Leading edge shape for flat plate boundary layer studies

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Summary In experimental boundary layer studies, a flat plate with some shaped nose piece is generally used; this is often prone to flow separation at the junction. By analysing the development of a laminar boundary layer on a two-parameter family of nose shapes, it is found that a cubic super-ellipse of axis ratio 6 or higher is a reasonable optimum shape for avoiding separation on or due to such nose-pieces.

1 Introduction

In laboratory investigations of boundary layers, it is common practice to use a flat plate with an elliptical nose. Such a configuration is however prone to flow separation near the junction of the nose piece with the plate, associated with the formation of a long or short separation bubble depending on Reynolds number among other parameters. The reason for such separation is a rapid increase in the pressure near the junction, where the curvature is not continuous although the slope is: each fluid particle suffers a sudden loss in centrifugal force when it crosses the junction, and the higher free stream pressure bears down more heavily on the body surface just downstream.

Such separation can be avoided by making the nose very long, but this also makes it relatively sharp and hence sensitive to incidence, and furthermore reduces the region of undistorted constant-pressure flow available for experiment on the plate. One therefore needs a nose shape that can alleviate the adverse pressure gradient without being excessively long.

If the nose shape were a super-ellipse, which is given by the curve (see Fig. 1)

$$[(a-x)/a]^n+(y/b)^n=1, \quad 0\leqslant x\leqslant a \quad n>2, \tag{1}$$

where a is the length of the nose and $2b$ is the plate thickness, it is easy to show that the second derivative d^2y/dx^2 (and hence the curvature) is zero at the junction $x=a$. Consequently the curvature is continuous there and the associated adverse pressure gradient should be less severe.

In this note, the actual variations of the pressure and velocity on the surface of a family of bodies consisting of a flat plate fitted with various super-elliptic noses (including the conventional ellipse) have been obtained by solving the potential flow problem using a simple panel method (Hess and Smith 1968). A laminar boundary layer calculation using the method of Thwaites (1949) is then performed, to determine the least separation-prone nose shape.

We first discuss briefly the family of nose shapes considered.

2 The super-ellipse

The shapes that a super-ellipse can take for various values of n are shown in Fig. 1, for a nose length of 3 units (i.e. for $a/2b=3$). It will be seen that as the exponent n increases the point of maximum curvature moves away from the stagnation point and tends to a sharp corner, the curves going continuously (as n varies) from the ellipse ($n=2$) to the rectangle ($n=\infty$). The associated increase in maximum curvature introduces an additional point of possible separation upstream of the junction. Thus, the flow is prone to separation near the junction for $n\simeq 2$, and near the corner for $n\Rightarrow\infty$; there must therefore be an optimum value of n between these limits that minimises chances of separation. This optimum is the object of the present study.

3 Computational procedure

To eliminate the effects of the trailing edge the plate length was chosen, after a few trials, to be 200 times the thickness $2b$. To obtain accurate results for the surface velocity distribution $q(s)$ on the nose without excessive computing effort, it is essential to find a good scheme for dividing the boundary into elements of graded length. Such a scheme should provide short elements on the nose and near the junction, and longer ones further downstream where the source strengths will be small.

The scheme finally adopted is illustrated in Fig. 2. Defining

$$x=a(1-\xi), \quad y=b\eta,$$

the nose region is divided into elements subtending equal polar angles in the $\xi\eta$ plane: i.e. the intervals in the angle

$$\varphi=\tan^{-1}\eta/\xi$$

are chosen to be equal. On the flat plate, equal intervals in a polar angle $\varphi=\tan^{-1}y/x$ are chosen, and the surface elements determined by projection of intersections of the radii with a line $y=$const. (say t), as illustrated in Fig. 2. The value of t is so selected that the length of the first element on the flat plate is

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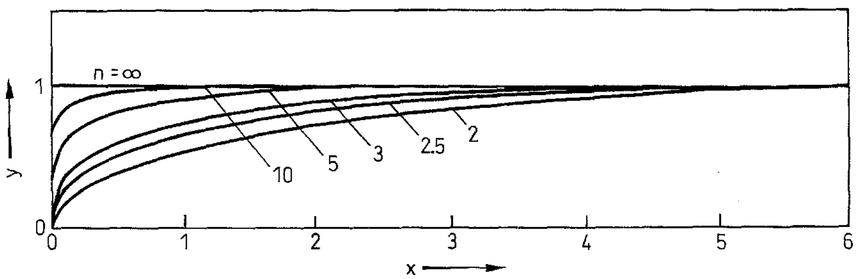

Fig. 1. Super-elliptic nose-shapes for an axis ratio of 6, going from an ordinary ellipse for $n=2$ to a rectangle as $n \Rightarrow \infty$

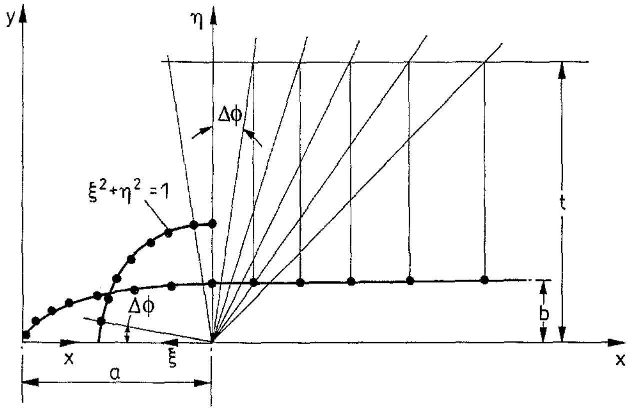

Fig. 2. Construction of panels or boundary elements for solution of potential flow problem

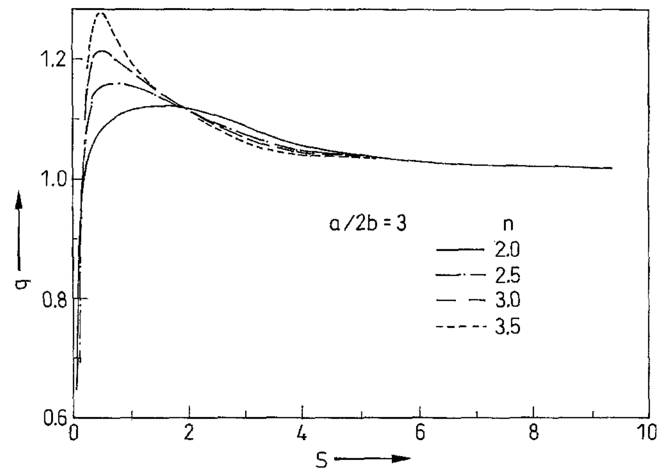

Fig. 3. Velocity distribution on flat plate with various superelliptic noses of axis ratio $a/b=6$. q is non-dimensionalized by the free-stream velocity

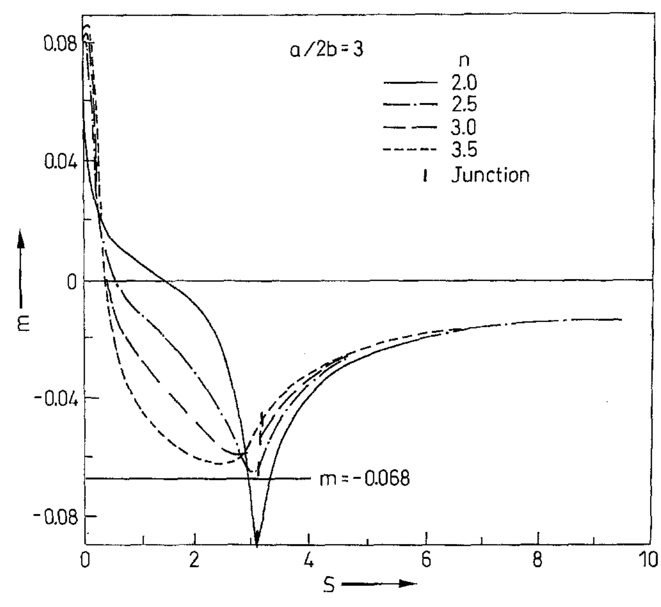

Fig. 4. The Thwaites pressure gradient parameter along a flat plate with various superelliptic noses of axis ratio $a/b=6$

equal to that of the last element on the nose. This method of dividing the flat plate gives short elements near the junction and very large elements towards the trialing edge, as desired.

The closeness of the boundary layer to separation is assessed on the assumption that it is laminar, which should be realistic as the Reynolds number is usually small on the nose. Further, as the leading edge is not sharp, the boundary layer approximations will be valid right from the stagnation point.

The development of the laminar boundary layer on the nose is calculated using the well-known Thwaites (1949) method. This gives eventually the pressure gradient parameter

$$m \equiv (dq/ds)\, \theta^2/\nu \qquad (2)$$

where s is distance along the surface from the leading edge stagnation point, q the velocity at s, θ the momentum thickness at s, and ν the kinematic viscosity.

4
Results and discussion

To illustate the results obtained, the velocity q and the parameter m are shown plotted against s for one nose-piece with $a/2b=3$ in Figs. 3 and 4. It is seen from Fig. 3 that the velocity drops rather suddenly at the junction for the elliptic nose, whereas it is more gradual for the super-ellipses. On the other hand, the peak velocity also increases as the index n increases, and is followed by high adverse pressure gradients near the point of maximum curvature.

Figure 4 shows the pressure gradient parameter m. It is recommended by Curle (1962) that the value of m at separation be taken as -0.09; however comparison with exact solutions shows that in different flows the value of m at separation has ranged from -0.068 to -0.16 (Curle 1962, p. 46). For the present analysis, we are going to assume the conservative value of -0.068 as the critical value that has to be exceeded to be certain of no separation. This critical value is also shown in the figure. It is seen that while the elliptic nose may lead to separation, the super-ellipses should avoid it; furthermore $n=3$

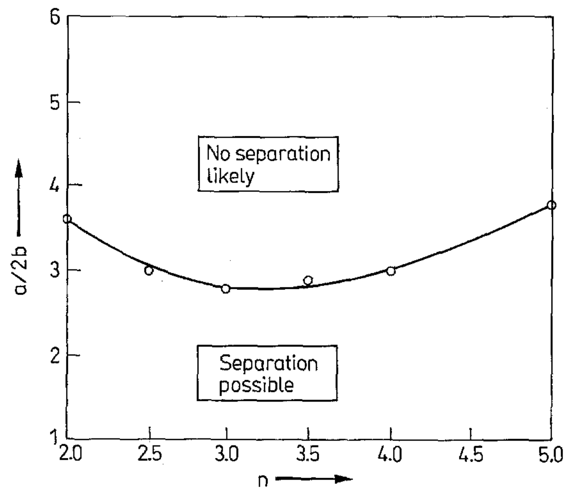

Fig. 5. The boundary between separated and unseparated flow in the plane of the two nose shape parameters, axis ratio a/b and exponent n

yields a higher $m_{\min}$ than either $n=2.5$ or 3.5, suggesting that a further increase in n will bring no benefit.

Similar analyses for other axis ratios (Narasimha and Prasad 1984) show that, for $a/b=1$, separation is possible at any value of n: while for $n=2$, 2.5 the junction presents the most severe problem, for $n=3$ and beyond the lowest m occurs upstream of the junction, near the region of highest surface curvature. For $n=3.5$ the value of $m_{\min}$ is lower than that for $n=3.0$.

Figure 5 summarizes the results, in the form of a boundary of separated flow in the plane of the parameters $a/2b$ and n. It is seen that, for a given $a/2b$, $m_{\min}$ is lowest for $n=2$ (the ellipse), and that separation may be avoided by using a super-elliptic nose with index 2.5 or 3 having a nose length of at least 3 times the flat plate thickness. The minimum axis-ratio for no separation is obtained for $n=3.15$.

From this diagram it is clear that if the axis-ratio is sufficiently large separation can always be avoided. As already pointed out, long noses are separation-prone at incidence, and delay the attainment of the constant pressure boundary layer solution, which is often a major objective in experiments on flat plates. If however tunnel flow quality is superior, in particular the span-wise variation of pitch in the free stream is negligible, a long nose piece may be affordable and desirable. If on the other hand we seek the shortest nose-piece that avoids separation, we should use a super-ellipse of axis ratio 6 and exponent $n=3.15$. However, the minimum in Fig. 5 is sufficiently flat that $n=3$ is almost as good, and a cubic super-ellipse with a nose length of thrice the plate thickness is a good practical optimum. This configuration has been found useful in various flat plate experiments in our laboratory (e.g. Narasimha et al. 1984).

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