Multipole Effects on the Dielectrophoretic Force in an 'Isomotive' Field

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ABSTRACT: The derivation of the isomotive field geometry for dielectrophoresis assumes that the field is uniform relative to the dimensions of the particle of interest. For this situation the particle may be modelled as a point dipole.

A more rigorous attempt has been made in this study to include contributions due to multiple effects. The error introduced by the finite nonuniformity is investigated and presented in a general analytical form. A numerical result, calculated for the case of isomotive field chamber, devoid of edge effects, indicated that contributions due to multipole effects are negligible outside of a region within a few particle radii about the origin.

INTRODUCTION

Dielectrophoresis involves the force exerted on a polarized body in a nonuniform field. The derivation of the isomotive electrode geometry (Pohl and Pollock, 1977; Pohl, 1978) (in which force is independent of position along one axis) assumes that the particle undergoing dielectrophoresis is small relative to local field nonuniformities.

Jones (1984) has investigated higher-order effects through multipole corrections for the case of a cylindrically symmetrical, cusped electric field. This approach presents a series solutions for the dielectrophoretic force in terms of dipole, quadrupole, octupole, and higher-order terms. Though originally applied to a cusped field, this result may be extended to more loosely symmetrical systems.

ISOMOTIVE FIELD GEOMETRY

In many of the field geometries studied for use in dielectrophoretic particle separation (Pohl and Pollock, 1977) the force on a body is strongly dependent upon its position. If the difference in polarizability between particles is small then the force difference due to these variations may be overwhelmed by minor positional differences. By making the force independent of position along an axis parallel to the force the isomotive field geometry allows identification of much more subtle variations between particles.

As presented elsewhere (Pohl and Pollock, 1977; Pohl, Kaler, and Pollock, 1978; Pohl, 1978) the electrodes required to produce an isomotive field may be described, in cylindrical coordinates, by

$$r = r_{60} \sin(3\phi/2)^{-2/3} \qquad (1)$$

where r is the radial distance measured from the origin, r_{60} is the value of r at $\phi = 60°$, and ϕ is the angle measured from the positive horizontal axis (see Fig. 1).

The resulting electric field is described by

$$E_o = \frac{(-3/2)\ V}{r_{60}^{3/2}} r^{\frac{1}{2}} (\hat{r} \sin (3\phi/2) + \hat{\phi} \cos (3\phi/2) \qquad (2)$$

It is important to note that these equations apply only to the region where $-2\pi/3 < \phi < 2\pi/3$ and where $r > 0$. In the region where $2\pi/3 < \phi < 4\pi/3$ the field is undefined.

MULTIPOLE EFFECTS

By making the assumption of a cusped field Jones (1984) developed a general expression for the force on a conductive dielectric sphere in a conductive dielectric medium. The approach used assumes a cylindrically symmetrical field and a spherical body.

By replacing the original problem involving a dielectric sphere with finite conductivity with one involving a shell with free charge over its surface, the net force on the body may be calculated via the force density (see Landau and Lifshitz, [illegible], sections 15 and 16; Stratton 1941). A particle in a uniform electric field may be modelled exactly as a dipole. As the field becomes nonuniform, higher-order terms become significant; these may be modelled as a series of axial multipoles (Jones, 1984). An effective multipolar moment of order n may then be calculated as

$$P_{eff}(n) = \frac{4\pi\varepsilon_1^{*}(\varepsilon_2-\varepsilon_1)a^{2n+1}}{(n-1)!\,[n\varepsilon_2+(n+1)\varepsilon_1]}\cdot\frac{\partial^{n-1}E_o}{\partial r^{n-1}} \qquad (3)$$

which leads to a general force expression

$$F_r(n) = \frac{4\pi\,\varepsilon_1^{*}\,(\varepsilon_2-\varepsilon_1)a^{2n+1}}{n!\,(n-1)!\,[n\varepsilon_2+(n+1)\varepsilon_1]}\;\frac{\partial^{n-1}E_o}{\partial r^{n-1}}\cdot\frac{\partial^{n}E_o}{\partial r^{n}} \qquad (4)$$

where a is the radius of the spherical body, ε_1 is the permittivity of the medium, ε_2 is the permittivity of the body, and r is the radial distance in the direction of interest.

The total force [illegible] thus be

$$F_T(r) = \sum_{n=1}^{\infty} F_r(n) \qquad (5)$$

If $\partial^n E_o/\partial r^n$ is zero then the resulting series is truncated at order n. From Eq. 2 however it becomes clear that Eq. 4 will result in an infinite series because of the [illegible] dependence.

MULTIPOLE EFFECTS IN THE ISOMOTIVE CASE

The derivation of the multipole expressions assumes a [illegible] field. In fact this restriction may be relaxed to allow geometries with cylindrical symmetry in the gradient of the field. In the case of an [illegible] isomotive field, the force components in the plane perpendicular to the [illegible] axis sum to zero. For this reason we may apply the axisymmetric result provided the sphere lies on the [illegible] axis. This results in an expression for the force on a spherical particle of radius a in the isomotive field:

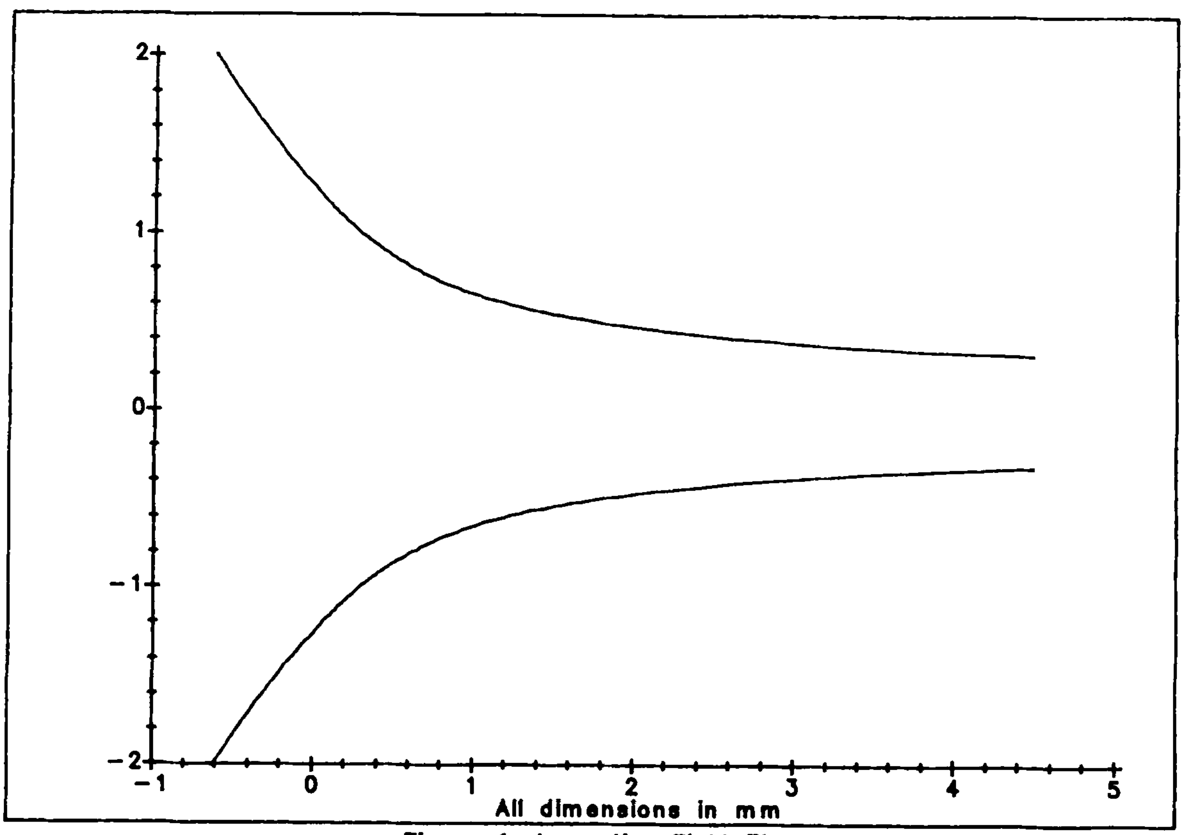

Figure 1: Isomotive Field Electrodes

$$F_T(r) = \frac{9\pi a^3 \varepsilon_1 (\varepsilon_2 - \varepsilon_1) V^2}{2(\varepsilon_2 + 2\varepsilon_1) r_{60}^3} \tag{6}$$

$$- \frac{9\pi\varepsilon_1 (\varepsilon_2 - \varepsilon_1) V^2}{8(2\varepsilon_2 + 3\varepsilon_1) r_{60}^3} \frac{a^5}{r^2} - \frac{9\pi\varepsilon_1 (\varepsilon_2 - \varepsilon_1) V^2}{128(3\varepsilon_2 + 4\varepsilon_1) r_{60}^3}$$

$$\frac{a^7}{r^4} - \cdots$$

which may be rearranged as the well known dipole force equation plus higher-order error terms thus

$$F_T(r) = \frac{9\pi a^3 \varepsilon_1 (\varepsilon_2 - \varepsilon_1) V^2}{2(\varepsilon_2 + 2\varepsilon_1) r_{60}^3} \tag{7}$$

$$\left[1 - \frac{(\varepsilon_2 + 2\varepsilon_1)}{8\,(2\varepsilon_2 + 3\varepsilon_1)} \left(\frac{a}{r}\right)^2 - \frac{(\varepsilon_2 + 2\varepsilon_1)}{64(3\varepsilon_2 + 4\varepsilon_1)} \left(\frac{a}{r}\right)^4 - \cdots \right]$$

Note that as the ratio of r/a becomes large the higher-order terms rapidly approach zero though the dipole term is independent of r.

It is also important to recognize that the force expression is meaningless for all values of $r/a < 1$, as this would require part of the body to protrude into a region of undefined dielectric field.

NUMERICAL RESULTS

We consider the case of a dielectric sphere in water passing through an isomotive field chamber with

r_{60} = 1.0 mm
a = 2.5 um
ε_1 = 80 (deionized water)
ε_2 = 400

Though the values of ε_1 and ε_2 would generally both be complex the deionized water has a very low conductivity and so we may approximate it to a real dielectric permittivity. For computational convenience we have chosen a real dielectric particle though this does not affect the generality of the analytical result. The first four terms of Eq. 7 thus become

$$F_T(r) = F_{dipole}\,[1 - .067 - .0058 - .0014] \text{ at } r/a = 1$$

$$F_T(r) = F_{dipole}\,[1 - .030 - .0011 - .0001] \text{ at } r/a = 1.5$$

Figures 2 and 3 are plots of the error introduced by quadrupole and octupole terms as a function of r. The error is expressed as a percentage of the dipole term.

DISCUSSION

Because of the r^{-2} dependence of the electric field term in the isomotive chamber al. partial derivatives of $\vec{E}$ in the $\hat{r}$ direction

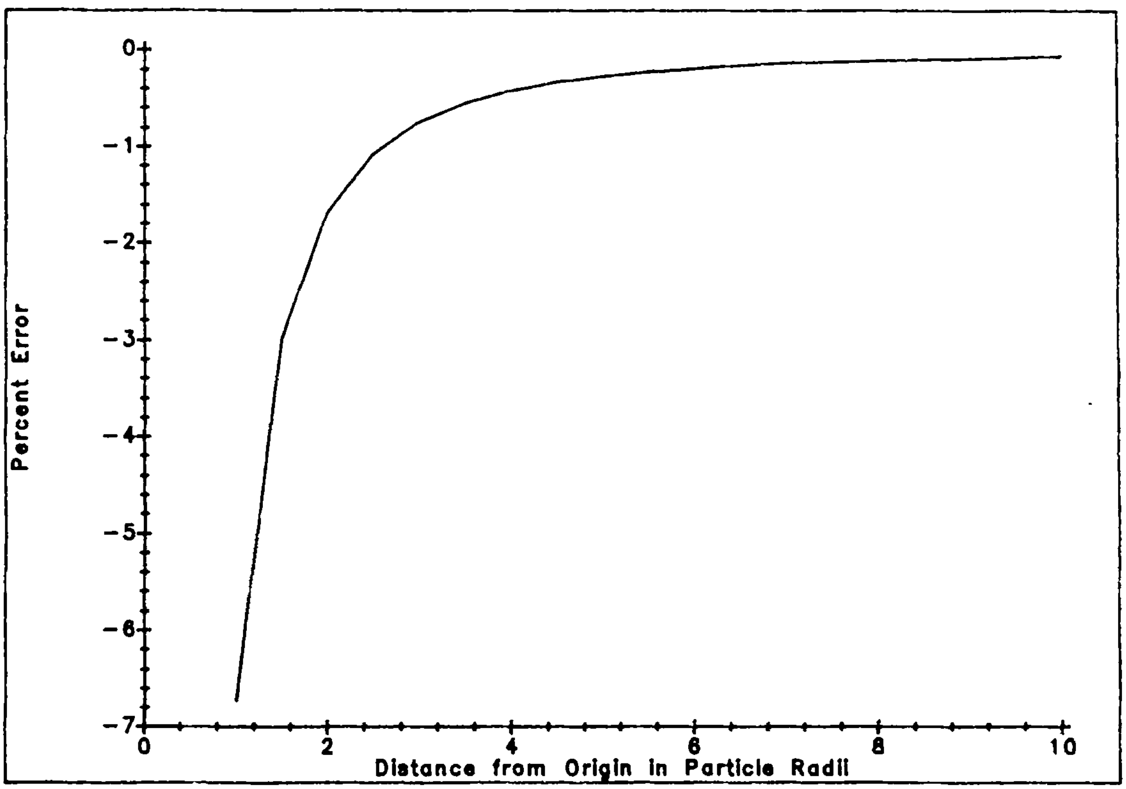

Figure 2 : Quadrupole Error

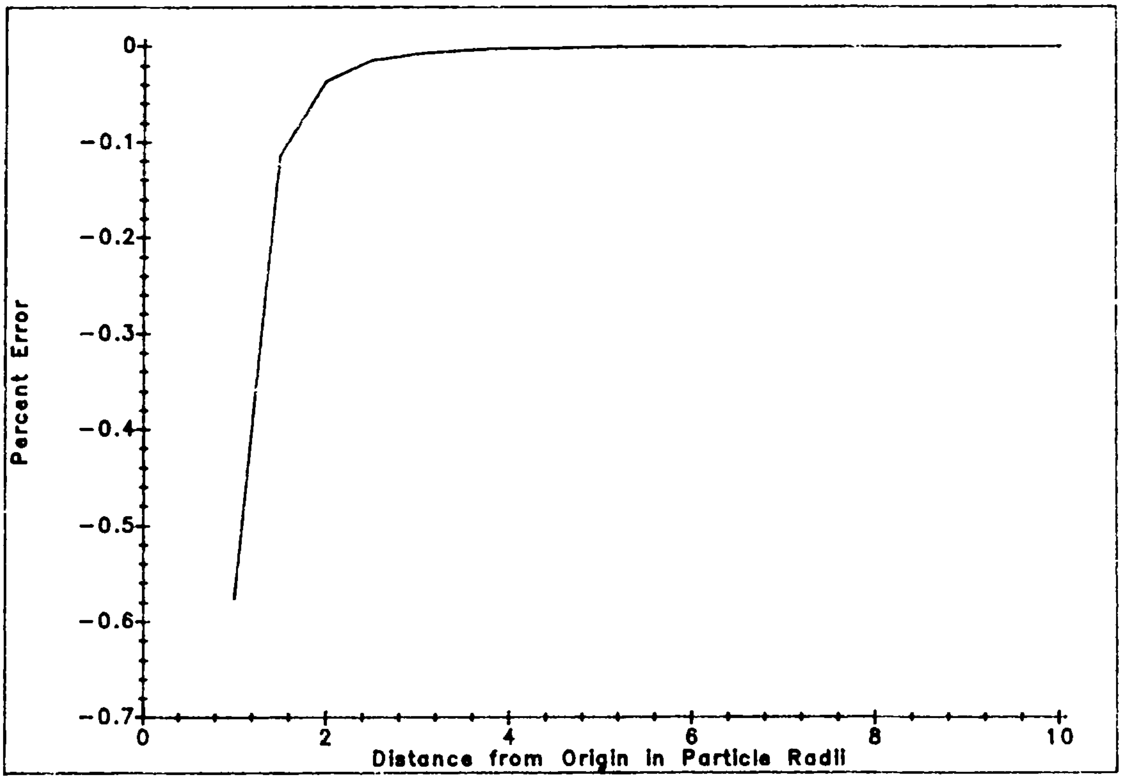

Figure 3 : Octupole Error

exist. However, because the error becomes small and extremely dependent on r for terms above order 2 (octupole), these may be neglected for all cases where $r > 1.5\ a$. If $r > 10\ a$ then all terms above dipole may be ignored (total error < 0.2% for the case cited here). The isomotive field case may thus be confirmed to be valid within a very small error (<<1%) for all regions outside an area within a few particle diameters of the origin.

These findings confirm that the isomotive chamber represents a practical geometry for dielectrophoretic separation and characterization of particles and, particularly, biological cells. In this vein a continuous, automated separation and characterization system is currently under development at this institution using the isomotive field geometry described.

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