bounded by the number f(k), which is independent of the choice of $p \in \pi(G)$. By Remak's Theorem 4.3.9 of [14], G can be imbedded in a complete direct product $G \simeq G_1 < H = \tilde{\prod}_{p \in \pi(G)} H_p$. Let m be a number divisible by the period of the group H_p/Y_p for all p. As shown above, such a number exists.

We consider the subgroup $H^m = \operatorname{gr}(g^m / g \in H)$. Since Y_p is an Abelian group for any p, obviously H^m is an Abelian group and H/H^m is a periodic group of period m. But then G has an Abelian normal subgroup R such that G/R is a periodic group of period m. By Lemma 10, G/R is a finite group and by Schmidt's Theorem 22.3.1 of [14], G is locally finite, which contradicts the assumption.

The theorem is proved.

THEOREM 2. A periodic group is locally solvable and of finite rank if and only if it is binary solvable and its Abelian subgroups have finite ranks.

Proof. The *necessity* of the conditions of the theorem is obvious. Let G be a periodic binary solvable group in which all Abelian subgroups have finite ranks. In view of Proposition 20 and [17], all Sylow p-subgroups in G are Chernikov groups. By Lemma 2, G has a complete part R and G/R is a binary solvable group with finite Sylow p-subgroups for all p. Further, by Kargapolov's Theorem (Proposition 3), in G/R all Abelian subgroups have finite ranks. Consequently, without loss of generality one can assume that R = (1). By Lemma 3, in this case G is finitely approximable. But then by Theorem 1, G is a locally finite group, and in view of Lemma 12, it is locally solvable, while its rank is finite (Proposition 4).

The theorem is proved.

From Theorem 2 follows the analog of a familiar result of Gorchakov [7]:

COROLLARY. A periodic binary solvable group of infinite rank has an Abelian subgroup of infinite rank.

For finitely layered groups several characterizations are known [15]. A new characterization of locally solvable finitely layered groups is given by

THEOREM 3. A periodic group is a locally solvable finitely layered group if and only if it is binary solvable and any locally solvable subgroup is finitely layered.

Proof. It is easy to show that G is a binary solvable group with Chernikov Sylow p-subgroups for all p. In view of Lemma 2, it suffices to prove the theorem under the condition that the Sylow p-subgroups [for any $p \in \pi(G)$] are finite. We give a proof by contradiction. Let G not be finitely layered. Then one of the conditions of Lemma 8 must fail for us. Let the first condition fail to hold, i.e., $\pi(G) \setminus \pi(C_G(a))$ is infinite. Consequently, there eixsts an infinite set $\mathfrak{L}_1 = \{P_1, P_2, \ldots, P_n, \ldots\}$ of finite Sylow p_i-subgroups, $p_i \in \pi(G)$ $\pi(C_G(a))$, $i = 1, 2, \ldots, n, \ldots$. To this set we adjoin the Sylow p-subgroup P containing the element $a : \mathfrak{L} = \{\mathfrak{L}_1, P\}$. By Lemma 6, there exists a strictly increasing chain of Hall subgroups starting with $P : P \simeq B_1 < B_2 < \ldots < T$, a periodic locally solvable group. It is finitely layered by the hypotheses of the theorem, and consequently $p \in N_1, N_1$ being some finite normal subgroup of T. But then in the centralizer of a there is an infinite set of subgroups of $\mathfrak{L}$, and this is contrary to our assumption. Consequently, the first condition does not fail. Let us now assume that the second condition is false, i.e., in $C_G(a)$ there is an infinite set of Sylow subgroups $P_1', P_2', \ldots$, which are not Sylow subgroups in G. We include each P_i' in a p_i-Sylow subgroup P_i of G and we consider the set $\mathfrak{M} = \{P, P_1, \ldots, P_n, \ldots\}$. Again by Lemma 6 there exists a locally finite group $T > \ldots > B_2 > B_1 \simeq P$, which, by the hypotheses of the theorem, is finitely layered, and $a \in K$, some finite normal subgroup of T. Then an infinite set of subgroups from $\mathfrak{M}$ lands in $C_G(K)$, which contradicts our assumption.

The theorem is proved.

THEOREM 4. A periodic group is locally solvable with the primary minimality condition if and only if it is binary solvable and any locally solvable subgroup satisfies the primary minimality condition.

Proof. The necessity of the hypotheses of the theorem is obvious. Let G be a binary solvable group in which any locally solvable subgroup satisfies the primary minimality condition. Obviously G is a binary solvable group with Chernikov Sylow p-subgroups for all p. By Lemma 2 it has a complete part R, and G/R satisfies all the hypotheses of the theorem. By Proposition 18, any locally solvable subgroup $K < B$ is finitely layered, and by Theorem 3 we conclude that B is also locally solvable and finitely layered. But then G is locally solvable and satisfies the primary minimality condition.

The theorem is proved.

4. Periodic F*-Groups with Abelian Subgroups of Finite Ranks

The material of this section was included in the paper after it was ready for press. As it turned out, considering the specific properties of F*-groups the proof of Theorem 1 can be generalized to these groups. We recall the definition of an F*-group [21]. The group G is said to be an F*-group if for any chain of subgroups $K < H < G$, where K is a finite subgroup, and any pair of elements a, b of the same prime order of $M = N_H(K)/K$, one can find in M an element c such that the subgroup gr $(a, c^{-1}bc)$ is finite. This class of groups was introduced by V. P. Shunkov. As follows from [22], there exist infinite finitely generated periodic F*-groups with Abelian subgroups of finite ranks. In connection with this the following result, giving necessary and sufficient conditions for the local finiteness of F*-groups with Abelian subgroups of finite ranks, is of interest.

THEOREM 5. A periodic F*-group with Abelian subgroups of finite ranks is locally finite with Sylow p-subgroups for all p, if an only if it is finitely approximable.

(The validity of this theorem and its corollary was pointed out to the author by V. P. Shunkov.)

Proof. Necessity. Let G be a locally finite group with finite Sylow p-subgroups and with Abelian subgroups of finite ranks. By Shunkov's theorem [23], G is an almost locally solvable group and by Lemma 3 it is finitely approximable.

Sufficiency. Let G be a periodic F*-group with Abelian subgroups of finite ranks and G be finitely approximable. We shall show that the Sylow p-subgroups of G are finite. Let P be a Sylow p-subgroup of G. Let us assume that P is infinite. Obviously P satisfies all the hypotheses of the theorem and it is finitely approximable. From this, in view of Proposition 11, the maximal locally finite subgroups of P are finite. Let Q be one of them. Since P is finitely approximable, it has a normal subgroup N_1 of finite index and $Q \cap N_1 = (1)$. On the basis of [21] it is easy to prove that maximal locally finite subgroups are conjugate both in P and in N_1. Consequently, the orders of all maximal locally finite subgroups both of P and of any of its subgroups are bounded by the number $|Q|$.

Let S_1 be a maximal locally finite subgroup of N_1. As already noted it is finite and all such subgroups are conjugate in N_1. By Frattini's lemma, $P = N_P(S_1)N_1 = P_1 N_1$, where $P_1 = N_P(S_1)$ and $T = P_1 \cap N_1$. By Theorem 4.2.4 of [14] on isomorphisms, $P_1/T \simeq P/N_1$ and since $Q \simeq QN_1/N_1$, one can find in P_1/T_1 a subgroup isomorphic with Q. From this it follows that if $P_1 = N_P(S_1)$ were a finite group, then its order would be strictly greater than $|Q|$, and this, as noted above, is impossible. Thus, P_1 is an infinite group. But then $\overline{P}_1 = P_1/S_1$ is also infinite and satisfies all the hypotheses of the theorem, while it is finitely approximable with respect to $\overline{P}_1$ we argue analogously to the preceding, and so on. As a result, we get a strictly increasing chain of finite subgroups $S_1 < S_2 < S_3 < \ldots$, which does not stop at a finite index, which is impossible. Consequently, the Sylow p-subgroups of G are finite, and in view of [21], they are conjugate for each p.

LEMMA 13. If G is a periodic finitely approximable F*-group with finite Sylow p-subgroups, then any quotient group of it has the same properties.

Proof. Let N be an arbitrary normal subgroup. First we shall show that G/N is an F*-group. Using the conjugacy of Sylow p-subgroups [21], the outline of the proof of Proposition 13, and the finite approximability, it is easy to prove that G/N is an F*-group. The proof of Lemma 9 carries over word for word to F*-groups and consequently, G/N is finitely approximable (the Sylow p-subgroups in G are finite and they are conjugate both in G and in N). Obviously the analog of Proposition 14 is valid for quotient-groups of F*-groups with finite Sylow p-subgroups. Then using this fact, the finite approximability of G/N and the arguments given at the beginning of the proof of Theorem 5, we prove the finiteness of the Sylow p-subgroups. This completes the proof of the lemma.

Using Lemma 13 and finite approximability, it is easy to prove analogs of Lemmas 7, 8, 10, 11. The remaining lemmas are also valid under the condition of finite approximability.

In view of the remarks made above and Lemma 13, the proof of Theorem 1 carries over word for word to finitely approximable periodic F*-groups with finite Sylow p-subgroups for all p.

The theorem is proved.

The following corollary allows us to single out finite groups from an arbitrary class of groups.

COROLLARY. The group G is finite if and only if it satisfies the following conditions:

1) G is a finitely generated group;

2) G is a periodic group;

3) G is an F*-group;

4) G is a finitely approximable group;

5) in G the ranks of Abelian subgroups are finite.

We shall show by examples that none of the conditions listed follows from the other four. The direct product of an infinite number of cyclic p-groups for different p obviously satisfies conditions 2-4, but it is not finitely generated. It is also obvious that an infinite cyclic group satisfies conditions 1, 3-5, but is not a periodic group. Sushanskii [24] announced that in groups of Aleshin type [26] Abelian subgroups are finite. Hence they satisfy conditions 1, 2, 4, 5, but are not F*-groups. Further, the group of Ol'shanskii [22] is not finitely approximable, but all the other conditions hold in it. Finally, in Golod's group [25], which is binary finite, conditions 1-4 hold, but the ranks of Abelian subgroups in it are infinite.

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