Foundations of Physics, Vol. 20, No. 6, 1990

The Paradox of Two Bottles in Quantum Mechanics

Bogdan Mielnik[1]

Received February 9, 1990

A class of retrospective measurements analogous to the "delayed choice experiments" of Wheeler and Greenberger is considered. A new argument shows that the reduction of the wave packet must affect the past states of the system. As a side product, our argument implies that the axiom about the reduction of the wave packet in relativistic space-time cannot be consistently introduced.

1. INTRODUCTION

With few exceptions,(1–3) the axiom about the reduction of the wave packet in quantum mechanics has been formulated in abstract terms, devoid of space-time aspects.(4-7) In attempts at including the reduction postulate into relativistic theory, the main complication was the Einstein–Podolsky–Rosen paradox (EPR).(8,9) In recurrent discussions of EPR, interest has been focused on correlations and distant consequences of a local measurement.(10–13) The consistency of quantum theory could be saved by proving that EPR does not allow one to transmit information at distance. (see, e.g., Ref. 14). Yet, some fundamental questions persisted (e.g., Refs. 15–22). A quite new challenge was Wheeler's idea of the "delayed choice experiment"(23–25) in which the reduction is retrospective. i.e., affects the past states of a system. Below, we shall discuss more situations of the same kind. Our point of departure is a transformed version of the "delayed choice experiment" described by Greenberger(24) (see also Greenberger and Ya Sin(25)). However, we focus attention on its different

[1] Departamento de Física, CINVESTAV, Apdo. Postal 14-740, 07000 México, D.F., and Institute of Theoretical Physics, Warsaw, Hoza 69, Poland.

0015-9018/90/0600-0745$06.00/0

aspects, which apparently evaded attention in the previous papers. We think our arguments show a fundamental flaw in the reduction axiom, at least in its present-day form.

2. ONE PARTICLE IN TWO BOTTLES

We shall stick to the orthodox idea that the *reduction* is an instantaneous act that concerns the wave function of a microparticle all over the space (otherwise, it would make little sense to attribute the meaning of the localization probability to the square of the wave function $\psi^+(x)\,\psi(x)$). Thus, given a wave packet composed of several space-separated components, the certainty that the particle is actually in one of them reduces simultaneously the other components to nonexistence. We shall, moreover, adopt the doctrine that the particle presence in one of the wave packet components can be verified indirectly, e.g., by checking its *absence* in the other components (Dicke[26]). In relativistic theory, the instantaneous character of the reduction means that the "collapse phenomenon" affects the particle state all over a certain three-dimensional space-like hyperplane in the four-dimensional space-time (if not, all the discussions concerning the EPR paradox would be pointless.) Until now, it is also believed that the reduction concerns only the state of a micro-object *after* the measurement and does not alter its *past*; however, this is precisely the point which we would like to discuss. To do this, we shall consider a somewhat exagerated but not impossible physical situation (generically, a kind of grotesquely distorted version of Greenberger's experiment[24]).

Imagine a micro-object (e.g., an atom) in a container composed of two bottles. Suppose that at the begining (i.e., below a certain space-like hyperplane denoted by Σ_{past}) the bottles are joined by their necks and the wave packet of the micro-object is spilled smoothly all over their common interior. Then a double partition is introduced. Next the bottles are neatly sealed and separated (so that the micro-object could not escape) and then transported carefully (lest the coherence be affected) to two widely separated places, for instance, two corners of our Galaxy (I don't dare to imagine a technology, but such a coherent transport is not fundamentally impossible; cf. recent works on the Berry phase[27]). There, the bottles remain sealed for a long time, the wave packet of the micro-object being divided into two distant, coherent parts: $\psi = \psi_A + \psi_B$. Many epochs later, an experimenter in one of the Galaxy corners opens one of the bottles (e.g., the bottle A) and finds that it is empty. How does this affect the wave function of the micro-object?

According to the orthodox philosophy, the answer to our question is simple. Once one of the bottles is opened, the whole wave packet becomes reduced. Finding the bottle A empty is strictly equivalent to finding the particle at the other end of the Galaxy, and hence, the whole state is similtaneously reduced to the distant component ψ_B. (Conversly, were the micro-object found in the open bottle, the distant component would simultaneously disappear.) As happens in EPR arrangement, this conclusion does not involve any faster-than-light transmission of a message. However, we would like to turn our attention to a different circumstance.

The concept of simultaneity in the Minkowski space-time is not unique. If the absence of the micro-object in the bottle A has been verified by an inertial observer, it means that the wave packet has been reduced (to its component ψ_B) on a certain spacelike hyperplane Σ_1 (see Fig. 1). Denoting by A_1 and B_1 the intersections of Σ_1 with the world tubes A and B, this is equivalent to saying that the micro-objects was found *absent* on the intersection A_1 while *present* on B_1. The consequences do not end here, however. Once its *absence* on A_1 has been verified, the micro-object has to *stay absent* on all tube intersections above A_1 (the repeatability of the measurement!). Moreover, it seems obvious that the absence of the microsystem cannot depend on the Lorentz transformations. Thus, should

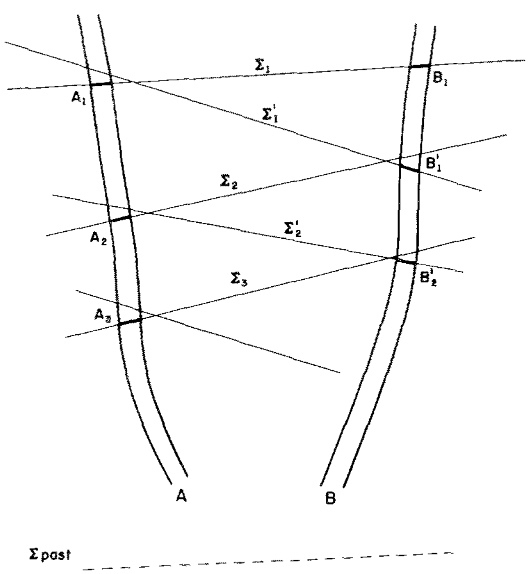

Fig. 1. Retrospective reduction: The certainty that the particle was absent on the intersection A_1 of the world tube A permits one to show that it had been present with probabilities 0 and 1 on the sequence of intersections A_1, A_2,... and B_1, B_2,..., respectively.

another (moving) observer look into the same bottle (above Σ_1), he must also find it empty with the probability 1. This immediately implies that the packet has been reduced not only on Σ_1 but on a much wider family of spacelike hyperplanes, all of which intersect the tube A above A_1. Let Σ'_1 be one of them; suppose it intersects the tube A *above* A_1 and B *below* B_1. As the *absence* of the micro-object is certain on the intersection $A'_1 = A \cap \Sigma'_1$, its *presence* is, henceforth, proved on $B'_1 = B \cap \Sigma'_1$ (and on the whole B tube above B'_1). By choosing, in turn, a hyperplane Σ_2 intersecting B *above* B'_1 but A *below* A_1, we cannot refrain from concluding that the state of the micro-object has been reduced on Σ_2 as well: The object is now *certainly present on* B_2, and so, *certainly absent* on A_2.[2] By iterating the argument, one comes to the conclusion that the reduction must affect not only the state on Σ_1 but the whole past of the micro-object: thus, the micro-object *was truly in the bottle B from the very beginning*, i.e., from the moment of the separation on Σ_{past}. The component ψ_A of its wave packet has never existed, although the check was not yet done.

In a way, this conclusion is logical. Since the bottles were sealed, one is tempted to believe that the presence of the micro-object in the bottle B at the moment of the final test means that it has always been there. As in EPR, the *"retrospective reduction"* does not imply the transmission of a message into the past. (Indeed, in order to check whether the components ψ_A and ψ_B conserved the ability to interfere, the observer on the spacelike Σ_{past} would have to open the bottles, henceforth spoiling the reduction process to be performed on Σ_1).

On the other hand, our conclusion is difficult to accept! An idea of a superposed state (wave packet) in which all components exist and all alternatives are potentially present until the very end is very deeply incrusted in the philosophy of quantum theory. Suppose a theoretical physicist living in our space-time somewhere around the spacelike hyperplane Σ_{past} presences the *preparation* of the micro-object state. He checks carefully all the conditions assuring that the micro-object wave function indeed fills the common interior of the joined bottles. He then checks the act of splitting and shipping the bottles, and he sees that, in agreement with all known criteria of laboratory art, the coherence between both components of the wave packet was never spoiled. As is well known, the *state* of a micro-object is uniquely defined by the *preparation* (and the preparation cannot

[2] At this moment one might formally object that the indirect reduction mechanism of Dicke type[26] cannot be iterated. However, the argument seems rather weak. Indeed, if the wave packet of our micro-object were reduced on Σ'_1: $\psi'_1 = \psi'_{1B}$, but not on Σ_2: $\psi_2 = \psi_{2A} + \psi_{2B}$, then the microobject would be present in the bottle B with certainty in the eyes of one inertial observer (associated with Σ'_1), but only with probability 1/2, shortly afterwards, in the eyes of another observer, which seems rather unphysical.

include the reduction act on Σ_1 which is still epochs away). Is'nt our theoretician entitled to infer that the micro-object in the two bottles is in a pure state $\psi = \psi_A + \psi_B$? Moreover, he might be very easily right! The future cannot be predicted. Should the experimenter on Σ_1 fail to be born or forget to open the bottle, the state of the micro-object actually would conserve coherence, and so it would remain superposed over all the future spacelike sections! We thus arrive at a certain "blind spot" of quantum theory: No state (neither pure nor mixed) can be assigned to the micro-object in the pair of bottles on the basis of the preparation procedure alone, without knowing a distant future. This might seem an exceptional situation, due to artificially prepared circumstances, but we think it is not. Indeed, we shall show that all the elements of the two-bottle paradox exist as well in the traditional EPR arrangement.

3. TWO-BOTTLE ASPECT IN THE EPR EXPERIMENT

Let us consider again the polarization variant of the EPR experiment described in Ref. 9. Suppose a source emits two photons of correlated (opposite) polarizations in two opposite spatial directions, so that the polarization state of the photon pair defined as[9,11]

$$\varphi = \frac{1}{\sqrt{2}} (|\uparrow\rangle\, |\leftarrow\rangle - |\leftarrow\rangle\, |\uparrow\rangle) \tag{1}$$

To simplify considerations, we shall neglect the wavelike propagation of the photons and assign to them two null world tubes A and B in Minkowski space-time (see Fig. 2), as if they were two classical particles endowed with quantum polarizations. This picture is justified if the photons propagate in a wave guide or optical fiber in the form of finite field pulses.[28] (It neither seems incorrect in empty space-time if the photons are not exactly monochromatic and their emission time is finite. The photon wave function is then between two finite wave fronts expanding with velocity c, and the part arriving at the polarization analyzers is additionally limited in the angular space, thus marking two null tubes in the Minkowski space-time).

Suppose now that far from the source, in a distant future, the polarization of the left photon is measured on the intersection A_1 of the tube A with a spacelike hyperplane Σ_1 (where Σ_1 is the simultaneity hyperplane of the inertial observer holding the polarization analyzer). Assume the polarization of the left A photon is found to be $|\uparrow\rangle$. Then the simultaneous polarization of the right B photon is decided into $|\rightarrow\rangle$ (on the intersection

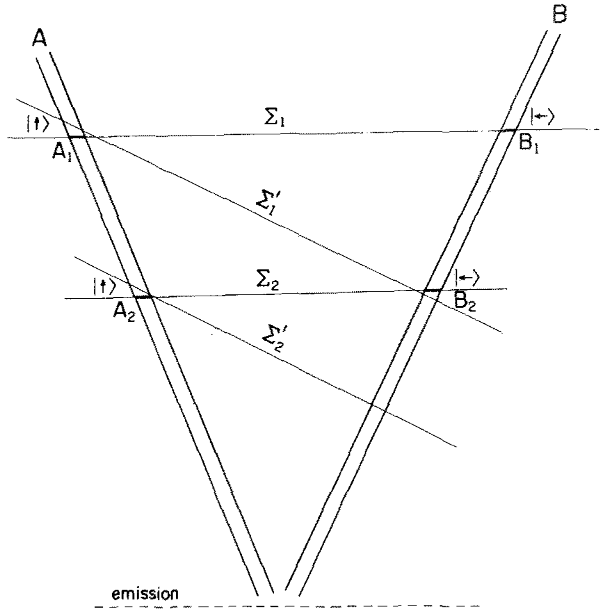

Fig. 2. The retrospective reduction of the photon polarizations in the Bohm–Aharomov variant of the EPR experiment.

B_1). Now remember that the photon polarization is Lorentz invariant (for the class of Lorentz frames moving parallel to the photon propagation). Hence, the polarizations of both photons turn out to be defined for a wider class of observers: They become fixed on any spacelike hyperplane intersecting the A tube above A_1 or the B tube above B_1. By choosing now a sequence of hyperplanes Σ_j and Σ'_j, as indicated on Fig. 2 (analogous to the system of hyperplanes of Fig. 1), and using only the argument that, whenever the polarization of a left photon becomes certain on a spacelike hyperplane, the polarization of its right-hand counterpart becomes certain too, one can show by induction that the polarization state of the photon pair has been reduced on the sequence of the hyperplanes Σ_1, Σ'_1, Σ_2, Σ'_2,..., reaching arbitrarily deep into the past.[3] If so, the state of the photon pair has also been reduced immediately after the emission act, long before the measurement was performed. Should a source emitting an ensemble of many photons be activated on the Σ_{past} plane, it might create only a mixed ensemble of the photon pairs, even if all known conditions assuring the

[3] The conterargument might again be attempted that the reduction mechanism does not permit iterations; but this argument fails for the same reason as in Sec. 2, i.e., due to the impossibility of a consistent construction of a wave packet below Σ_1.

coherence were carefully observed! This conclusion curiously coincides with the hypothesis that the collapse of the wave packet is caused by the very separation of the quantum system,[29–32] but it is hardly acceptable otherwise. Indeed, should the observer on Σ_1 fail to perform the measurement, the state of the photon pair would still be coherent! Once again, we are in a blind spot of the doctrine; no quantum state, either pure or mixed, can be assigned to the photon pair without the prior knowledge of the future! One of the important pillars of quantum philosophy is thus shaken: the conviction that the system *preparation* defines the system *state*.

The above conclusion is only a part of the trouble. Imagine, in turn, that *two* polarization measurements are performed on Σ_1: one on the left side, which ask the left A photon to choose between the polarization states $|\uparrow\rangle$ and $|\rightarrow\rangle$, and the other, on the right side, which ask the B photon to choose between the states $|\nwarrow\rangle$ and $|\nearrow\rangle$. This kind of measurement is sometimes discussed as one for which the definition of the reduced end state presents an ambiguity. This seems not to be the case, not at least if the state of the photon pair *after* both measurements is considered. One just has to remember that the polarization observable is not protected by any conservation law and can be perturbed during the act of measurement (an example is the simple transition of a photon through a Nicol prism). The two-sided polarization check on Σ_1, when the polarization directions measured on both sides are incompatible, is precisely one of the tests which do not conserve the total polarization of the system. The end state obtained after the measurement, however, is no mystery: It is one of the four reduced states $|\uparrow\rangle|\nearrow\rangle$, $|\uparrow\rangle|\searrow\rangle$, $|\rightarrow\rangle|\nearrow\rangle$, and $|\rightarrow\rangle|\searrow\rangle$, with probabilities $\frac{1}{2}|\langle\rightarrow|\nearrow\rangle|^2$, $\frac{1}{2}|\langle\rightarrow|\searrow\rangle|^2$, $\frac{1}{2}|\langle\uparrow|\nearrow\rangle|^2$, and $\frac{1}{2}|\langle\uparrow|\searrow\rangle|^2$, respectively (some wider aspects of the joint probability distribution were recently discussed in Refs. 33 and 34). The difficulty starts when one tries to reconstruct both photon states *below* the hyperplane Σ_1, applying consistently the reduction axiom. To fix the attention, suppose the two-sided measurement has detected the polarization $\uparrow$ of the A photon and $\nearrow$ of the B photon. Then consider again the situation on the sequence of hyperplanes Σ_j, Σ'_j. For the inertial observer of the hyperplane Σ'_1 *the only measurement already done* is that for the left photon: Henceforth, the state of the photon pair on Σ'_1 should be reduced to $|\uparrow\rangle|\leftarrow\rangle$, and so the state of the right photon becomes $|\leftarrow\rangle$ everywhere in the B tube above the section B'_1 and below B_1. By repeating the argument as before, we conclude that the polarizations of both left and right photons are reduced to $|\uparrow\rangle$ and $|\leftarrow\rangle$ on the sequence of the left and right intersections A_1, A_2,... and B_1, B_2,... retreating arbitrarily deep into the past. This result contains already a germ of desaster, due to its obvious asymmetry. In fact, by constructing an analogous sequence of hyperplanes starting from the B tube instead of the A tube, one can infer that the polarizations of both photons have been reduced to $|\nwarrow\rangle$ and $|\swarrow\rangle$ on another sequence of intersections of A and B tubes. These two implications cannot hold simultaneously. Indeed, an open contradiction is obtained by noticing that nothing forbids the intersections B_1 and A_2 to be space separated. If so, there can exist a bundle of spacelike hyperplanes $\{\Sigma''_\lambda\}$ intersecting the B tube *above* B_1, such that the intersections $A''_\lambda = A \cap \Sigma''_\lambda$ cover A_2 completely (see Fig. 3). By virtue of the polarization test performed on B_1, the polarization state of the left photon on any intersection A''_λ has to be $|\nwarrow\rangle$. However, by tracing the reduction acts subsequently on the hyperplanes Σ'_1 and Σ_2 (i.e., along the way $A_1 \to B_2 \to A_2$), we have already shown that the polarization state on the intersection A_2 has to be $|\uparrow\rangle$. Due to the invariance of the photon polarization with respect to the Lorentz transformations mapping Σ_2 on Σ''_λ, both statements cannot be simultaneously true. Thus the orthodox reduction postulate leads to a self-contradiction in Minkowski space-time.

Observation 1. It seems that the above inconsistency admits no simple solution in the spirit of the "semantic diplomacy" of the Copenhagen school.[6] Indeed, in the arrangement of Fig. 3, all the external macroscopic conditions around the system, both in the past and in the future (i.e., the system preparation on Σ_{past} and the kind of the measurement to be performed on Σ_1), are carefully fixed. Yet, the system state between Σ_{past} and Σ_1 eludes any definition consistent with the rules of quantum mechanics. (In

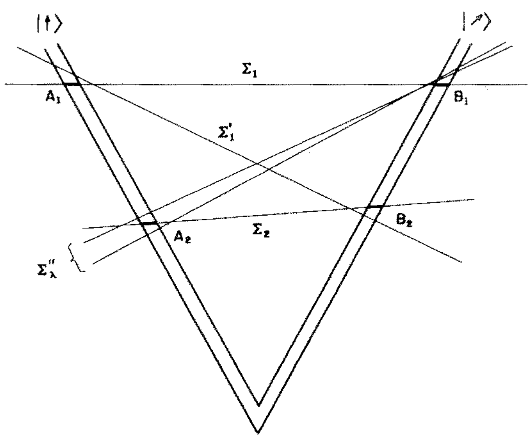

Fig. 3. The inconsistency of the reduction postulate in the Minkowski space-time. On the segment of the world tube A covered by the hyperplanes Σ''_λ, the polarization of the left photon has to be $|\nwarrow\rangle$ with probability 1, but simultaneously it has to be $|\uparrow\rangle$ with probability 1.

alternative terms: The *manual*[35] exists, but no quantum state can be constructed.) Not even the confidential information about the future output of the polarization measurement on Σ_1 can help: we still continue without the possibility of assigning to the photon pair in between Σ_{past} and Σ_1 any polarization state, either mixed or pure, either straight or obligue!

Observation 2. A deeper option in answering the puzzle is that one cannot, anyway, assign a *quantum state* (pure or mixed) to a *single* quantum system (in this case, to a single photon pair), since the states are meaningful only for statistical ensembles.[36] Let us therefore recall that the only reason why the reduction postulate was accepted is that it permits us to explain the *stability* and *repeatability* properties of each *single* result of measurement (including the simultaneous observability of the same macroscopic effect in the eyes of many independent observers; see discussions by A. Shimony,[37] p. 767 and H. Primas,[38] Chap. 3.5, p. 123). These facts can be explained *only* if one assumes that each single quantum system jumps to one of the possible eigenstates of the measured quantity. Should this statement be dropped or weakened, the whole explanation is nullified. Thus, the reduction postulate leads to an absurdity precisely in the only case which might be of physical interest!

4. CONCLUSIONS

There are two possible conclusions to be derived from our paradox. In the first place, one might infer that the very concept of the quantum mechanical state vector (or density matrix) is an element of a rather incomplete "predictive game," much more fragmentary than originally supposed.[6] There may be situations where the macroscopic conditions around the system (i.e., the system preparation and the type of measurement to be performed) are specified, and yet, the game of constructing the "quantum state" is frustrated! While such a solution cannot be *a priori* rejected, the good applicability of the orthodox theory does not seem to point in this direction.

We thus suspect that the whole *quid pro quo* lies rather in the reduction axiom. Without dismissing too quickly some less naive versions, we think, however, that our paradox might be a final *coup de grace* for that idea. What truly happens during the measurement most probably is a loss of coherence between the superposition components, rather than the disappearance of some of them.[39–43] The assumed "collapse" of the wave packet might be a sufficient but not necessary condition to explain the persistence of the visible macroscopic effects. The problem of an alternative explanation might be one of our best hints leading beyond the present-day quantum theory.

ACKNOWLEDGMENTS

The author is indebted to his colleagues at the Institute of Theoretical Physics of Warsaw University in Poland and at the Departamento de Física del CINVESTAV in México for their interest in this paper and for critical discussions.

REFERENCES

1. R. Haag and D. Kastler, *J. Math. Phys.* **5**, 848 (1964).
2. J. P. Antoine and A. Gleit, *Int. J. Theor. Phys.* **4**, 197 (1971).
3. J. Kijowski, *Rep. Math. Phys.* **6**, 361 (1974).
4. P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1947).
5. J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, (1955).
6. N. Bohr, *Atomic Physics and Human Knowledge* (Science Editions, New York, 1961).
7. F. London and E. Bauer, *La theorie de l'observation en mécanique quantique* (Hermann & Cie, Paris, 1939).
8. A. Einstein, B. Podolsky, and N. Rosen, *Phys. Rev.* **47**, 777 (1935).
9. D. Bohm and Y. Aharonov, *Phys. Rev.* **108**, 1070 (1957).
10. J. S. Bell, *Physics* **1**, 195 (1965).
11. J. F. Clauser and A. Shimony, *Rep. Prog. Phys.* **41**, 1881 (1978).
12. A. Aspect, P. H. Grangier, and G. Roger, *Phys. Rev. Lett.* **47**, 460 (1981); **49**, 91 (1982).
13. A. Aspect, J. Dalibard, and G. Roger, *Phys. Rev. Lett.* **49**, 1804 (1982).
14. C. D. Cantrell and M. O. Scully, *Phys. Rep.* **43**, 499 (1978).
15. H. Margenau and J. L. Park, *Int. J. Theor. Phys.* **1**, 211 (1968).
16. K. R. Popper, Quantum mechanics without an observer," in *Quantum Theory and Reality*, M. Bunge, ed. (Springer, Berlin, 1967).
17. J. S. Bell, "Are there quantum jumps?" in *Speakable and Unspeakable in Quantum Mechanics*. (Cambridge University Press, Cambridge, 1987).
18. J. L. Park, *Found. Phys.* **1**, 23 (1970).
19. J. R. Croca, in *Problems in Quantum Physics; Gdańsk 87* (World Scientific, Singapore, 1988), pp. 24–42.
20. J. P. Vigier, in *Open Questions in Quantum Physics* (Reidel, Dordrecht, 1985), pp. 297–322; in *Problems in Quantum Physics; Gdańsk 87*, pp. 317–346.
21. A. Posiewnik and J. Pykacz, *Phys. Lett. A* **128**, 5 (1988).
22. D. Home and M. Whitaker, *Phys. Lett. A* **128**, 1 (1988).
23. J. A. Wheeler, in *Mathematical Foundations of Quantum Theory*, R. A. Marlow, ed. (Academic Press, New York, 1978).
24. D. M. Greenberger, *Rev. Mod. Phys.* **55**, 875 (1983).
25. D. M. Greenberger and A. YaSin, *Found. Phys.* **19**, 679 (1989).
26. R. H. Dicke, *Am. J. Phys.* **49**, 925 (1981).
27. M. V. Berry, *Proc. R. Soc. London A* **392**, 45 (1984).
28. W. D. Jackson, *Classical Electrodynamics* (John Wiley and Sons, New York, 1975).
29. E. Schrödinger, *Proc. Camb. Philos. Soc.* **31**, 555 (1935).
30. W. H. Furry, *Phys. Rev.* **49**, 393 (1936).
31. G. Ghirardi, A. Rimini, and T. Weber, *Phys. Rev. D* **34**, 479 (1986).

32. A. Kent, *Mod. Phys. Lett. A*, **4**, 1839 (1989).
33. M. Scully, *Phys. Rev. D*, **28**, 2477 (1983).
34. K. Wodkiewicz, *Phys. Lett. A* **112**, 276 (1985); *A* **129**, 415 (1988).
35. C. H. Randall and D. J. Foulis, "The operational approach to quantum mechanics," in *Physical Theory as a Logico-Algebraic Structure*, C. A. Hooker, ed. (Reidel, Dordrecht, 1978).
36. J. L. Park, *Am. J. Phys.* **36**, 211 (1968).
37. A. Shimony, *Am. J. Phys.* **31**, 755 (1963).
38. H. Primas, *Chemistry, Quantum Mechanics and Reductionism* (Springer, Berlin, 1983).
39. E. P. Wigner, *Am. J. Phys.* **31**, 6 (1963).
40. H. D. Zeh, *Found. Phys.* **1**, 69 (1970).
41. K. Hepp, *Helv. Phys. Acta* **45**, 237 (1972).
42. G. Ludwig, *Foundations of Quantum Mechanics II*, W. Beiglböck *et al.*, eds. (Springer, New York, 1985).
43. B. Mielnik, "Is the reduction of the wave packet indeed necessary in quantum mechanics?" in *Problems in Quantum Physics, Gdańsk 89*. (World Scientific, Singapore, 1990)

Printed in Belgium