MODELING

Quantity and quality management of groundwater: An application to irrigated agriculture in Iraklion, Crete

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A quantity-quality problem in which pollution generates production externalities is analyzed empirically. Water is pumped by farmers from a common access aquifer, and deep percolation resulting from the irrigation causes accumulation of pollutants in the aquifer. Pollution negatively affects the production of the agricultural output through the deterioration of the groundwater quality. By comparing the cooperative with the noncooperative solution, an optimal policy scheme in the form of water taxes is determined. The scheme induces farmers acting noncooperatively to follow policies that correspond to the regulator's optimum. The model is applied to the case of groundwater management in the Iraklio prefecture of Crete. Agricultural production functions are estimated using an externality variable as explanatory variable. An optimal control model that corresponds to the cooperative solution is solved using multiple shooting methods. Paths for water stock, salinity stock, and water use at the regulator's optimum are derived. The optimal water tax is calculated in the final stage.

Keywords: Groundwater management, production externality, common access, fixed effects models, multiple shooting, optimal tax.

1 Introduction

The management of the quantity and quality of groundwater corresponds to a so-called quantity-quality (q-q) problem. The solution to this problem is characterized by the inability of competitive markets to achieve effective resource allocation. This is due to the impossibility of exclusion and the absence of clearly defined property rights.

In a q-q problem the deterioration of quality due to pollution results in the reduction of the effective use of the resource. Thus the management of the resource should account for both its use and the emissions of pollutants that influence the effectiveness of its use. The q-q problem has been analyzed mainly in a framework where pollution affects the growth rate of a renewable resource on the one hand, and generates consumption externalities that affect the utility of consumers on the other (e.g., Siebert [16], Tahvonen and Kuuluvainen [17], Xepapadeas [18]).

The purpose of the present paper is to model and empirically analyze a q-q problem in which pollution generates production externalities (see for example Dinar and Zilberman [7]). In this case, water is pumped by farmers from a common access aquifer, and deep percolation resulting from the irrigation causes accumulation of pollutants in the aquifer. Pollution negatively affects the production of the agricultural output through the deterioration of the groundwater quality.

The basic analytical framework for modeling the problem is based on determining cooperative and noncooperative solutions to the q-q problem. The cooperative solution is determined at a first stage. At this solution each farmer uses water such that the total present value of the region's profits is maximized. Since the problem is inherently dynamic, the maximization is constrained by the relationships reflecting the accumulation of water in the aquifer and the accumulation of pollutants in the water stock. This cooperative solution could be regarded as being the solution of a regulatory agency that seeks to maximize regional net profits by managing the quantity and the quality of water used for irrigation. The noncooperative solution is derived in the second stage. The solution examined in this stage is the myopic one, in which each farmer maximizes profits without taking into account the dynamics characterizing water and pollution accumulation. By comparing the cooperative with the noncooperative solution, a policy scheme in the form of water taxes is determined. The scheme induces farmers acting noncooperatively to follow policies that correspond to the regulator's optimum.

The model is used to analyze groundwater management in the Iraklio prefecture of Crete. Agricultural production functions are estimated for three crops – grapes, olive oil and citrus – using panel data. In the production functions, water and the externality variable (salinity in our case) are used as explanatory variables. Using information about water inflows in the area and salinity accumulation, the equations describing the accumulation of water and salinity are specified. The estimated production relationships are used to solve the optimal control problem corresponding to the cooperative solution. Multiple shooting methods are used to solve the two point boundary problem and to derive paths for water stock, salinity stock, and water use at the regulator's optimum. Similar paths are derived for the myopic private equilibrium. The estimated models use Cobb-Douglas production relationships which can be regarded as more relevant for empirical applications, although more difficult to handle, than the often used linear-quadratic approximations, when numerical solutions are required. The obtained numerical solutions are used to approximate smooth functions of the state and the costate variables in order to calculate the optimal water tax at the final stage.

2 The empirical problem

The empirical problem is to investigate the issue of groundwater management under production externalities, due to salinity concentration in the underground aquifer, in the Iraklio prefecture of Crete, Greece, and to quantify a regulatory framework which could help towards achieving efficient allocation of the water resources in the area. We start by providing a description of the area under investigation, which will be helpful in the modeling of the water management problem. Thus the main objectives are the identification of the water supply and water consumption conditions in the area.

2.1 *Area description*

Water consumption in Greece, according to estimates of the Ministry of Agriculture, was 5,440 mil m^3 in 1986. Of this amount 4,520 mil m^3 were used for irrigation, 750 mil m^3 to supply houses, 90 mil m^3 for industrial use and 80 mil m^3 for the remaining uses. In Crete, for the same year, total consumption of water was estimated at 280 mil m^3 (233.5 mil m^3 for irrigation, 39 mil m^3 to supply houses, 3.5 mil m^3 for industrial use and 4 mil m^3 for the remaining uses). These numbers are expected to increase by 70–75% by the year 2000.

According to research done by the National Statistical Service of Greece in 1990, 40% of the municipalities and communities of Greece have a shortage of water all summer, 7% have a shortage all year round and only 33% have sufficiency (the remaining 20% did not give any information). In Crete, 56.5% of the municipalities and communities (that represent 44.5% of the island's population) have sufficiency in water all year round, while the remaining 43.5% (that represent 55.5% of the population) have problems with the water supply. In Greece approximately 30% of the agricultural land was irrigated in 1990, while in Crete, for the same year, irrigated land represented 19.5% of total agricultural land (Angelakis [1]).

The Iraklio prefecture is the largest of the four prefectures of the island of Crete, with an area of 2641 km^2, compared to 8260 km^2 for the whole island, and is located approximately in the center of the island. It is mainly mountainous with two big plains, the so-called plain of Messara and the plain of Iraklio. The plain of Messara is the most fruitful and the biggest one on the whole island (40 × 6–12 km).

Table 1
Total cultivated land in Crete (in stremmas). Source: National Statistical Service of Greece.

Area	Total cultivated land (incl. fallow land)	Fallow land
Crete	3,256,597	709,768
Iraklio	1,500,181	263,042
Lassithi	622,567	261,080
Rethimno	511,392	120,761
Chania	622,457	64,885

The total cultivated land, including fallow land, in Crete and in each prefecture of the island is shown in table 1. In Iraklio, crops on arable land occupy an area of 157.536 stremmas, garden area uses 38,216 stremmas, vines (grapes and raisins) take up 309,228 stremmas, and the area under trees in compact plantations is 732,159 stremmas. Fallow land represents 263,042 stremmas. Table 2 shows irrigated area in Crete and the Iraklio prefecture in the year 1990.

2.1.1 *Supply and consumption of water in the Iraklion prefecture*

It is estimated that the island of Crete receives approximately 9 billion m^3 of precipitation per year. Of this amount approximately 70% is lost due to seepage in the sea or evaporation, while the rest replenishes underground aquifers. Thus there is an approximate inflow of 3 billion m^3 of water in the underground aquifers of the island (Monopolis [13]). Assuming that the geological structure of the island is homogeneous, this implies an approximate inflow of 1 billion m^3 of water in the underground aquifers of the Iraklio prefecture. This inflow, which is not of course exploitable in full due to technical or economic reasons, is concentrated in 18 underground aquifers [1] which are used to pump water for irrigation, urban, and industrial uses.

[1] The underground aquifers are: Timpaki, Moires, Protoria, Perama, Arvis, Vianos, Emparos, Thraphanos, Mallia, Episcopi, Filisis, Arhanes, Agia Irini, Finikia, Tyllisos, Fidele, Hersonisos, and Siva. Source: Internal report of the Water Authority of Crete, 1993.

Table 2
Distribution of irrigated land, by use (in stremmas). Source: National Statistical Service of Greece.

Area	Total irrigated crops	Crops on arable land	Garden area	Vines	Area under trees
Crete	738,314	103,375	75,996	116,513	442,430
Iraklio	353,589	44,940	33,235	103,572	171,842

During the period 1909–1987, the annual average rainfall in the prefecture of Iraklio was 489.8 mm while the seasonal precipitation for winter was 252.6 mm, for spring 94.9 mm, for summer 5.3 mm and for autumn 138.2 mm (Mahairas and Koliva-Mahaira [11]). During this time there were observed two moist periods, the first lasting for 16 years (1917–1932) and the second for 8 years (1961–1969), and one dry period lasting 23 years (1938–1960). During the first moist period there was an increase in precipitation of 13% while in the second one the increase was around 28%. It was also concluded that the dry season in Iraklio lasts for five months and the moist for seven. The dry period was accompanied by a 13% decrease in rainfall (Mahairas and Koliva-Mahaira [11]). Between the years 1963–1987, a continuous water deficit was observed in the area of Iraklio since the difference between precipitation and potential evapotranspiration remained negative (Balafoutis [2]).

While on the supply side the above results indicate the possibility of shortages, on the demand side water has acquired great importance for the agricultural economy of the prefecture of Iraklio, and irrigation has become a necessity even for such traditional crops as grapes/raisins and olives. In oil production, especially, proper fertilization and irrigation are among the factors that have contributed to substantial increases in productivity. Regarding vineyards, the widespread replanting, combined with the installation of new vineyards with American rootstocks, has made water one of the most important factors in vine production.

With an estimated annual water use of 650 m^3 per stremma of irrigated land, 200 liters per person, and considering some limited industrial use for bottling water and olive oil processing, the current annual consumption amounts to approximately 230 million m^3 in the Iraklio prefecture. Although comparison of this estimated consumption with the expected annual inflows seems to indicate a water surplus, this is not the case since the exploited underground aquifers are not sufficient to cover demand. Thus long periods of low rainfalls during spring and droughts during summer in recent years, along with the increased use of irrigation water and urban demand, have intensified the problem of finding irrigation water. According to Maliarakis [12], there is an intense exploitation of the existing underground aquifers in the whole area of the prefecture. In 1990 alone, approximately 500 licenses for drilling and wells were issued by the responsible public service. It is certain that many additional unauthorized drillings have also been carried out, intensifying the exploitation of the already exploited underground aquifers.

The intensive exploitation of existing underground aquifers has also increased salinity concentrations as indicated by recent reports [2]. As a consequence the quality of irrigation water is reduced and this could potentially have negative external effects on agricultural production.

Thus, the area description suggests that the most important aspects with respect to water conditions are the potential water overexploitation and the increase in the salinity concentration. This indicates the need for a regulatory policy. The design and the quantification of this policy are addressed in the remainder of the paper.

Having described the water conditions in the area we turn to the development of an economic model with a structure appropriate, given the informational constraints imposed by data limitations, for the analysis of the empirical problem.

3 The economic model

The economic model draws on infinite horizon, optimal control models of irrigated agriculture [3], developed by Dinar and Xepapadeas [5,6] and Xepapadeas [19]. The model can be described as follows.

3.1 *A model of irrigated agriculture*

A region with a fixed amount of homogenous land occupied by $i = 1, \ldots, n$ farmers is considered. To simplify things, we assume that land is completely utilized in the production of a crop and is equally divided among farmers. Assuming that water is the only variable input in production, each farmer i's production function is specified as [4]:

$$y^i(t) = f(w^i(t), S(t)), \quad f_w > 0, f_{ww} < 0, f_S < 0, f_{wS} \leqslant 0,$$

where $w^i(t)$ is water applied by farmer i at time t, and $S(t)$ is the amount of salinity accumulated in the aquifer from which water is pumped at time t. Salinity negatively affects production ($f_S < 0$), and also does not increase the marginal productivity of water ($f_{wS} < 0$).

Water is pumped by an aquifer of stock $W(t)$, with maximum capacity $\bar{W}$ [5]. The accumulation of water in the aquifer is determined as:

$$\dot{W}(t) = F(t) - \sum_{i=1}^{n} w^i(t) - \delta W(t), \quad W(0) = W^0 > 0, \tag{1}$$

[2] Internal report of the Water Authority of Crete, 1993.

[3] There is extensive literature on this issue. For a survey, see for example, Dinar and Zilberman [7].

[4] When appropriate, subscripts associated with functions denote partial derivatives.

[5] When water w^i is applied, hw^i is the amount effectively used by the plants while $(1 - h)w^i$ is the amount returned to the aquifer as deep percolation, $h \in (0, 1]$, where h is irrigation efficiency. In our case, and in the absence of information about irrigation efficiency we implicitly consider the case of $h = 1$. This assumption means that in the empirical estimation the marginal product of effective water is underestimated if the actual irrigation efficiency is less than one.

where $F(t)$ is net exogenous water inflow in the aquifer, that is, exogenous inflows less outflows which are regarded as exogenous to the specific water management problem, and δ is the rate of natural water losses (seepage, evaporation).

Salinity in the aquifer accumulates according to:

$$\dot{S}(t) = \sum_{i=1}^{n} mw^i(t), \quad S(0) = S^0 \geqslant 0, \tag{2}$$

where m is a coefficient measuring the rate of salinity accumulation in the aquifer per unit of applied water [6].

The pumping cost per unit of applied water is assumed to be constant c [7].

If $p(t)$ denotes the exogenous and stationary price of output at each instant in time, then regional benefits are defined as:

$$\pi_R(t) = \sum_{i=1}^{n}[p(t)y^i(t) - cw^i(t)],$$

while individual farmers' benefits are defined as:

$$\pi_F(t) = p(t)w^i(t) - cw^i(t).$$

The model developed can be used to analyze the regulator's problem (cooperative solution), and noncooperative solutions in the management of water use and salinity.

3.2 *The social optimum*

The regulator's problem is to choose time paths for applied water in order to maximize discounted regional profits over a fixed time horizon $t \in [0, T]$ [8]. At the end of the planning horizon, the regulator requires a minimum amount of water and a maximum amount of accumulated salts in the aquifer. The problem is:

$$\max_{\{w^i(t)\}} \int_0^T \mathrm{e}^{-\mathrm{it}} \left[\sum_{i=1}^{n}(pf(w^i, S) - cw^i)\right] dt$$

subject to

$$\dot{W} = F - \sum_{i=1}^{n} w^i - \delta W, \quad W(0) = W^0, \quad W(T) \geqslant W^T, \tag{P1}$$

$$\dot{S} = \sum_{i=1}^{n} mw^i, \quad S(0) = S^0, \quad S(T) \leqslant S^T,$$

$$0 \leqslant W \leqslant \bar{W},$$

where W^T and S^T represent exogenously determined terminal values. These values can be regarded as indicating minimum acceptable water stock and maximum acceptable salinity accumulation respectively.

The current value Lagrangean for the problem is:

$$\mathcal{L} = \sum_i [pf(w^i, S) - cw^i] + \mu\left(F - \sum_i w^i - \delta W\right) + \lambda\left(\sum_i mw^i\right) + k_1(\bar{W} - W) + k_2 W,$$

where the costate variables μ and λ denote the shadow values of the water stock and salinity respectively ($\lambda < 0$).

Assuming interior solutions for applied water use, the optimality conditions for problem (P1) are given for $i = 1, \ldots, n$ by:

$$pf_w(w^i, S) - c - \mu + \lambda m = 0 \tag{3}$$

and the adjoint equations. Assuming that we have an interior solution with respect to the water stock constraint, the constraint is not binding at the solution and $k_1 = k_2 = 0$. Thus the adjoint equations are:

$$\dot{\mu} = (r + \delta)\mu, \tag{4.1}$$

$$\dot{\lambda} = r\lambda - \sum_i pf_S(w^i, S) \tag{4.2}$$

along with the state equations (1) and (2). The transversality conditions require:

$$\lambda(T) \leqslant 0 \ (= 0 \text{ if } S(t) < S^T), \quad \mu(T) = 0$$

by the assumption of interior solution for the water stock.

Using (3), the short-run demand function for applied water is defined as:

$$w^{*i} = w^*(p, S, c, \mu, \lambda, m). \tag{5}$$

Short-run comparative statics are easily obtained from (3), using the implicit function theorem, as:

$$w_p^* > 0, \quad w_S^* < 0, \quad w_c^* < 0, \quad w_\mu^* < 0, \quad w_\lambda^* > 0, \quad w_m^* < 0. \tag{6}$$

Thus, an increase of the pollution content in deep percolation, due for example to a change in the type of fertilizers or pesticides used, will reduce the applied water used.

Since m represents an exogenous parameter in our model, the effects of its changes on the maximum discounted value of regional benefits can be obtained by applying comparative dynamic analysis. Let $V^*(m)$ denote the maximum discounted value of regional benefits, defined as:

$$V^*(m) \equiv \max \int_0^T e^{-rt}\pi_R(t)dt.$$

[6] The linearity is a simplifying assumption that is used due to data limitations. A more general function $m(w^i, W)$ including also existing water stock could be a better representation of salinity accumulation.

[7] Again, in a more general model unit, pumping costs can be regarded as a non-increasing function of the existing water stock: $c(W)$, $c' \leqslant 0$, $c'' \geqslant 0$.

[8] A fixed time horizon is used instead of the infinite horizon assumption of the optimal growth type of models, since this concept better fits the planning process of a regulating agency.

Using the dynamic envelope theorem (Caputo [3]) we obtain:

$$\frac{\partial V^*}{\partial m} \equiv \int_0^T e^{-rt} \frac{\partial \mathcal{L}^*}{\partial m} dt$$
$$= \int_0^T e^{-rt} \left[\lambda^*(t) \sum_i w^{*i}(t) \right] dt < 0 , \qquad (7)$$

with all expressions evaluated along the optimal path determined by optimality conditions (3) and (4). The comparative dynamic derivative (7) indicates that an increase in the pollution content of the deep percolation reduces the present value of regional profits.

Substitution of (5) into (1), (2), (4.1) and (4.2) will provide a system of differential equations that determine the optimal time paths of water stock pollution stock and their corresponding shadow values.

3.3 *The private optimum and the optimal water tax*

Having determined the necessary conditions for the solution to the regulator's problem, we examine policy schemes under which individual farmers can be induced to undertake the applied water use that is consistent with the regulator's choices. When farmers act myopically, each one solves the problem:

$$\max_{w^i} pf(w^i, S) - c^p w^i ,$$

where c^p is the private cost of pumping water, which might be different than the true pumping cost associated with the regulator's problem, due to water subsidies or other market distortions. In general $c \geqslant c^p$.

Water use is determined by the first-order condition:

$$pf_w(w^i, S) - c^p = 0 . \qquad (8)$$

Comparing (8) to (3), it follows that water tax per unit time is determined as:

$$\tau^M(t) = \mu^*(t) - m\lambda^*(t) + (c - c^p) , \qquad (9)$$

where $\mu^*(t)$, $\lambda^*(t)$ are the optimal paths for the shadow values of the water stock and the pollution, as determined by the solution of the regulator's problem. That is, the optimal tax accounts for the water and pollution stock values not accounted for by the myopic farmers, as well as for possible discrepancies in unit pumping costs due to subsidies. The assumption of myopic farmers – as opposed to the more sophisticated assumption of strategic behavior by farmers – is used, since it seems more realistic and empirically relevant [9].

4 Results

The application of the above model requires the determination of production functions and water and salinity accumulation relationships. These empirical relationships will determine the optimal water use and the optimal regulatory framework.

4.1 *Estimation of production relationships under production externalities*

The production functions are estimated for three different crops: grapes, olive oil and citrus ($j = 1, 2, 3$), using data on output, water consumption and salt concentration for the period 1980–1990. The data set is used as a panel of data to estimate the agricultural production function [10]. The production function model is defined as:

$$y_{jt} = \alpha_j + \beta w_{jt} + \gamma S_t + u_{jt} , \qquad (10.1)$$

$$\mathcal{E}(u_{jt} \mid w_{jl}, \ldots, w_{jT}, S_1, \ldots, S_T, \alpha_j) = 0 , \qquad (10.2)$$
$$(j = 1,2,3 ; \quad t = 1, \ldots, T ; \quad T = 11) ,$$

where all variables are measured in logarithms, and: y_{jt} is the value of output, w_{jy} is applied water, and S_j is the average salinity concentration in the underground aquifers in the prefecture [11]. In this model the salinity concentration is the same for all three crops, thus the parameter γ, which is the output elasticity with respect to salinity, reflects the externality parameter. If the externality has a negative effect on production, as is assumed in the theoretical model, then γ is expected to have a negative value. On the other hand, β is the output elasticity with respect to water with an expected positive value. When α_j is treated as non random, model (10) is the *fixed-effects* model, with α_j capturing crop specific, time invariant effects in the production process. When α_j is treated as a random variable, model (10) is the *random-effects* model. Estimation results are presented in table 3.

Table 3
Production function estimation (fixed effects models).

Variable	Coefficient	t-Statistic
α_1	3.984853	1.461176
α_2	3.572015	1.306706
α_3	2.158996	0.789793
w	0.785801	6.590481
S	−0.199005	−1.669045
$F(2,28)$	21.926 P-value = 0.0	
$F(2,28)$ [a]	359.9795 P-value = 0.0	

[a] F-statistic for the equality of dummy variables in the fixed effects model.

The above results indicate that the externality parameter has the correct sign and is significant at the 10%

[9] For the analysis of the problem when the farmers act strategically, and in particular follow linear Markov strategies, see Xepapadeas [19].

[10] Agricultural production functions were the earliest application of panel data estimation (Mundlak [14]). For an exposition of panel data econometrics see for example Chamberlain [4], Maddala [9].

[11] Output is measured in constant 1980 drachmas, applied water is measured in thousand m^3, and salinity concentration is measured in parts per thousand (ppt) Cl-ions.

level, so the hypothesis of negative production externalities due to salinity concentrations seems to be supported by the data. The water coefficient also has the correct sign and is significant at any level. Its value suggests diminishing returns in water use for the three crops examined. The intercepts appear not to be statistically significant, nevertheless the rejection of the null hypothesis about the equality of the intercepts suggests the existence of time invariant differences in the production process among the three crops [12], if it is accepted that the largest intercept in value could be nonzero.

4.2 *Water use under optimal regulation and under market equilibrium*

The estimated production functions are used, along with water and salinity accumulation equations, to determine the time paths of water stock, salinity and water use under optimal regulation and in market equilibrium.

4.2.1 Optimal regulation

In order to solve the optimal control model developed in section 3.2, a water accumulation and a salinity accumulation relationship need to be specified.

As indicated above, the annual water inflow in the Iraklio prefecture's aquifers is approximately 1 bil m^3. The total annual agricultural, domestic, and industrial consumption is estimated to be in the range of 230 million m^3. Regarding as exogenous all other water uses except irrigation water for citrus, olives and vines, and using an annual rate of water losses of 10%, the water accumulation relationship can be defined as [13]:

$$\dot{W} = (1000 - 160) - \sum_{j=1}^{3} w_j - 0.1W. \tag{11}$$

To derive a salinity accumulation relationship annual changes in the salinity levels are related to total water used for the three crops. The resulting equation is:

$$\dot{S} = 0.00295\left(\sum_{j=1}^{3} w_j\right). \tag{12}$$

The current value Hamiltonian for the regulator's optimum is defined as:

$$H = \sum_{j=1}^{3}(A_j w_j^{\beta} S^{\gamma}) - c\left(\sum_{j=1}^{3} w_j\right) + \mu\left(F - \left(\sum_{j=1}^{3} w_j\right) - \delta W\right) + \lambda m\left(\sum_{j=1}^{3} w_j\right),$$

where: $A_j = \exp(\alpha_j), F = 840, \delta = 0.10, m = 0.00295$.

The costate variables μ and λ reflect the shadow values (or costs) of water stock and salinity respectively, and c is the unit pumping cost. There were no available data to estimate pumping cost functions. Based on personal interviews with the water authorities of the prefecture, an estimated cost of $c = 100$ dr/m^3 was used. Using these parameter values, the estimated short-run demand for water at the regulator's optimum for each of the three crops, derived from Pontryagin's maximum principle, are defined as:

$$\hat{w}_1 = \left[\frac{(100000 + \mu - 0.002995\lambda)}{42.23827}\right]^{1/(\beta-1)} S^{-0.92908},$$

$$\hat{w}_2 = \left[\frac{(100000 + \mu - 0.002995\lambda)}{27.96523}\right]^{1/(\beta-1)} S^{-0.92908},$$

$$\hat{w}_3 = \left[\frac{(100000 + \mu - 0.002995\lambda)}{6.80695}\right]^{1/(\beta-1)} S^{-0.92908}.$$

Substituting the above functions into the differential equations (11) and (12), and those describing the evolution of the costate variables, we obtain the modified Hamiltonian dynamic system for the problem as:

$$\dot{\mu} = (r + \delta)\mu, \quad r = 0.10, \tag{13.1}$$

$$\dot{\lambda} = r\lambda - \gamma S^{\gamma-1} \sum_i \alpha_i \hat{w}_i^{\beta}, \tag{13.2}$$

$$\dot{W} = F - \sum_i \hat{w}_i - \delta W, \quad W(0) = 10000 \text{ bil m}^3, \tag{13.3}$$

$$\dot{S} = m \sum_i \hat{w}_i, \quad S(0) = 200 \text{ ppt}. \tag{13.4}$$

The above dynamic system is solved for a time horizon of twenty years, $t \in [0, T]$, $T = 20$. To solve the system, two more boundary conditions – apart from the initial conditions on the water and the pollution stock – are required. The additional conditions are derived by the transversality conditions of the maximum principle. Assume that the regulator leaves free the terminal (at $T = 20$) values of the water and the salinity stocks. Then the transversality condition of the maximum principle (e.g. Seierstad and Sydsaeter [15]) requires that $\lambda(T) = 0$, $\mu(T) = 0$ [14]. It follows thus from (13.1) that $\mu(t) = 0$ for all $t \in [0, T]$. The MHDS is reduced now to a

[12] The random-effects model was not estimated because the estimate of the variance of the indirect effects was negative. The number of groups is small, equal to the number of independent variables including the constant, and large sample formulas did not provide satisfactory results.

[13] The relationship between the discrete time data used in the estimations and the continuous time formulations of the problem can be described as follows: Let there be a continuous time index $s \in R$. The continuous time differential equation can be defined as: $\dot{x} = ds/dt = f(x(s))$. The state at time $t + \Delta t$ where $\Delta t = 1$ so that time is measured in "natural" units (a year) is defined as $x_{t+1} = x_t + \int_t^{t+1} f(x(s))ds$.

[14] The same transversality conditions could have been used if the regulator's terminal conditions had been $W(T) \geqslant 8500$, $S(T) \leqslant 201$, and we had interior solutions with respect to the stock variables.

Table 4
Smooth approximations of the solutions for W, S, and λ.

Equation [a]	R^2
$W(t) = 9747.795 - 68.4257t$	0.93407
$S(t) = 200.0001 + 0.020539t$	0.99999
$\lambda(t) = 0.37129 + 0.000873t + 0.000623t^2$	0.99807

[a] All coefficients are significant at the 1% level.

system of three differential equations with one terminal and two initial conditions. The system is nonlinear and is solved numerically using Mathematica 2.2 [10]. Since, however, one condition is terminal, the numerical solutions are obtained using the multiple shooting approach (Lipton et al. [8]). An initial value is specified for λ and then the system of (13.2)–(13.4) is solved forward or "shot". The values of $\lambda(0)$ are adjusted iteratively until the terminal value $\lambda(T)$ is sufficiently close to zero.

In order to determine, however, the optimal water use under the regulator's problem, as a function of time alone, the time paths for W, S, and λ should be substituted back into the short-run water demand functions, $\hat{w}_i$. To obtain smooth functions $W(t)$, $S(t)$ and $\lambda(t)$, low degree polynomials in t are fitted to the discrete values of W, S, and λ for $t = 1, 2, \ldots, 20$ derived from the numerical solution of the system (13.2)–(13.4). The results are presented in table 4. Substituting the above functions into the short-run definitions of $\hat{w}_i(t)$, we obtain the open-loop water use corresponding to the regulator's optimum.

4.2.2 Market equilibrium

In market equilibrium the farmers maximize their profit function defined in section 3.3. Using a value for $c^p = 50$ drh per m^3, which implies a 50% subsidy of true pumping costs, the short run demand for water at the private optimum is obtained as:

$$\hat{w}_i^p = \left[\frac{50000}{\alpha_i \beta}\right]^{1/(\beta-1)} S^{-0.92908}, \quad i = 1, 2, 3.$$

These functions, when substituted into the differential equations (11) and (12), determine the evolution of the water and the pollution stock in market equilibrium. The smooth function for the salinity stock is determined, in a similar way to the regulator's optimum as: $S(t) = 200.0379 + 0.51085t$, which when substituted back into $\hat{w}_i^p$ describes the long-run water use at the private optimum.

The different time paths of water stock, salinity stock and water used, at the regulator's optimum and at the market equilibrium, are presented in figures 1–6.

From figures 1 and 2, it is clear that market equilibrium leads to an overexploitation and excess accumulation of salts, as compared to the regulator's optimum. The overexploitation of water resources can also be seen in figures 4–6, where water use is shown. The excess water use is created mainly by the deviations between the true unit pumping costs and the actual costs paid by the farmers. On the other hand, the excess accumulation of salts is due to the fact that the farmers do not take into account the shadow cost of salinity concentration. An idea of the cumulative social costs imposed by increased salinity concentration can be obtained by using the comparative dynamic result (7). Numerical integration using the estimated functions indicates that a small increase in the salinity concentration coefficient m will reduce the present value of regional profits by 56,000 drachmas.

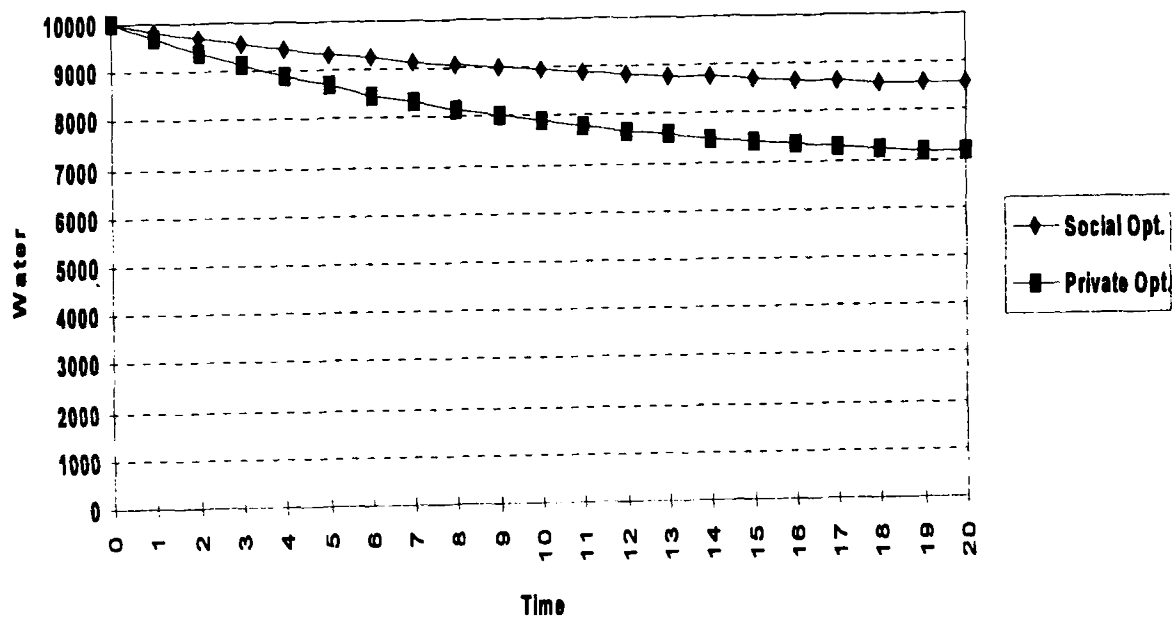

Figure 1. Time path for water stock at the regulator's optimum and the private optimum (market equilibrium).

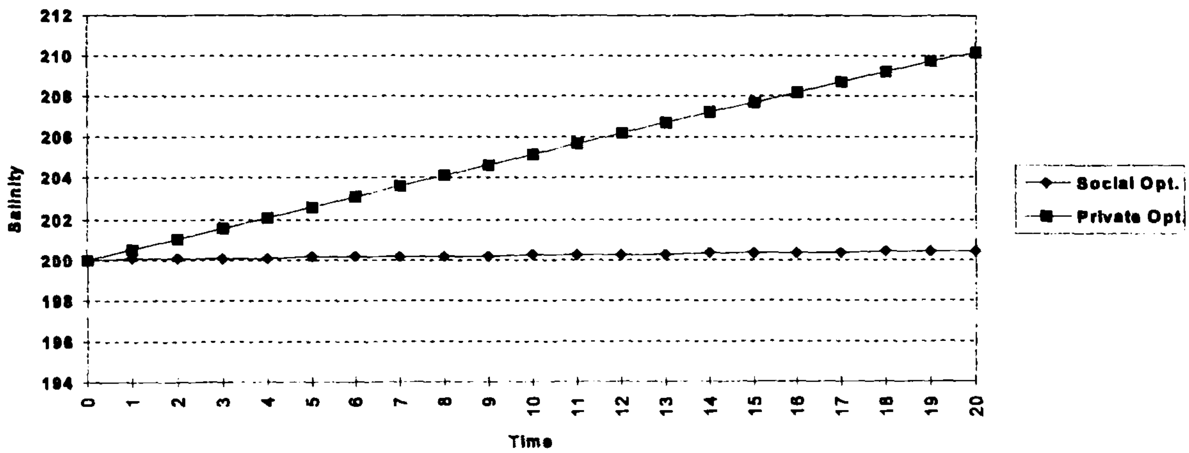

Figure 2. Time path for salinity at the regulator's optimum and the private optimum (market equilibrium).

4.2.3 The optimal water tax

Using (9), the optimal water tax is determined as: $\tau(t) = 50000 - 0.00295\lambda(t)$, where $\lambda(t)$ is defined by the smooth function of table 4. Thus the optimal water tax consists of two parts: a constant part, corresponding to the difference between true pumping costs and pumping costs paid by the farmers; and the time dependent part, that reflects the social costs of salinity concentration. The path of the water tax is shown in figure 7.

5 Summary and conclusions

The present paper analyzed a quantity-quality problem in the context of irrigated agriculture, which can be regarded as a typical example of such a problem. In modelling the empirical problem, the regulator's optimum was analyzed in an optimal control context, while the market equilibrium was analyzed under the assumption that farmers act myopically and do not take into account

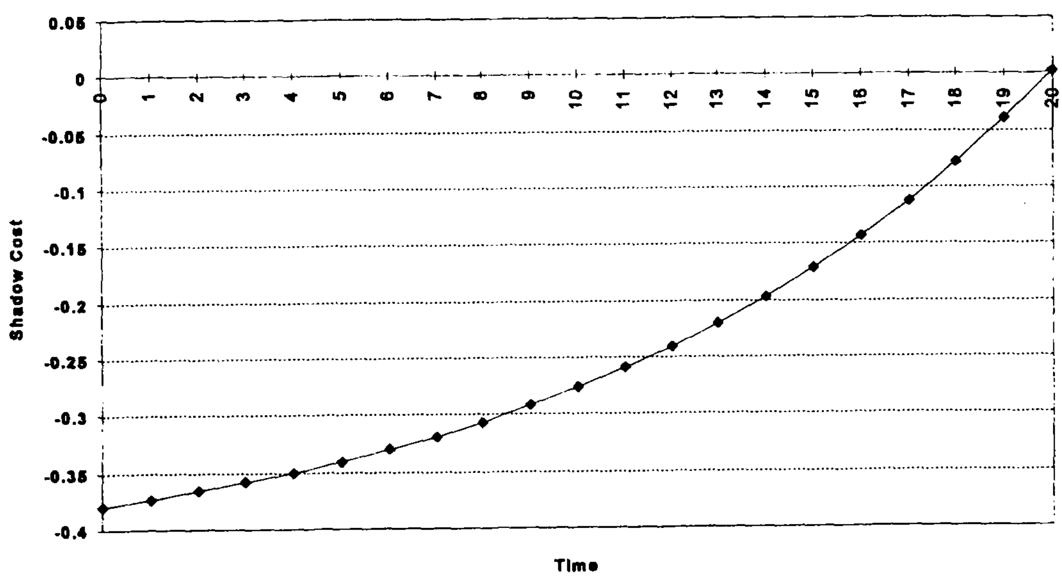

Figure 3. Shadow cost of salinity at the regulator's optimum.

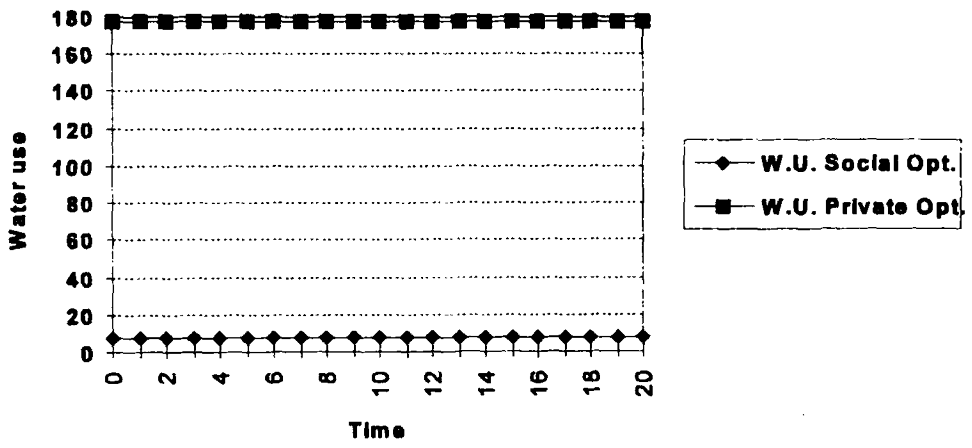

Figure 4. Water use at the regulator's and the private optimum.

the dynamics of water and pollution accumulation. Analyzing the optimality conditions in each case, the optimal water tax is derived.

In the empirical part, agricultural production functions are estimated using as explanatory variables water used and the stock of salinity in the aquifer. The results support the idea that the negative externality due to salinity concentration in the aquifer is statistically significant. The estimated agricultural production functions are used, along with the specified water and salinity accumulation equations, to determine the water and salinity paths. It is shown that the myopic equilibrium involved excess water use and excess pollution. The water tax is also estimated. The largest proportion of the water tax corrects distortions due to subsidization of the unit pumping costs, while the rest corrects distortions due to the farmers' myopic behavior.

The caculated optimal water tax, if applied, will substantially increase the farmer's pumping costs and existing tax burdens, mainly due to the elimination of the

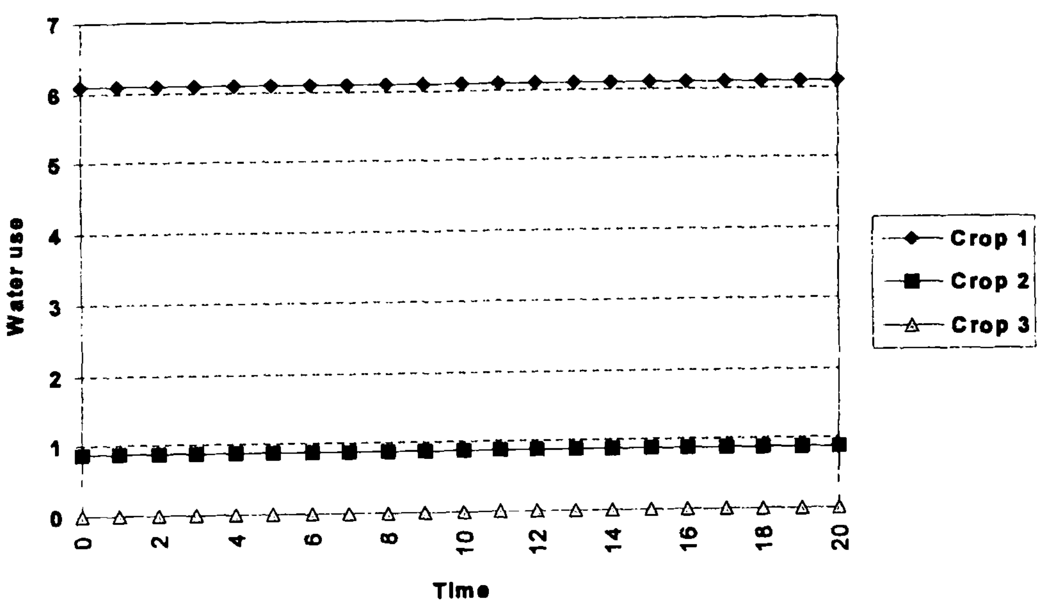

Figure 5. Water use for each crop at the regulator's optimum.

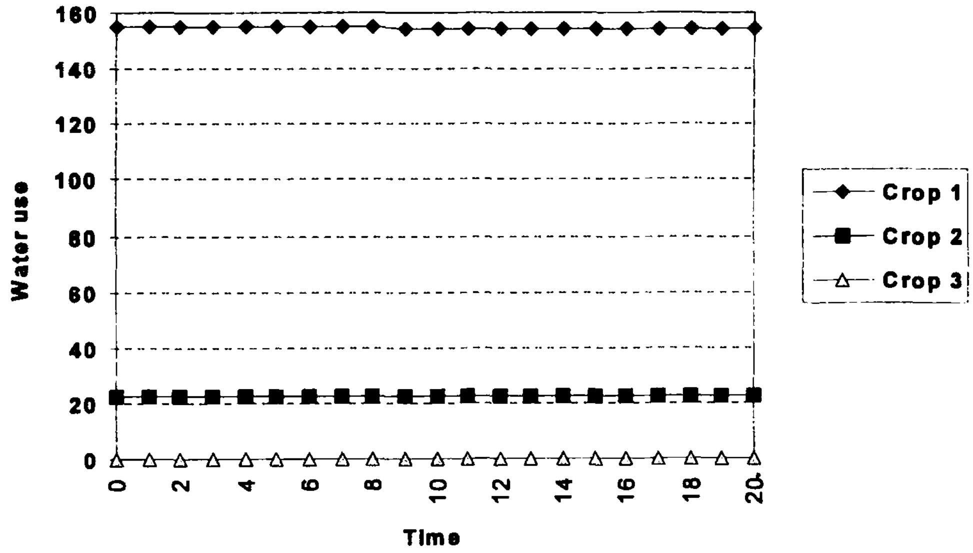

Figure 6. Water use for each crop at the private optimum.

implied subsidy. Although political constraints may impede the implementation of the water tax at the present time, a continuing deterioration of the underground water quantity and quality conditions could make such a tax scheme a viable policy in the future.

The above analysis, apart from providing useful information for water policy in the region, provides additionally a framework for the empirical analysis of these types of problems and gives indications about the information requirements for applied analysis. The required information relates basically to the agricultural production functions with pollution stock as an additional explanatory variable, the characteristics of the aquifer, capacity, inflows-outflows, and the characteristics of pollution accumulation in the aquifer.

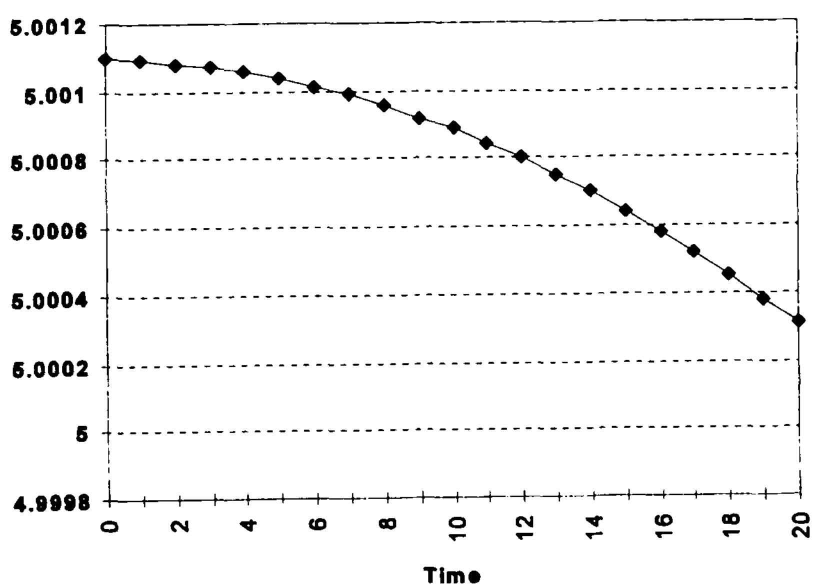

Figure 7. Optimal water tax.

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