

# Quantum Gravity, the Origin of Time and Time's Arrow

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Received August 12, 1992; revised September 8, 1992

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*The local Lorentz and diffeomorphism symmetries of Einstein's gravitational theory are spontaneously broken by a Higgs mechanism by invoking a phase transition in the early universe, at a critical temperature  $T_c$  below which the symmetry is restored. The spontaneous breakdown of the vacuum state generates an external time, and the wave function of the universe satisfies a time-dependent Schrödinger equation, which reduces to the Wheeler-deWitt equation in the classical regime for  $T < T_c$ , allowing a semiclassical WKB approximation to the wave function. The conservation of energy is spontaneously violated for  $T > T_c$ , and matter is created fractions of seconds after the big bang, generating the matter in the Universe. The time direction of the vacuum expectation value of the scalar Higgs field generates a time asymmetry, which defines the cosmological arrow of time and the direction of increasing entropy as the Lorentz symmetry is restored at low temperatures.*

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## 1. INTRODUCTION

Because of the general covariance of Einstein's gravitational theory, time is an arbitrary parameter and in the canonical Dirac constraint theory the super-Hamiltonian vanishes, reflecting the time-translational invariance of the theory. The quantum mechanical operator equation for the wave function of the universe leads to the Wheeler-DeWitt (WD) equation,<sup>(1)</sup> which is a second-order hyperbolic differential equation in the dynamical phase space variables and which possesses only stationary solutions. The wave function is time independent and there is no temporal development in a spatially closed universe. In effect, time has disappeared from the theory. The Schrödinger equation is only meaningful in a fixed-frame situation and

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quantum mechanics seems to require an external time in order that quantum mechanical measurements can be made of time-dependent observables. Thus, we are faced with a fundamental conflict between quantum mechanics and relativity, and it would appear that we may be forced to give up one or the other of the two fundamental pillars of modern physics.

In quantum mechanics, the universe is correctly described by the first-order Schrödinger wave equation, which leads to a positive-definite, conserved probability current density, but as we have seen the concept of general covariance is in serious conflict with this picture. Most attempts to extract a time variable identify time as some combination of field variables and rely on the WKB approximation to the WD equation.<sup>(2-5)</sup> Such an approach is at best approximate, assuming a special form of the WKB wave function, and being valid only in certain regions of superspace, in which the classical regime is known to hold. The problem in this approach is to explain why certain domains of spacetime have a classical Lorentzian structure such that the wave function has an oscillatory behavior.<sup>(6)</sup> Another recent attempt<sup>(7-9)</sup> to solve the problem of time in quantum gravity abandons general covariance at the classical level by generalizing Einstein's gravitational theory to a unimodular theory with  $\sqrt{-g} = 1$ .

In the following, we shall propose a solution to the problem, in which we spontaneously break local Lorentz invariance and diffeomorphism invariance of the vacuum state of the early universe, with the symmetry-breaking pattern  $SO(3, 1) \rightarrow O(3)$ . Within this symmetry-breaking scheme, we shall retain the three-momentum operator constraint equations but relax the super-Hamiltonian constraint for the wave function, thereby, obtaining a time-dependent Schrödinger equation. In this framework, quantum mechanics and gravitation are united in a meaningful observational scheme. The local Lorentz invariant structure of the gravitational Lagrangian is maintained as a "hidden" symmetry. After the spacetime symmetries are restored in the early Universe for a temperature  $T < T_c$ , the wave function has an oscillatory behavior, and it is peaked about a set of classical Lorentzian four-geometries. One may then use the WKB approximation and the tangent vector to the configuration space—for paths about which  $\Psi$  is peaked—to define the proper time  $\tau$  along the classical trajectories.<sup>(6)</sup> Thus, once the mechanism of spontaneous symmetry breaking of the spacetime symmetries has taken place in the early universe, then the problems of time and time's arrow are solved by means of the classical Hamilton-Jacobi equation, and the classical trajectories define a time and time's direction in the symmetric phase. The universe is then clearly divided into a quantum gravity regime and a classical regime, making the WKB solution to the origin of the time variable unambiguous without arbitrary boundary conditions.

The problems of quantizing gravity and treating it as a Yang–Mills type gauge theory are critically assessed in Sections 4, 6, and 7, without attempts being made to resolve many of the fundamental problems in quantum gravity. In Sections 5, 8, 9, and 10, a solution is proposed for the origin of time and time’s arrow, the second law of thermodynamics, and the unity of gravity and quantum mechanics. We end with a summary of our results in Section 11.

## 2. CANONICAL FORMALISM FOR GRAVITY

The action in Einstein’s gravitational theory appropriate for a fixed three-geometry on a boundary is<sup>(1)</sup>

$$S_E = \frac{1}{16\pi G} \left[ \int_{\partial M} d^3x h^{1/2} 2K + \int_M d^4x (-g)^{1/2} (R + 2A) \right] \quad (2.1)$$

where the second term is integrated over spacetime and the first over its boundary,  $K$  is the trace of the extrinsic curvature  $K_{ij}$  ( $i, j = 1, 2, 3$ ) of the boundary three-surface, and  $A$  is the cosmological constant. We write the metric in the usual 3 + 1 form:

$$ds^2 = (N^2 - N_i N^i) dt^2 - 2N_i dx^i dt - h_{ij} dx^i dx^j \quad (2.2)$$

and the action becomes

$$S_E = \frac{1}{16\pi G} \int d^4x h^{1/2} N [-K_{ij} K^{ij} + K^2 - R(h)^{(3)} + 2A] \quad (2.3)$$

where

$$K_{ij} = \frac{1}{N} \left[ -\frac{1}{2} \frac{\partial h_{ij}}{\partial t} + N_{(i|j)} \right] \quad (2.4)$$

$R^{(3)}$  denotes the scalar curvature constructed from the three-metric  $h_{ij}$  and a stroke denotes the covariant derivative with respect to the latter quantity. The matter action  $S_M$  can also be constructed from the  $N$ ,  $N_i$ ,  $h_{ij}$  and the matter field.

The super-Hamiltonian density is given by

$$H = NH_0 + N^i H_i = H_0 \sqrt{h} + N^i H_i \quad (2.5)$$

where  $H_0$  and  $H_i$  are the usual Hamiltonian and momentum constraint functions, defined in terms of the canonically conjugate momenta  $\pi^{ij}$  to the dynamical variables  $h_{ij}$ :

$$\pi^{ij} = \frac{\delta L_E}{\delta(\partial h_{ij}/\partial t)} \quad (2.6)$$

where  $L_E$  is the Einstein–Hilbert Lagrangian density. Classically, the Dirac constraints are

$$H_i = 0, \quad H_0 = 0 \quad (2.7)$$

These constraints are a direct consequence of the general covariance of Einstein’s theory of gravity.

### 3. WAVE FUNCTION OF THE UNIVERSE AND THE EUCLIDEAN PATH INTEGRAL FORMALISM

Following Hartle and Hawking,<sup>(10)</sup> we define the wave function of the universe  $\Psi$  by

$$\Psi[h_{ij}, \phi] = - \int [dg][d\phi] \mu[g, \phi] \exp(iS[g, \phi]) \quad (3.1)$$

where  $\phi$  denotes a matter field,  $S$  is the total action, and  $\mu[g, \phi]$  is an invariant measure factor. The integral (or sum) is over a class of spacetimes with a compact boundary on which the induced metric is  $h_{ij}$  and field configurations which match  $\phi$  on the boundary. In the quantum mechanics of a closed universe, a problem arises with the definition of a natural energy, since in a sense the total energy is zero, i.e., the gravitational energy cancels the matter energy. Moreover, there arises the problem of the meaning of an “external” observer outside the universe. We shall not attempt to resolve this problem here.

It is reasonable to be able to expect to define a wave function of the universe in terms of an Euclidean functional integral of the form

$$\Psi[h_{ij}, \phi] = \int [dg][d\phi] \mu[g, \phi] \exp(-I_E[g]) \quad (3.2)$$

where  $I_E$  is the Euclidean action for gravity obtained from  $S_E$  by letting  $t \rightarrow -it$ . But this leads to a well-known problem, namely, Euclidean gravity is a “bottomless” theory, i.e., a theory whose action is unbounded from below. If we write the Euclidean metric  $g_{\mu\nu}^{(E)}$  as a product of the conformal factor times a unimodular metric:

$$g_{\mu\nu}^{(E)} = \Omega^2 \bar{g}_{\mu\nu} \quad (3.3)$$

where

$$\det(\bar{g}) = 1 \quad (3.4)$$

then the action

$$I_E = -\frac{1}{16\pi G} \int d^4x \sqrt{g} R = \frac{1}{16\pi G} \int d^4x [ -(\partial_\mu \Omega)^2 - \Omega^2 \bar{R} ] \quad (3.5)$$

is unbounded from below, since it can be made arbitrarily negative by a suitable choice of  $\Omega$ , due to the “wrong” sign of the kinetic term for the conformal factor  $-(\partial_\mu \Omega)^2$ . It has been suggested that the conformal factor should be integrated over a complex contour in field space.<sup>(11)</sup> But it can be shown that contour deformations in functional integrals can lead to complex nonperturbative contributions to the correlation functions, even when all the perturbative contributions are real.<sup>(12)</sup> Other methods to get around this problem such as compactification of the field space<sup>(13)</sup> or adding higher derivative terms to stabilize the action<sup>(14)</sup> lead to violations of unitarity, and the vacuum of such stabilized theories may have nothing to do with the vacuum of the corresponding Minkowski theory. A recent promising suggestion by Greensite<sup>(15)</sup> is to use the method of a fifth-time action to formulate Euclidean quantum gravity.

#### 4. QUANTUM GRAVITY

The problem of quantizing the gravitational field remains a serious issue, which has not yet been satisfactorily resolved. The standard approach based on perturbation theory using a fixed background metric such as the Minkowski spacetime metric, leads to the divergence of the loop integrals in the quantized theory. The first-order loops for pure vacuum gravity are renormalizable, but this is due to the existence of an identity in four-dimensional pseudo-Riemannian spacetime which does not lead to renormalizable higher-order loops. Moreover, the diagrams yield nonrenormalizable contributions in all orders when matter is present. The choice of a fixed classical background goes against the spirit of the diffeomorphism invariance of general relativity.

Further serious problems arise when the path integral formalism is adopted, based on the generating function:

$$Z = \int [dg][d\phi] \mu[g, \phi] \exp(-I_E) \quad (4.1)$$

where  $\mu[g, \phi]$  is a measure factor which is chosen to guarantee local gauge invariance to all orders. This measure factor is not well defined in four-dimensional spacetime because the set of all topological measures cannot be classified. Thus, the measure is *intrinsically* undefined and the path

integral formalism cannot be mathematically formulated. In addition to this, one has to contend with the problem of the unboundedness of the functional path integral in Euclidean space.

Attempts have been made to put quantum gravity on a gauge lattice using Wilson-type loop methods.<sup>(16)</sup> This approach could lead to a nonperturbative solution to quantum gravity. A measure can now be uniquely chosen for a 4-sphere, and using the Wilson link variables  $U$  and choosing a lattice size  $a$ , the problem can be formulated in Euclidean space in a gauge invariant manner. However, manifest diffeomorphism invariance is lost, and it is not clear how one would recover it in the continuum limit. In addition, there appears to be no explicit method by which one can numerically realize a transition to the physical Minkowski spacetime. How does one numerically perform a rotation in the momentum variable  $p_0$  by  $\pi/2$  during a Monte Carlo simulation? Moreover, the problem of whether the S-matrix is unitary cannot be satisfactorily answered in this type of formulation. Attempts to solve quantum gravity by using the canonical formalism run into serious difficulties because the Dirac constraint equations are nonpolynomial equations which are hard to solve. To overcome this problem, Ashtekar<sup>(17)</sup> has proposed transforming to a complex connection and reformulating the problem with this connection. It is found that the Hamiltonian constraint equations become polynomial equations which are easier to solve. However, an equally serious problem now arises, for the inner products of state vectors in the linear Hilbert space are complex and cannot be normalized in a meaningful way. Thus, the quantum mechanical formalism is rendered unusable, unless some way is found to circumvent this problem.

Since the gauge theory formalism in quantum field theory has been so successful, it would seem fruitful to formulate quantum gravity as a gauge theory using the vierbein formalism. A quadratic term in the curvature can be added to the Lagrangian, yielding a Yang–Mills type of structure. However, as will be shown in Section 7, the whole gauge formalism is intrinsically nonviable since the Hamiltonian in the physical Minkowski space-time is unbounded from below, due to the noncompact nature of the local Lorentz gauge group  $SO(3, 1)$  in the flat tangent space (fiber bundle tangent space). Even after the standard longitudinal ghosts have been removed by choosing a suitable Faddeev–Popov ghost fixing, there remain additional negative metric transverse components, generated by the noncompact metric. After a choice of Faddeev–Popov gauge fixing is performed to get rid of these components, it can be proved that the S-matrix is not unitary. Thus, the whole gauge quantization program for gravity fails. This problem has its roots in the attempt to quantize the metric of spacetime with its intrinsically noncompact structure due to the existence

of a light cone. Standard quantization of Yang–Mills theories does not, of course, suffer this problem, for these Yang–Mills theories are associated with compact groups such as  $SU(n)$  or  $O(n)$ . The Hamiltonian is bounded from below after suitable gauge fixing, and the S-matrix is unitary. It appears that a new way to consistently quantize gauge theories based on noncompact groups is needed to make physical sense of quantum gravity.

## 5. THE PROBLEM OF TIME AND THE SCHRÖDINGER EQUATION

In quantum mechanics, a suitably normalized wave function is defined by the path integral

$$\psi(\vec{x}, t) = - \int [d\vec{x}(t)] \exp[iS(\vec{x}(t))] \quad (5.1)$$

We obtain

$$\frac{\partial \psi}{\partial t} = -i \int [d\vec{x}(t)] \frac{\partial S}{\partial t} \exp(iS) \quad (5.2)$$

which leads to the Schrödinger equation

$$i \frac{\partial \psi}{\partial t} = H\psi \quad (5.3)$$

A differential equation for the wave function of the universe,  $\Psi$ , can be derived by varying the end conditions on the path integral (5.1). Since the theory is diffeomorphism invariant, the wave function is independent of time, and we obtain

$$\frac{\partial \Psi}{\delta N} = -i \int [dg][d\phi] \mu[g, \phi] \left[ \frac{\delta S}{\delta N} \right] \exp(iS[g, \phi]) = 0 \quad (5.4)$$

where we have taken into account the translational invariance of the measure factor  $\mu[g, \phi]$ . Thus, the value of the integral should be left unchanged by an infinitesimal translation of the integration variable  $N$  and leads to the operator equation:

$$H_0 \Psi = 0 \quad (5.5)$$

The classical Hamiltonian constraint equation takes the form

$$H_0 = \delta S / \delta N = h^{1/2} (-K^2 + K_{ij} K^{ij} - R^{(3)} + 2\Lambda + 16\pi G T_m) = 0 \quad (5.6)$$

where  $T_{mn}$  is the stress-energy tensor of the matter field projected in the direction normal to the surface. By a suitable factor ordering (ignoring the well-known “factor ordering” problem), the classical equation  $\delta S/\delta N=0$  translates into the operator identity

$$\left\{ -\gamma_{ijkl} \frac{\delta^2}{\delta h_{ij} \delta h_{kl}} + h^{1/2} \left[ R^{(3)}(h) - 2A - \frac{16\pi}{M_P^2} T_{mn} \left( -i \frac{\delta}{\delta \phi}, \phi \right) \right] \right\} \Psi[h_{ij}, \phi] = 0 \quad (5.7)$$

where  $\gamma_{ijkl}$  is the metric on superspace,

$$\gamma_{ijkl} = \frac{1}{2} h^{-1/2} (h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl}) \quad (5.8)$$

and  $M_P^2 = 1/G$  is the (Planck mass)<sup>2</sup>. This is the familiar WD equation for a closed universe.<sup>(1)</sup>

Because time has disappeared in the diffeomorphism-invariant Einstein gravity theory, the universe is stationary and we do not obtain the familiar Schrödinger equation. This obviously creates difficulties, since time plays such a central role in quantum mechanics.<sup>(18)</sup> Attempts to identify dynamical phase variables in the WD equation with time lead to problems, for they lack a conserved, positive-definite probability current for the wave function  $\Psi$ , since the WD equation is a second-order hyperbolic differential equation in the ordinary Minkowski metric signature. This has led to “third quantization” of the wave function, treated as a dynamical field variable,<sup>(19–23)</sup> and to a “wormhole” field theory.<sup>(24–26)</sup> We do not view this as a satisfactory situation, since we would expect that the wave function of the universe should be time dependent and lead to a complex Schrödinger equation or its covariant counterpart—the Tomonaga–Schwinger equation:

$$i \frac{\delta \Psi}{\delta \tau} = \mathcal{H} \Psi \quad (5.9)$$

which leads to the ordinary time-dependent Schrödinger wave equation for global time variations, with a positive-definite probabilistic interpretation. We shall therefore propose a new definition of the wave function of the universe which takes the form

$$\Psi[h_{ij}, \phi] = - \int [dg][d\phi] M[g, \phi] \exp(iS[g, \phi]) \quad (5.10)$$

where  $M[g, \phi]$  is a measure factor that breaks the time translational invariance of the path integral and makes the wave function  $\Psi$  explicitly time dependent. We now obtain

$$\begin{aligned} \frac{\delta \Psi}{\delta N} = & - \int [dg][d\phi] \frac{\delta M}{\delta N} \exp(iS) \\ & - i \int [dg][d\phi] M[g, \phi] \frac{\delta S}{\delta N} \exp(iS) \end{aligned} \tag{5.11}$$

This leads to the time-dependent Schrödinger equation

$$i \frac{\delta \Psi}{\delta N} = \tilde{H}_0 \Psi \tag{5.12}$$

where  $\tilde{H}_0$  denotes

$$\tilde{H}_0 = -i \frac{\delta \ln M}{\delta N} \tag{5.13}$$

A simple example of a measure factor that brings in an explicit time dependence (or  $N$  dependence) is

$$M[g, \phi] = \mu[g, \phi] N^b \tag{5.14}$$

This measure factor  $M[g, \phi]$  retains the momentum constraint equation  $H_i = 0$  as an operator equation:

$$H_i \Psi = 0 \tag{5.15}$$

while keeping the invariance of the spatial three-geometry at the quantum mechanical level as well as at the classical level. If the measure  $M[g, \phi]$  is chosen so that the diffeomorphism group  $\mathcal{D}$  is broken down to a subgroup  $\mathcal{S}$ , then there will exist a minimal choice of  $M[g, \phi]$  which will break time translational invariance. The choice of  $M[g, \phi]$  is not unique and some, as yet, unknown physical principle is needed to determine  $M[g, \phi]$ . At the classical level, we shall continue to maintain general covariance, and the classical constraint equation (5.5) holds. The Bianchi identities

$$G_{\mu}{}^{\nu}{}_{;\nu} = 0 \tag{5.16}$$

are valid, where  $G_{\mu}{}^{\nu} = R_{\mu}{}^{\nu} - \frac{1}{2} \delta_{\mu}{}^{\nu} R$  and  $;$  denotes covariant differentiation with respect to the connection. It is only the quantum mechanical wave function that breaks the diffeomorphism invariance, i.e.,  $N$  is no longer a free variable for the wave function of the universe. This leads naturally to

a cosmic time which can be used to measure time-dependent quantum mechanical observables. We find that for any operator  $O$ , we get

$$\frac{\delta}{\delta N} \langle O \rangle = i \langle [H, O] \rangle \quad (5.17)$$

which constitutes the quantum mechanical version of Hamilton's equation. In contrast to the WD equation, Ehrenfest's theorem follows directly from (5.17).

The probability to find the system in configuration space at time  $t$  is given by

$$dP = d\Omega_q |\Psi(q_i, t)|^2 \quad (5.18)$$

where  $d\Omega_q$  is the configuration space element. Provided  $\Psi$  is well behaved at infinity, the integral of  $|\Psi|^2$  taken over the whole of configuration space is independent of time and can be normalized to one:

$$\int d\Omega_q |\Psi(q_i, t)|^2 = 1 \quad (5.19)$$

This follows from the fact that  $|\Psi|^2$  is the time component of a conserved probability current. We also have that  $dP \geq 0$ .

The semiclassical WKB approximation is valid for  $M_p \rightarrow \infty$ , and the Einstein–Hamilton–Jacobi equation is now solved using an expansion in powers of the wave function

$$\Psi = \exp(iS) \quad (5.20)$$

We require that the wave function in the classical limit  $M_p \rightarrow \infty$  obey the operator equation:

$$H_0 \Psi_{\text{WKB}} = 0 \quad (5.21)$$

which is consistent with the classical constraint equation (5.6).

Can we break spacetime translational invariance at the quantum mechanical level? This does not violate any known fundamental physical law. It solves the problem of time in quantum gravity and makes gravity consistent with quantum mechanics, which ultimately requires a “real” external observable time in order to make the theory physically meaningful. There is no violation of general covariance at the classical level, guaranteeing that the standard macroscopic experimental tests are not violated. The principle of general covariance is held by most physicists to be sacred, but to retain this symmetry at the quantum mechanical level for the wave

function is too high a price to pay, since it means giving up the possibility of formulating a physically sensible theory of quantum gravity. It is not clear how we can perform an experimental test to detect a quantum mechanical violation of time translational invariance in the wave function of the universe, since quantum gravity effects only become significant at the Planck energy  $\sim 10^{19}$  GeV, or in very early universe cosmology. However, it is important to unify quantum mechanics and gravity within a conceptually logical picture, since both of these pillars of modern physics are with us to stay.

### 6. GAUGE FORMALISM FOR GRAVITY

Let us define the metric in any noninertial coordinate system by

$$g_{\mu\nu}(x) = e_\mu^a(x) e_\nu^b(x) \eta_{ab} \tag{6.1}$$

where

$$e_\mu^a(X) = \left( \frac{\partial \zeta_X^a}{\partial x^\mu} \right)_{x=X} \tag{6.2}$$

The  $\zeta_X^a$  are a set of locally inertial coordinates at  $X$ . The vierbeins  $e_\mu^a$  satisfy the orthogonality relations:

$$e_\mu^a e_b^\mu = \delta_b^a, \quad e_a^\mu e_\nu^\mu = \delta_a^\nu \tag{6.3}$$

which allow us to pass from the flat tangent space coordinates (the fiber bundle tangent space) labeled by  $a, b, c, \dots$  to the world spacetime coordinates (manifold) labeled by  $\mu, \nu, \rho, \dots$ . The fundamental form (6.1) is invariant under Lorentz transformations:

$$e_\mu^{\prime a}(x) = L_b^a(x) e_\mu^b(x) \tag{6.4}$$

where  $L_b^a(x)$  are the homogeneous  $SO(3, 1)$  Lorentz transformation coefficients that can depend on position in spacetime, and which satisfy

$$L_{ac}(x) L_d^a(x) = \eta_{cd} \tag{6.5}$$

For a general field  $f_n(x)$  the transformation rule will take the form

$$f_n(x) \rightarrow \sum_m [D(L)(x)]_{nm} f_m(x) \tag{6.6}$$

where  $D(L)$  is a matrix representation of the (infinitesimal) Lorentz group.

The  $e_\mu^a$  will satisfy

$$e_{\mu,\sigma}^a + (\Omega_\sigma)_c^a e_\mu^c - \Gamma_{\sigma\mu}^\rho e_\rho^a = 0 \quad (6.7)$$

where  $e_{\mu,\nu}^a = \partial e_\mu^a / \partial x^\nu$ ,  $\Omega_\mu$  is the spin connection of gravity, and  $\Gamma_{\mu\nu}^\lambda$  is the Christoffel connection. Solving for  $\Gamma$  gives

$$\Gamma_{\sigma\lambda\rho} = g_{\delta\rho} \Gamma_{\sigma\lambda}^\delta = \eta_{ab} (D_\sigma e_\lambda^a) e_\rho^b \quad (6.8)$$

where

$$D_\sigma e_\mu^a = e_{\mu,\sigma}^a + (\Omega_\sigma)_c^a e_\mu^c \quad (6.9)$$

is the covariant derivative operator with respect to the gauge connection  $\Omega_\mu$ . By differentiating (6.1), we get

$$g_{\mu\nu,\sigma} - g_{\rho\nu} \Gamma_{\mu\sigma}^\rho - g_{\mu\rho} \Gamma_{\nu\sigma}^\rho = 0 \quad (6.10)$$

where we have used  $(\Omega_\sigma)_{ca} = -(\Omega_\sigma)_{ac}$ .

The (spin) gauge connection  $\Omega_\mu$  remains invariant under the Lorentz transformations provided:

$$(\Omega_\sigma)_b^a \rightarrow [L \Omega_\sigma L^{-1} - (\partial_\sigma L) L^{-1}]_b^a \quad (6.11)$$

A curvature tensor can be defined by

$$([D_\mu, D_\nu])_b^a = (R_{\mu\nu})_b^a \quad (6.12)$$

where

$$(R_{\mu\nu})_b^a = (\Omega_\nu)_{b,\mu}^a - (\Omega_\mu)_{b,\nu}^a + ([\Omega_\mu, \Omega_\nu])_b^a \quad (6.13)$$

The curvature tensor transforms like a gauge field strength:

$$(R_{\mu\nu})_b^a \rightarrow L_c^a (R_{\mu\nu})_d^c (L^{-1})_b^d \quad (6.14)$$

In holonomic coordinates, the curvature tensor is

$$R_{\sigma\mu\nu}^\lambda = (R_{\mu\nu})_b^a e_a^\lambda e_\sigma^b \quad (6.15)$$

and the scalar curvature takes the form

$$R = e^{\mu a} e^{\nu b} (R_{\mu\nu})_{ab} \quad (6.16)$$

The action is written as

$$S_G = \int d^4x e \left\{ -\frac{1}{16\pi G} [R(\Omega) + 2\Lambda] + \frac{1}{8\omega} (R_{\mu\nu}(\Omega))_b^a (R^{\mu\nu}(\Omega))_a^b \right\} \quad (6.17)$$

where  $e \equiv \sqrt{-g} = \det(e_\mu^a e_{av})^{1/2}$ ,  $R(\Omega)$  denotes the scalar curvature determined by the spin connection  $\Omega$ , and  $\omega$  is a dimensionless coupling constant. We have included a term quadratic in the curvature to ensure that the action has the familiar Yang–Mills form.<sup>(27–31)</sup>

The field equations take the form

$$R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} (R(g) + 2\Lambda) = 8\pi k^2 E_{\mu\nu} \tag{6.18}$$

where  $R(g)$  denotes the curvature scalar determined by the pseudo-Riemannian metric and connection. Moreover,

$$E_{\mu\nu} = \frac{1}{2} ((R_{\mu\lambda}(\Omega))_b^a (R_\nu^\lambda(\Omega))_a^b - \frac{1}{4} g_{\mu\nu} (R_{\lambda\rho}(\Omega))_b^a (R^{\lambda\rho}(\Omega))_a^b) \tag{6.19}$$

and  $k^2 = G/\omega$ . These equations can be written as

$$\frac{1}{8\pi k^2} B_{\mu\nu}(g) = C_{\mu\lambda\nu\rho} B^{\lambda\rho}, \quad R(g) = 4\Lambda \tag{6.20}$$

where

$$B_{\mu\nu} = R_{\mu\nu}(g) - \frac{1}{4} g_{\mu\nu} R(g) \tag{6.21}$$

and  $C_{\lambda\mu\nu\rho}$  is the Weyl tensor. These field equations are satisfied if the vacuum equations hold:

$$R_{\mu\nu}(g) = 0 \tag{6.22}$$

We have restricted ourselves to a torsion-free  $SO(3, 1)(SL(2, C))$  gauge theory. However, our conclusions regarding such gauge theories apply equally well to the more general theories based on the Poincaré group<sup>(29,30)</sup> and conformal gauge symmetries.<sup>(31)</sup>

## 7. QUANTIZATION OF GAUGE GRAVITY THEORY

Some comments about the quantization of the action (6.17) are in order. Let us introduce the notation:  $E_i = R_{0i} (i = 1, 2, 3)$ ,  $H_i = \frac{1}{2} \varepsilon_{ijk} R^{jk}$ ,  $\vec{\mathcal{E}}_E^i = \vec{\mathcal{E}}^{i0}$ ,  $\vec{\mathcal{E}}_H^i = \vec{H}^{i0}$ ,  $\vec{\mathcal{H}}_E^i = \frac{1}{2} \varepsilon^{ikl} \vec{E}_{kj}$ ,  $\vec{\mathcal{H}}_H^i = \frac{1}{2} \varepsilon^{ikj} \vec{H}_{kj}$ . Then we obtain

$$E_{00} = \frac{1}{2} (\mathcal{H}_E^2 + \mathcal{H}_H^2 - \mathcal{E}_E^2 - \mathcal{E}_H^2) \tag{7.1}$$

The noncompactness of the homogeneous Lorentz group  $SO(3, 1)$ , with the associated indefiniteness of the group metric, leads to the indefiniteness of the energy.<sup>(32,33)</sup> The Hamiltonian is not bounded from

below and we lose the ground state, and the S-matrix is not unitary. To check unitarity in a Yang–Mills theory of gravity, one considers the lowest order nontrivial diagrams—the one-loop diagrams.<sup>(33)</sup> The one-loop diagram amplitude is denoted by  $A(\text{loop})$ . Let us denote the contribution to the intermediate states by the transverse components in  $\text{Im } A(\text{loop})$  by  $\text{Im } C(\text{loop})$ . Then we should have

$$\text{Im } A(\text{loop}) = \text{Im } C(\text{loop}) \quad (7.2)$$

But this does not guarantee that unitarity is satisfied, because  $\text{Im } C(\text{loop})$  contains the contributions of *negative metric transverse components*. If we gauge fix away the negative transverse components in the external states, then we see that

$$\text{Im } A(\text{loop}) \neq \text{Im } C(\text{loop}) \quad (7.3)$$

and unitarity is violated and the  $SO(3, 1)(SL(2, C))$  gauge symmetry is no longer respected. If we insist on maintaining gauge symmetry for the amplitudes, then the theory cannot be unitary. This problem does not occur for Yang–Mills theories based on compact groups. The invariant quadratic forms, which appear in the energy or the norm of the perturbative quanta, are not positive definite if the group is not semisimple and compact. The Killing form  $K_{ab} = f_{ac}^d f_{bd}^c$  (where  $f_{bc}^a$  are the group structure coefficients) has no definite sign in this case. The quadratic invariant  $\eta_{\mu\nu}$  for  $SU(p, q)$  or  $O(p, q)$  has this property, and ghost particles occur in any representation.

One possible way out of this dilemma<sup>(32)</sup> is to realize the noncompact gauge group invariance *nonlinearly*, its maximal compact subgroup  $H$  being realized linearly on the fields. Thus, the disease may be cured by forming field-dependent positive metrics. However, up till the present time, no concrete program has been developed to rid noncompact Yang–Mills theories of the ghost disease.

Although the path integral is defined in the tangent fiber bundle space, since a Wick rotation leads to  $SO(3, 1) \rightarrow O(4)$  and the Euclidean action in the tangent space is positive definite (reflection positivity), it is unbounded in the base space ( $g_{\mu\nu} \rightarrow g_{\mu\nu}^{(E)}$ ) due to the conformal mode. It is far from clear that the situation is healthy. There is no mathematically rigorous proof that we can define the quantum theory in the physical space, since singularities in the solutions of the field equations probably exist, which prevent a simple Wick rotation, and also forbid any kind of meaningful analytic continuation to take place. This is a subject which seriously requires a rigorous mathematical solution, before any sensible quantum gravity theory can be defined.

The gravitational gauge theories are in general not renormalizable. For our Euclidean  $SO(4)$  theory, a calculation of the one-loop counter term gives<sup>(34)</sup>

$$\Delta \mathcal{L} = \frac{1}{\varepsilon} \left[ \left( \frac{137}{60} + \frac{r_L - 1}{10} \right) R_{\mu\nu}^2(g) + C_{2L} \frac{11}{12} \left( -\frac{1}{2} \right) \text{tr}[R_{\mu\nu}(\Omega) R^{\mu\nu}(\Omega)] \right] \quad (7.4)$$

where  $\varepsilon = 8\pi^2(n - 4)$ ,  $r_L = 6$ , and  $C_{2L} = C_2(SO(4))$ . The zero-torsion version of the theory is renormalizable but nonunitary.<sup>(35)</sup> Torsion would restore unitarity but spoil the renormalizability of the theory with a quadratic Lagrangian. If the vierbein  $e_\mu^a$  is not quantized, then the theory becomes simply an  $SO(4)$  Yang–Mills gauge field theory in curved spacetime. The theory is not unitary in Minkowski spacetime. The one-loop counterterm has the form

$$\Delta \mathcal{L} = \frac{1}{\varepsilon} \left[ \frac{r_L}{20} C_{\lambda\mu\nu\rho}^2 + C_{2L} \left( -\frac{1}{2} \right) \text{tr} \left( \frac{11}{12} R_{\mu\nu}^2 \right) \right] \quad (7.5)$$

where the Weyl tensor  $C_{\lambda\mu\nu\rho}$  depends only on the external field and does not spoil the renormalizability of the theory. When  $e_\mu^a$  is quantized, then additional  $R^2(g)$  terms contribute, violating renormalizability when the torsion is zero. The theory in which the vierbein is not quantized simply avoids the problematic issue of quantizing the geometrical gravitational field, i.e., it evades the fundamental problem of seeking a consistent quantum gravity theory.

## 8. THE ARROW OF TIME AND SPONTANEOUS BREAKING OF THE GRAVITATIONAL VACUUM

Let us now address the fundamental problem of the origin of time and of the arrow of time and the second law of thermodynamics. To this end, we shall consider a specific kind of symmetry breaking in the early universe, in which the local Lorentz vacuum symmetry is spontaneously broken by a Higgs mechanism. We postulate the existence of a scalar field,  $\phi$ , and assume that the vacuum expectation value (vev) of the scalar field,  $\langle \phi \rangle_0$ , will vanish for some temperature  $T$  less than a critical temperature  $T_c$ , when the local Lorentz symmetry is restored. Above  $T_c$  the nonzero vev will break the symmetry of the ground state of the universe from  $SO(3, 1)$  down to  $O(3)$ . The domain formed by the direction of the vev of the Higgs field will produce a time arrow pointing in the direction of

increasing entropy and the expansion of the universe. Let us introduce four real scalar fields  $\phi^a(x)$  which are invariant under Lorentz transformations

$$\phi'^a(x) = L_b^a(x) \phi^b(x) \quad (8.1)$$

We can use the vierbein to convert  $\phi^a$  into a 4-vector in coordinate space:  $\phi^\mu = e_a^\mu \phi^a$ . The covariant derivative operator acting on  $\phi$  is defined by

$$D_\mu \phi^a = [\partial_\mu \delta_b^a + (\Omega_\mu)_b^a] \phi^b \quad (8.2)$$

If we consider infinitesimal Lorentz transformations

$$L_b^a(x) = \delta_b^a + \omega_b^a(x) \quad (8.3)$$

with

$$\omega_{ab}(x) = -\omega_{ba}(x) \quad (8.4)$$

then the matrix  $D$  in (6.6) has the form

$$D(1 + \omega(x)) = 1 + \frac{1}{2} \omega^{ab}(x) \sigma_{ab} \quad (8.5)$$

where the  $\sigma_{ab}$  are the six generators of the Lorentz group which satisfy  $\sigma_{ab} = -\sigma_{ba}$  and the commutation relations

$$[\sigma_{ab}, \sigma_{cd}] = \eta_{cb} \sigma_{ad} - \eta_{ca} \sigma_{bd} + \eta_{db} \sigma_{ca} - \eta_{da} \sigma_{cb} \quad (8.6)$$

The set of scalar fields  $\phi$  transforms as

$$\phi'(x) = \phi(x) + \omega^{ab}(x) \sigma_{ab} \phi(x) \quad (8.7)$$

The gauge spin connection which satisfies the transformation law (6.11) is given by

$$\Omega_\mu = \frac{1}{2} \sigma^{ab} e_a^\nu e_{b\nu;\mu} \quad (8.8)$$

We shall now introduce a Higgs sector into the Lagrangian density such that the gravitational vacuum symmetry, which we set equal to the Lagrangian symmetry at low temperatures, will break to a smaller symmetry at high temperature. The pattern of vacuum phase transition that emerges contains a symmetry antirestoration.<sup>(36-43)</sup> This vacuum symmetry breaking leads to the interesting possibility that exact zero temperature conservation laws, e.g., electric charge and baryon number, are broken in the early universe. In our case, we shall find that the spontaneous breaking of the Lorentz symmetry of the vacuum leads to a violation of the exact zero temperature conservation of energy in the early universe.

We shall consider the Lorentz invariant Higgs potential:

$$V(\phi) = -\frac{1}{2}\mu^2 \sum_{a=0}^3 \phi_a \phi^a + \lambda \sum_{a=0}^3 (\phi_a \phi^a)^2 \tag{8.9}$$

where  $\lambda > 0$  is a coupling constant such that  $V(\phi)$  is bounded from below. Our Lagrangian density now takes the form

$$\mathcal{L} = \mathcal{L}_G + \sqrt{-g} [\frac{1}{2}D_\mu \phi_a D^\mu \phi^a - V(\phi)] \tag{8.10}$$

If  $V$  has a minimum at  $\phi_a = v_a$ , then the spontaneously broken solution is given by  $v_a^2 = \mu^2/4\lambda$  and an expansion of  $V$  around the minimum yields the mass matrix:

$$(\mu^2)_{ab} = \frac{1}{2} \left( \frac{\partial^2 V}{\partial \phi_a \partial \phi_b} \right)_{\phi_a = v_a} \tag{8.11}$$

We can choose  $\phi_a$  to be of the form

$$\phi_a = \begin{pmatrix} 0 \\ 0 \\ 0 \\ v \end{pmatrix} = \delta_{a0}(\mu^2/4\lambda)^{1/2} \tag{8.12}$$

All the other solutions of  $\phi_a$  are related to this one by a Lorentz transformation. Then, the homogeneous Lorentz group  $SO(3, 1)$  is broken down to the spatial rotation group  $O(3)$ . The three rotation generators  $J_i (i = 1, 2, 3)$  leave the vacuum invariant

$$J_i v_i = 0 \tag{8.13}$$

while the three Lorentz-boost generators  $K_i$  break the vacuum symmetry

$$K_i v_i \neq 0 \tag{8.14}$$

The  $J_i$  and  $K_i$  satisfy the usual commutation relations

$$[J_i, J_j] = i\epsilon_{ijk} J_k, \quad [J_i, K_j] = i\epsilon_{ijk} K_k, \quad [K_i, K_j] = -i\epsilon_{ijk} K_k \tag{8.15}$$

The mass matrix  $(\mu^2)_{ab}$  can be calculated from (8.11):

$$(\mu^2)_{ab} = (-\frac{1}{2}\mu^2 + 2\lambda v^2) \delta_{ab} + 4\lambda v_a v_b = \mu^2 \delta_{a0} \delta_{b0} \tag{8.16}$$

where  $v$  denotes the magnitude of  $v_a$ . There are three zero-mass Goldstone bosons, the same as the number of massive vector bosons, and there are three massless vector bosons corresponding to the unbroken  $O(3)$  symmetry. After the spontaneous breaking of the vacuum, one massive physical

Higgs particle  $\phi^H$  remains. No ghost particles will occur in the unitary gauge. The vector boson mass term is given in the unitary gauge by

$$\mathcal{L}_\Omega = \frac{1}{2}\sqrt{-g} (\Omega_\mu)^{ab} v_b (\Omega^\mu)_a^c v_c = \frac{1}{2}\sqrt{-g} \sum_{i=1}^3 ((\Omega_\mu)^{i0})^2 (\mu^2/4\lambda) \quad (8.17)$$

We could have extended this symmetry breaking pattern to the case where we have two sets of vector representations,  $\phi_{a1}$  and  $\phi_{a2}$ . The invariant spin connection can depend on the length of each Lorentz vector and the angle between them,  $|\phi_{a1}|$ ,  $|\phi_{a2}|$ , and  $|\phi_{a1}\phi_{a2}|$ . The solutions for the minimum must be obtained from the conditions imposed on these three quantities. We can choose  $\phi_{a1}$  with only the last component nonzero and  $\phi_{a2}$  with the last two components nonzero in order to satisfy these conditions. The Lorentz  $SO(3, 1)$  symmetry is then broken down to  $O(2)$  (or  $U(1)$ ) symmetry.<sup>(44)</sup>

A phase transition is assumed to occur at the critical temperature  $T_c$ , when  $v_a \neq 0$  and the Lorentz symmetry is broken and the three gauge fields  $(\Omega_\mu)^{i0}$  become massive vector bosons. Below  $T_c$  the Lorentz symmetry is restored, and we regain the usual classical gravitational field with massless gauge fields  $\Omega_\mu$ . The symmetry breaking will extend to the singularity or the possible singularity-free initial state of the big bang, and since quantum effects associated with gravity do not become important before  $T \sim 10^{19}$  GeV, we expect that  $T_c \leq 10^{19}$  GeV.

In most known cases of phase transitions of the first and second kind, the more symmetrical phase corresponds to higher temperatures and the less symmetrical one to lower temperatures. A transition from an ordered to a disordered state usually occurs with increasing temperature. Examples of two known exceptions in nature are the “lower Curie point” of Rochelle salt, below which the crystal is orthorhombic, but above which it is monoclinic. Another example is the gapless superconductor. A calculation of the effective potential for the Higgs breaking contribution in (8.10) shows that extra minima in the potential  $V(\phi)$  can occur for a noncompact group such as  $SO(3, 1)$ . This fact has been explicitly demonstrated in a model with  $O(n) \times O(n)$  symmetric four-dimensional  $\phi^4$  field theory.<sup>(43)</sup> This model has two irreducible representations of fields,  $\vec{\phi}_1$  and  $\vec{\phi}_2$ , transforming as  $(n, 1)$  and  $(1, n)$ , respectively. The potential is

$$V = \sum_i \frac{1}{2} m_i^2 \vec{\phi}_i^2 + \sum_{i,j} \frac{1}{8} \vec{\phi}_i^2 \lambda_{ij} \vec{\phi}_j^2 \quad (8.18)$$

The requirement of boundedness from below gives  $(\lambda_{12} = \lambda_{21})$

$$\lambda_{11} > 0, \quad \lambda_{22} > -(\lambda_{11} \lambda_{22})^{1/2} \quad (8.19)$$

If we have  $\lambda_{12} < -(1 + 2/n) \lambda_{22}$ , then the one-loop free energy predicts spontaneous symmetry breaking to  $O(n) \times O(n-1)$  at sufficiently high temperatures without symmetry breaking at small temperatures. The standard symmetry breaking restoration theorems can be broken in this case because the dynamical variables  $\vec{\phi}_i$  do not form a compact space.

When the symmetry is restored, the entropy will rapidly increase and for a closed universe will reach a maximum at the final singularity, provided that no further phase transition occurs which breaks the Lorentz symmetry of the vacuum. Thus, the symmetry breaking mechanism explains in a natural way the low entropy at the initial singularity and the large entropy at the final singularity. It is claimed that the entropy increase can be explained in inflationary models but, as yet, no satisfactory inflationary model has been constructed.<sup>(45-48)</sup> There does not exist in the standard inflationary scenario an ingredient that can explain the time asymmetry between the initial and final singularity in a closed universe.<sup>(49)</sup>

Since the ordered phase is at a much lower entropy than the disordered phase and due to the existence of a domain determined by the direction of the vev of the Higgs field, a natural explanation is given for the cosmological arrow of time and the origin of the second law of thermodynamics. Thus, the spontaneous symmetry breaking of the gravitational vacuum corresponding to the breaking pattern,  $SO(3, 1) \rightarrow O(3)$ , leads to a manifold with the structure  $R \times O(3)$ , in which time appears as an absolute external parameter. The vev of the Higgs field,  $\langle \phi \rangle_0$ , points in a chosen direction of time to break the symmetry creating an arrow of time. The evolution from a state of low entropy in the ordered phase to a state of high entropy in the disordered phase explains the second law of thermodynamics.

### 9. BROKEN ENERGY CONSERVATION AND CREATION OF MATTER

We shall define the energy-momentum tensor by<sup>(50)</sup>

$$T_{\mu\nu} = e_{a\mu} v_\nu^a \tag{9.1}$$

where  $v_\mu^a$  is a coordinate vector and a Lorentz vector, and  $T_{\mu\nu}$  is a coordinate tensor and a Lorentz scalar. In classical general relativity  $T_{\mu\nu}$  satisfies

$$T_{\mu\nu} = T_{\nu\mu} \tag{9.2}$$

and

$$T^\nu_{\mu;\nu} = 0 \tag{9.3}$$

Under the infinitesimal Lorentz transformations (8.3) with  $|\omega_b^a| \ll 1$ , the matter action  $S_M$  must be stationary with respect to variations in the variables except the vierbein, which is treated as an external field variable. Thus, we consider only the change

$$\delta e_a^\mu(x) = \omega_a^b(x) e_b^\mu(x) \quad (9.4)$$

The invariance of the matter action under position-dependent Lorentz transformations leads to

$$\int d^4x (-g)^{1/2} v_\mu^a(x) e^{b\mu}(x) \omega_{ab}(x) = 0 \quad (9.5)$$

Since  $\omega_{ab}$  satisfies (8.4), we have for arbitrary  $\omega$  that the coefficient of  $\omega$  must be symmetric:

$$v_\mu^a e^{b\mu} = v_\mu^b e^{a\mu} \quad (9.6)$$

or that

$$v_v^a e_{a\mu} = v_\mu^b e_{bv} \quad (9.7)$$

which establishes the symmetry condition (9.2). To show (9.3), we use the invariance of the matter action under the infinitesimal coordinate transformations:

$$x'^\mu = x^\mu + \xi^\mu(x) \quad (9.8)$$

with  $|\xi^\mu| \ll 1$ . Then we get

$$\delta e_a'^\mu \equiv e_a^\nu \xi^\mu_{,\nu} - e_{a,\lambda}^\mu \xi^\lambda \quad (9.9)$$

After integration by parts, we obtain for arbitrary  $\xi^\mu$ :

$$(\sqrt{-g} v_\lambda^a e_a^\nu)_{,\nu} + \sqrt{-g} v_\mu^a e_{a,\lambda}^\mu = 0 \quad (9.10)$$

This can be written as

$$(\sqrt{-g} T_\lambda^v)_{,\nu} + \sqrt{-g} T_{v\mu} e^{av} e_{a,\lambda}^\mu = 0 \quad (9.11)$$

From (6.1) and (6.10) and using (6.3), we obtain the usual conservation law

$$(\sqrt{-g} T_\lambda^v)_{,\nu} + \frac{1}{2} \sqrt{-g} T_{\mu\nu} g^{\mu\nu}{}_{,\lambda} = 0 \quad (9.12)$$

which can easily be shown to be identical to (9.3).

When we enter the broken local Lorentz symmetry phase for  $T > T_c$ , the action  $S$  will violate local Lorentz invariance in a fixed gauge and  $T^{\mu\nu}$  will no longer be symmetric. As a consequence, the conservation of  $T^{\mu\nu}$  will be spontaneously broken, and this means that the diffeomorphism invariance of the theory has been spontaneously broken, since (9.12) originates from the assumption that  $S_M$  is invariant under the group of diffeomorphism transformations.

In general, the action  $S$  is invariant under Lorentz and diffeomorphism transformations, but when a specific direction of symmetry breaking along the time axis is chosen, energy conservation is spontaneously broken. In the symmetry-restored phase of the universe, conservation of energy is always satisfied, since diffeomorphism invariance and local Lorentz invariance are strictly obeyed both for the action and the vacuum. Let us consider small oscillations about the true minimum and define a shifted field:

$$\phi'_a = \phi_a - v_a \quad (9.13)$$

Then, the action becomes

$$S' = S + S_b \quad (9.14)$$

where  $S_b$  is no longer invariant under the local Lorentz gauge transformations and the energy-momentum tensor is no longer conserved. After Faddeev–Popov ghost fixing, we can define a new set of extended Becchi–Rouet–Stora (BRS) Lorentz gauge transformations under which  $S'$  is invariant, and a set of Ward–Takahashi identities can be found. The Higgs mass and graviton mass contributions proportional to  $v^a = \langle 0 | \phi^a | 0 \rangle$  are given by (8.16) and (8.17).

In the broken-symmetry phase, in a fixed gauge the wave function of the universe,  $\Psi$ , is no longer time-translationally invariant. A real external time has been created in the ordered phase  $T > T_c$ . From this follows that we obtain a time-dependent Schrödinger equation (5.12). We can now make sense of the time dependence of quantum mechanical operators in quantum cosmology, and the Ehrenfest theorem follows. In the low-energy classical region for  $T < T_c$ , the wave function in the WKB approximation satisfies a Wheeler–deWitt equation<sup>(1)</sup>

$$H_0 \Psi_{\text{WKB}} = 0 \quad (9.15)$$

How do we now reconcile the existence of a real time variable and a time evolution in the classical domain for  $T < T_c$ ? We shall adopt the approach of Halliwell,<sup>(6)</sup> in which the quantum and classical regimes are distinguished according to whether the wave function  $\Psi$  has an exponential or oscillatory behavior, respectively. The regions in which the wave func-

tion is exponential are regarded as classically forbidden and  $\Psi$  cannot be associated with a Lorentzian geometry. On the other hand, the regions in which the wave function is oscillatory are regarded as classically allowed;  $\Psi$  is peaked about a classical Lorentzian four-geometry. A finite number of functions  $h^\alpha(t)$ , representing components of the three-metric, are defined in minisuperspace and their wave function  $\Psi(h)$  satisfies the WD equation:

$$H\Psi = (-\frac{1}{2}\nabla^2 + U(h)) \Psi(h) = 0 \quad (9.16)$$

where  $\nabla^2$  is the Laplacian operator in the minisuperspace. In the oscillatory region, the WKB approximation gives

$$\Psi(h) = F(h) \exp(iS(h)) \quad (9.17)$$

where  $S(h)$  is a rapidly varying phase and  $F(h)$  is a slowly varying function. From (9.16) and (9.17) we obtain

$$\frac{1}{2}(\nabla S)^2 + U(h) = 0 \quad (9.18)$$

$$\nabla S \cdot \nabla F + \frac{1}{2}\nabla^2 S = 0 \quad (9.19)$$

where a contribution of order  $\nabla^2 F$  has been neglected. Now it can be shown that  $\Psi$  is peaked about the set of trajectories which satisfies

$$f = \nabla S \quad (9.20)$$

where  $f$  is the momentum conjugate to  $h$ . To obtain a time variable, Halliwell uses the tangent vector in configuration space for the paths for which  $\Psi$  is peaked:

$$\frac{d}{d\tau} = \nabla S \cdot \nabla \quad (9.21)$$

where  $\tau$  is the proper time along the classical trajectories.

We see that time emerges as a parameter which labels points along the trajectories for which the wave function is peaked. Reparameterization invariance shows itself as the freedom to choose this parameter, such that

$$\frac{1}{N(t)} \frac{d}{dt} = \nabla S \cdot \nabla \quad (9.22)$$

where  $N(t)$  is an arbitrary function of  $t$ . The location and existence of the oscillatory region of spacetime is determined by the initial domain of ordered spontaneous symmetry breaking in the early universe, which imposes the boundary condition on the wave function. Thus, there is a

physical mechanism in the early universe which determines, for all time, the region in which a classical Lorentz geometry exists.

The presence of the Lorentz symmetry broken phase at high temperatures will spontaneously create matter at the beginning of the universe, due to the violation of the energy conservation. This could explain the origin of matter in the early universe. Energy conservation is restored as an exact law at lower energies. Also the presence of domains in the ordered phase will produce an arrow of time pointing in the direction of increasing entropy as the temperature drops during the expansion of the universe. Therefore, the Lorentz symmetry breaking of the gravitational vacuum has engendered a real time asymmetry in the universe. One of the interesting consequences of this symmetry breaking is that the standard proof of the CPT theorem fails.<sup>(51)</sup> This failure is probably inevitable in any new physical law that truly introduces a cosmological arrow of time and a time asymmetry. This could have important implications for CP or T violation observed in  $K^0$  decay.

We have arrived at a seemingly radical version of quantum gravity, which is fundamentally at odds with Einstein's vision of gravitational theory, based on the equivalence principle and the associated principle of general covariance. We have spontaneously broken the diffeomorphism group of transformations in order to understand the fundamental observational facts underlying thermodynamics, statistical physics, quantum mechanics, and the psychological arrow of time. It is now possible to explain the following physical phenomena:

1. The second law of thermodynamics
2. Schrödinger equation for the wave function of the universe
3. The real arrow of time (e.g., the aging of human beings)
4. The existence of matter

The price to pay for an explanation of these empirical laws of nature, according to our scenario, is:

5. The violation of Lorentz invariance and time translational invariance in the early universe<sup>(52)</sup>
6. Violation of the conservation of energy in the early universe
7. Breaking of CPT invariance

It is also necessary to postulate the existence of a short-range gravitational force (massive spin connection  $\Omega_\mu$ ) at very high energies  $\sim 10^{19}$  GeV.

## 10. FIELD EQUATIONS IN THE BROKEN SYMMETRY PHASE

We shall now investigate the gravitational field equations for cosmology in the broken phase of the early universe. We find that a Friedman–Robertson–Walker (FRW) cosmology can exist in which the massive vector gravitational gauge field  $\Omega_\mu^{0n} = V_\mu^n (n = 1, 2, 3)$  dominates the vacuum energy, which could drive the universe into an inflationary de Sitter phase, which ceases when the temperature drops below  $T_c$ . The mass of  $V_\mu^n$  is of order  $m \sim M_P \sim 10^{19}$  GeV.

The total action for the theory is

$$S = S_G + S_M + S_\phi \quad (10.1)$$

where  $S_G$  is given by (6.17) and  $S_M$  is the usual matter action for gravity. Moreover,

$$S_\phi = \int d^4x \sqrt{-g} \left[ \frac{1}{2} D_\mu \phi_a D^\mu \phi^a - V(\phi) \right] \quad (10.2)$$

Performing a variation of  $S$  leads to the field equations:

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi [G(T^{\mu\nu} + C^{\mu\nu}) + k^2 E^{\mu\nu}] + \Lambda g^{\mu\nu} \quad (10.3)$$

where  $T^{\mu\nu}$  is the matter tensor for a perfect fluid:

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu} \quad (10.4)$$

Moreover,  $E^{\mu\nu}$  is given by (6.19), and the scalar field energy-momentum tensor is of the usual form:

$$C^{\mu\nu} = D^\mu \phi_a D^\nu \phi^a - \mathcal{L}_\phi g^{\mu\nu} \quad (10.5)$$

The compatibility relation for the vierbeins is postulated to be

$$e_{\mu,\sigma}^a + (\Omega_\sigma)_c^a e_\mu^c - \Gamma_{\sigma\mu}^\rho e_\rho^a = 0 \quad (10.6)$$

where  $(\Omega_\mu)_{ab} = -(\Omega_\mu)_{ba}$ .

Since we assume that the symmetry breaking pattern is  $SO(3, 1) \rightarrow O(3)$ , there will be three massless gauge vector fields  $(\Omega_\mu)_{nm} = -(\Omega_\mu)_{mn}$  denoted by  $U_\mu^n$ , three massive vector bosons  $V_\mu^n$ , and one massive Higgs boson  $\phi^H$ . Because  $G^{\mu\nu}$  satisfies the Bianchi identities (5.16), we find in the broken symmetry phase after the shift of the scalar field  $\phi$  according to (9.13):

$$T^{\mu\nu}{}_{;\nu} = K^\mu \quad (10.7)$$

where  $K^\mu$  contains the mass terms proportional to  $v = \langle \phi \rangle_0$ . Thus the conservation of energy-momentum is spontaneously violated and matter can be created in this broken symmetry phase.

When the temperature passes below the critical temperature  $T_c$ ,  $v = 0$  and the action is restored to its classical form (10.1) with a symmetric degenerate vacuum and a massless spin gauge connection  $(\Omega_\mu)_b^a$ , and we regain the standard energy-momentum conservation laws:  $T^{\mu\nu}_{; \nu} = 0$ .

The manifold in the broken phase has the symmetry  $R \times O(3)$ . The three-dimensional space with  $O(3)$  symmetry is assumed to be homogeneous and isotropic and yields the usual maximally symmetric three-dimensional space:

$$d\sigma^2 = R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \tag{10.8}$$

where the spatial coordinates are comoving and  $t$  is the “absolute” external time variable. This is the Robertson–Walker theorem for our ordered phase of the vacuum and it has the correct subspace structure for the FRW universe with the metric

$$ds^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right] \tag{10.9}$$

The null geodesics of the metric (10.9) are the light paths of the subspace and  $ds$  measures the “absolute” time at each test particle.

## 11. CONCLUSIONS

By spontaneously breaking the gravitational vacuum at a critical temperature  $T_c$ , the gravitational gauge connection acquires a mass and the local Lorentz group of the ground state is broken:  $SO(3, 1) \rightarrow O(3)$ . When the temperature cools below  $T_c$ , the local Lorentz symmetry of the ground state of the universe is restored and the gauge connection  $\Omega_\mu$  becomes massless. In the ordered phase of the early universe, which extends to the singularity at  $t = 0$ , time becomes a physical external parameter. The vev of the Higgs field  $\phi$  chooses a direction in which to break the symmetry of the gravitational vacuum and this creates an arrow of time. The entropy undergoes a huge increase as the universe expands into the disordered phase, after it passes through the phase transition at the temperature  $T_c$ , explaining the second law of thermodynamics. The spontaneous violation of the conservation of energy in the first fractions of seconds of the birth of the universe explains the creation of matter.

The existence of an external time in the broken phase of the universe leads to a consistent quantum cosmology with a time-dependent Schrödinger equation and a conserved probability density for the wave function. When the local Lorentz symmetry and diffeomorphism invariance are restored for  $T < T_c$ , a time variable can be defined by means of the tangent vector in the classical configuration space, and the wave function has oscillatory behavior determined by a WKB approximation scheme. The initial broken symmetry phase in the early universe divides the universe into the quantum gravity regime and the classical regime that ensues when the spacetime symmetries are restored.

## ACKNOWLEDGMENTS

This work was supported by the Natural Sciences and Engineering Research Council of Canada. I thank N. Cornish, M. Clayton, and J. Greensite for helpful discussions. I also thank L. Mas and J. Carot, Universitat de les Illes Balears, Palma de Mallorca, for their kind hospitality while part of this work was in progress. I am also grateful for the hospitality of the High Energy Physics group at the Niels Bohr Institute, Copenhagen, Denmark, where part of this work was completed.

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