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MINIMAL PRIME IDEALS IN 0-DISTRIBUTIVE SEMILATTICES

by

N. K. THAKARE (Aurangabad) and Y. S. PAWAR (Aurangabad)

1. Introduction

Let S be a meet-semilattice. A nonempty subset I of S will be called an *ideal* if

(i) $x \leq y$ in S and $y \in I$ imply $x \in I$, and

(ii) if the join of any finite number of elements of I exists in S then it must be in I.

This definition of an ideal in a meet-semilattice is to be found in VENKATANARASIMHAN [8]. A *filter* F of S is a nonempty subset of S such that $a, b \in F$ is equivalent to $a \wedge b \in F$. A proper ideal I of S is called *prime* if $a \wedge b \in I$ implies $a \in I$ or $b \in I$. A proper ideal (filter) of S which is not contained in any other proper ideal (filter) of S is called a *maximal* ideal (filter). A minimal element in the set of all prime ideals of S is called a *minimal prime* ideal. A proper filter F is called prime provided that, whenever for any finite subset A of S, $\vee A$ exists and is in F, then $a \in F$ for some $a \in A$. A semilattice S with 0 is called 0-distributive if

$$a \wedge x_1 = a \wedge x_2 = \ldots = a \wedge x_n = 0$$

for $x_1, \ldots, x_n$ (n finite) in S imply

$$a \wedge (x_1 \vee \ldots \vee x_n) = 0,$$

whenever $x_1 \vee \ldots \vee x_n$ exists in S.

The authors [5] earlier studied such semilattices for the case $n = 2$. All the results obtained there are invariably valid for the 0-distributive semilattices introduced here.

The concept of minimal prime ideals was put to advantage by KIST [4] while investigating commutative semigroups. The purpose of this paper is to obtain some properties of minimal prime ideals in 0-distributive semilattices. The study that we shall carry out will, in many ways, be distinct from that of KIST [4] Our study has resulted in extending the findings of SPEED [6],

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and VENHATANARASIMHAN [9], [10]. We also study special types of minimal prime ideals the motivation of which stems from the investigations carried out by CORNISH and STEWART [3].

2. Minimal prime ideals

We begin this section with a characterization of minimal prime ideals in a 0-distributive semilattice in terms of maximal filters. It also provides us with a useful tool for establishing properties of minimal prime ideals. We use in its proof a characterization of 0-distributive semilattices obtained earlier by the authors ([5], Theorem 5).

THEOREM 1. *Let S be a 0-distributive semilattice. A subset M of S is a minimal prime ideal if and only if its set complement $S - M$ is a maximal filter.*

PROOF. Let M be a minimal prime ideal of S. As the set complement of a prime ideal is a proper filter in a semilattice, we get $S - M$ to be a proper filter in S. If $S - M$ is not a maximal filter, then it must be contained in some maximal filter, say F, in S. By 0-distributivity of S (see PAWAR and THAKARE [5]), F is prime. This makes $S - F$ to be a prime ideal contained in M. It then contradicts the minimality of M and hence $S - M$ must be a maximal filter.

Conversely, let $S - M$ be a maximal filters in S. As S is 0-distributive, $S - M$ is a prime filter; see [5]. Thus M is a prime ideal. To prove the minimality of M, assume to the contrary. If a prime ideal Q is contained in M, the filter $S - Q$ properly contains $S - M$ and it is against our assumption.

Our next result is an immediate consequence of Theorem 1.

COROLLARY 2. *In a 0-distributive semilattice every prime ideal contains a minimal prime ideal.*

PROOF. Let P be a prime ideal in a 0-distributive semilattice S. As $S - P$ is a proper fitler of S and $0 \in S$, $S - P$ must be contained in some maximal filter, say F, in S. Then $S - F$ is the minimal prime ideal contained in P and we are done.

Theorem 1 is also used to prove the following equivalent property for prime ideal to be a minimal prime. For any nonempty subset A of S,

$$A^* = \{x \in S : x \wedge a = 0, \text{ for all } a \in A\},$$

the set of all disjoint elements of A in S.

THEOREM 3. *A prime ideal M in a 0-distributive semilattice S is minimal prime if and only if $\{x\}^* - M \neq \emptyset$ for any $x \in M$.*

PROOF. Let M be a minimal prime ideal and x be any element in M. By Theorem 1, $S - M$ is a maximal filter; and as $x \notin S - M$ there exists (see [5]) an element y in $S - M$ such that $x \wedge y = 0$. Thus $y \in \{x\}^* - M$, proving that $\{x\}^* - M \neq \emptyset$.

Conversely, let a prime ideal M satisfy the given condition. Consider any element which is not in $S - M$. Then $x \in M$ and hence $\{x\}^* - M \neq \emptyset$, by assumption. Thus there eixsts $y \in \{x\}^*$ such that $y \notin M$. Hence, we get for any x not in $S - M$ an element y in $S - M$ such that $x \wedge y = 0$. By a result of PAWAR and THAKARE [5], we conclude that $S - M$ is a maximal filter. An appeal to Theorem 1 now leads to the minimal primeness of M.

The preceding theorem, in turn, permits us to state the following

THEOREM 4. *A prime ideal M is a minimal prime ideal in a 0-distributive semilattice S if and only if it contains precisely one of $\{x\}$, $\{x\}^*$ for every $x \in S$.*

PROOF. Let M be a minimal prime ideal in S. If $x \in M$ then by Theorem 3, $\{x\}^* - M \neq \emptyset$. Hence we obtain $\{x\}^* \not\subset M$. Suppose that $\{x\}^* \subseteq M$. Then $x \in M$ will yield that $x \notin S - M$. As $S - M$ is a maximal filter, there exists a y in $S - M$ such that $x \wedge y = 0$; see [5]. Thus $y \in \{x\}^*$ and $y \notin M$, contradicting our assumption $\{x\}^* \subseteq M$. Hence in this case $x \notin M$. This completes the proof of "only if" part.

Conversely, let a prime ideal M contain precisely one of $\{x\}$, $\{x\}^*$ for any $x \in S$. Consider any element y not in $S - M$. Then $y \in M$ gives that $\{y\}^* \not\subset M$. Hence there exists z in $\{y\}^*$ such that $z \notin M$. Thus we get that for any y not in $S - M$, there is $z \in S - M$ such that $y \wedge z = 0$; and we are led to the maximality of the filter $S - M$ on account of [5]. Since S is 0-distributive, we get M to be a minimal prime ideal by Theorem 1.

COROLLARY 5. *If M is a minimal prime ideal in a 0-distributive semilattice S and x is an element of M, then $\{x\}^{**} \subseteq M$.*

PROOF. As M is a minimal prime ideal and $x \in M$, there is an element $y \in S - M$ such that $x \wedge y = 0$ by Theorem 4. If $\{x\}^{**} \subsetneq M$, there would be an element z in $\{x\}^{**}$ with $z \notin M$. Then, since $S - M$ is a maximal filter (see Theorem 1), $y \wedge z \in S - M$. But as $y \in \{x\}^*$ and $z \in \{x\}^{**}$ we get $y \wedge z = 0$. Thus $0 \in S - M$, contradicting the maximality of $S - M$. Hence $\{x\}^{**}$ must be contained in M, where $x \in M$.

It follows immediately from Theorem 4 and Theorem 1 that

COROLLARY 6. *A filter F in a 0-distributive semilattice S is maximal if and only if F contains precisely one of $\{x\}$, $\{x\}^*$ for every $x \in S$.*

We know that I^* is the pseudocomplement of an ideal I in the ideal atltice $I(S)$ when S is a 0-distributive semilattice; see [5]. Further, we have

THEOREM 7. *In a 0-distributive semilattice S the pseudocomplement of any ideal I is the intersection of all minimal prime ideals not containing I.*

PROOF. Recall that P is a prime ideal in a semilattice S if and only if for any two ideals A and B of S, $\emptyset \neq A \cap B \subseteq P$ implies $A \subseteq P$ or $B \subseteq P$. As $I \cap I^* = \{0\} \subseteq M$ for any minimal prime ideal M, we get $I^* \subseteq M$ when $I \subseteq M$. Therefore $I^* \subseteq \bigcap \{M \in \mathfrak{m} : I \subsetneqq M\}$ where $\mathfrak{m}$ denotes the set of all minimal prime ideals of S. If $I^* \subset \bigcap \{M \in \mathfrak{m} : I \subsetneqq M\}$, then there exists $x \in \bigcap \{M \in \mathfrak{m} : I \subsetneqq M\}$ such that $x \notin I^*$. Hence for some $y \in I$, $x \wedge y \neq 0$. But as $x \wedge y \neq 0$, $x \wedge y$ must be contained in some maximal filter, say F, of S. Hence $y \notin S - F$, and we infer that $I \subsetneqq S - F$. As $S - F$ is a minimal prime ideal (see Theorem 1) of S, we obtain

$$\bigcap \{M \in \mathfrak{m} : I \subsetneqq M\} \subseteq S - F.$$

Therefore $x \in S - F$, a contradiction to the fact $x \in F$. Thus we must have

$$\bigcap \{M \in \mathfrak{m} : I \subsetneqq M\} = I^*.$$

As mentioned earlier the ideal lattice $I(S)$ of a 0-distributive semilattice S is pseudocomplemented, it will be interesting to discuss dense and normal elements in $I(S)$. An ideal I in a 0-distributive semilattice S is called *normal* if it is a normal element of $I(S)$, i.e., if $I = I^{**}$. As a natural consequence of Theorem 7, we get

THEOREM 8. *Any normal ideal in a 0-distributive semilattice is the intersection of all minimal prime ideals containing it.*

PROOF. Let I be any normal ideal in a 0-distributive semilattice S. Then $I = I^{**}$. By Theorem 7,

$$(I^*)^* = \bigcap \{M \in \mathfrak{m} : I \subsetneqq M\},$$

I^* being an ideal of S. As $I \cap I^* = \{0\} \subseteq M$ and M is prime for every $M \in \mathfrak{m}$, we obtain $I \subseteq M$, whenever $I^* \nsubseteq M$. Hence

$$I^{**} = I = \bigcap \{M \in \mathfrak{m}: I \subseteq M\}.$$

The converse of Theorem 8 happens to be true for principal ideals and is proved in the following

THEOREM 9. *A principal ideal of a 0-distributive semilattice is normal if and only if it is the intersection of all minimal prime ideals containing it.*

PROOF. In view of Theorem 8, we need to establish one way implication only. Let

$$(a] = \bigcap \{M \in \mathfrak{m}: (a] \subseteq M\}.$$

Then for any $a \in S$, we have

$$(a]^{**} = ((a]^*]^* = \bigcap \{M \in \mathfrak{m}: (a]^* \not\subset M\};$$

by Theorem 7,

$$\bigcap \{M \in \mathfrak{m}: (a]^* \not\subset M\} = \bigcap \{M \in \mathfrak{m}: (a] \subseteq M\},$$

since M is prime and

$$(0] = (a] \cap (a]^* \subseteq M,$$

and

$$\bigcap \{M \in \mathfrak{m}: (a] \subseteq M\} = (a],$$

by assumption. Thus $(a]^{**} = (a]$.

COROLLARY 10. *The intersection of all minimal prime ideals of a 0-distributive semilattice is $\{0\}$.*

An ideal I in a 0-distributive semilattice S is called *dense* if $I^* = \{0\}$, i.e., if I is a dense element of $I(S)$.

An interesting property of non-dense ideals in a 0-distributive semilattice is investigated in the following

THEOREM 11. *Any non-dense ideal of a 0-distributive semilattice is contained in a minimal prime ideal and the converse is true for principal ideals.*

PROOF. Let I be any non-dense ideal of a 0-distributive semilattice S. As $I^* \neq \{0\}$, there exists an $x \neq 0$ in I^*. Let this x be contained in the maximal filter F of S. As $S - F$ is a minimal prime ideal and $I^* \nsubseteq S - F$, we obtain $I \subseteq S - F$ by primeness of $S - F$.

Now for the second assertion, let the principal ideal $(a]$ be contained in a minimal prime ideal M in S. As $a \in M$, $\{a\}^* \nsubseteq M$; see Theorem 4. Hence $(a]^* = \{a\}^* \neq \{0\}$. This proves that $(a]$ is non-dense.

Next we have a rather interesting characterization.

THEOREM 12. *In a 0-distributive semilattice an element belongs to some minimal prime ideal if and only if it is non-dense.*

PROOF. Let S be a 0-distributive semilattice and $x \in M$, a minimal prime ideal of S. By Theorem 1, $\{x\}^* \nsubseteq M$. Hence $\{x\}^* \neq 0$ proving that x is non-dense.

Conversely, let x be a dense element. Then $\{x\}^* = 0$. If x belongs to some minimal prime ideal M, then $\{x\} \subseteq M$ and $\{x\}^* \subseteq M$, contradicting the minimality of M; see Theorem 4. Hence x will not be in any minimal prime ideal.

As every pseudocomplemented semilattice is 0-distributive, the result of VENKATANARASIMHAN ([9], Lemma VIII) follows as a corollary to the following

THEOREM 13. *The subsequent statements are equivalent in a 0-distributive semilattice S:*

(1) *Every prime ideal is minimal prime.*

(2) *Every prime filter is minimal prime.*

(3) *Every prime filter is maximal.*

PROOF.

(1) $\Rightarrow$ (2). Let there be a prime filter F that is not minimal. Then there exists a prime filter $F_1 \subset F$. But then $S - F$ is contained in $S - F_1$. As F and F_1 are prime fitlers, $S - F$ and $S - F_1$ are prime ideals in S. Hence, by assumption, $S - F_1$ must be a minimal prime ideal; and it is not the case. Thus the prime filter F must be a minimal prime filter.

(2) $\Rightarrow$ (3). Let F be a prime filter that is not maximal. Since $0 \in S$, F must be contained in some maximal filter, say M, in S. As S is 0-distributive, M must be a prime filter; see [5]. But then, by assumption, M must be a minimal prime filter. This is not true as $F \subset M$; thus F must be maximal.

(3) $\Rightarrow$ (1). By Corollary 2, if I is a prime ideal in S that is not minimal then I contains a minimal prime ideal, say I_1. But then $S - I \subset S - I_1$. Clearly, both $S - I$ and $S - I_1$ being prime filters of S must be maximal by assumption; however it leads to a contradiction.

Let us have a characterization of minimal prime ideals in the following

THEOREM 14. *A prime ideal P is a minimal prime ideal in a 0-distributive semilattice S if and only if P consists of all elements $x \in S$ such that $x \wedge y = 0$ for some $y \notin P$.*

PROOF. Let P be a minimal prime ideal in S and $x \in P$. Then by Theorem 3, there exists an element y in $\{x\}^* - P$. Clearly $y \wedge x = 0$ and $y \notin P$. Next, suppose that $z \wedge x = 0$ for some $x \notin P$. Then $z \wedge x = 0 \in P$; and primeness of P leads to the conclusion that $z \in P$. Thus P consists of all elements $x \in S$ such that $x \wedge y = 0$ for some $y \notin P$.

3. Minimal prime annihilator ideals

As before, let S be a 0-distributive semilattice. Denote by $\mathfrak{B}(S)$ the set $\{A^* : \emptyset \neq A \subseteq S\}$. As A^* is an ideal for any subset A of S, we call A^* to be an annihilator ideal. Thus $\mathfrak{B}(S)$ is the set of all annihilator ideals in S. $\mathfrak{B}(S)$ is partially ordered by set inclusion and the greatest lower bound is the set intersection. ADAMS [1] showed that $\mathfrak{B}(S)$ is a Boolean lattice. In an attempt to characterize minimal prime annihilator ideals, we need the following

LEMMA 15. *An annihilator ideal A^* is a prime ideal in a 0-distributive semilattice S if and only if A^* is a dual atom in $\mathfrak{B}(S)$)i.e., $A^* \subseteq B^* \neq S$ implies that $A^* = B^*$).*

PROOF. Assume that A^* is a prime ideal and $A^* \subseteq B^* \neq S$. The last assumption implies that $s \wedge b_1 \neq 0$ for some $s \in S$ and some nonzero $b_1 \in B$. For any $b \in B^*$ as $b \wedge b_1 = 0$, we get $b \wedge b_1 \in A^*$; this in turn implies that either $b \in A^*$ or $b_1 \in A^*$. Since $A^* \subseteq B^*$, we get $b_1 \notin A^*$. Thus $b \in A^*$ and we have $B^* = A^*$ leading to the conclusion that A^* is a dual atom of $\mathfrak{B}(S)$.

Conversely, let A^* be a dual atom in $\mathfrak{B}(S)$. As $A^* \neq S$, there exists an $s \in S$ such that $s \wedge a \neq 0$ for some nonzero $a \in A$. Again $s \notin \{a\}^*$ will imply that $\{a\}^* \neq S$. Since A^* is a dual atom, $S \neq \{a\}^* \supseteq A^*$ implies that $\{a\}^* = A^*$. For any $b \in S$, as $\{a \wedge b\}^* \supseteq \{a\}^*$, either $\{a\}^* = \{a \wedge b\}^*$ or $\{a \wedge b\}^* = S$. If $b \notin \{a\}^*$, then $\{a \wedge b\}^* = \{a\}^*$. For, if $\{a \wedge b\}^* = S$, then $a \in \{a \wedge b\}^*$ will yield that $a \wedge b = 0$; i.e., $b \in \{a\}^*$. To prove that $A^* = \{a\}^*$ is prime, let $x \wedge y \in \{a\}^*$ and $x \notin \{a\}^*$. Then as $\{a \wedge x\}^* = \{a\}^*$ and as $y \in \{a \wedge x\}^*$, we get $y \in \{a\}^*$.

Recall that an element x in a semilattice is meet-prime if $a \wedge b \leq x$ implies $a \leq x$ or $b \leq x$; see SZÁSZ [7], p. 51. It is well-known that in a Boolean algebra an element is a dual atom if and only if it is meet-prime; (see [8],

p. 51). This permits us to characterize prime annihilator ideals in a 0-distributive semilattice as

LEMMA 16. *In a* 0*-distributive semilattice an annhilator ideal* A^* *is prime if and only if* A^* *is a meet-prime element of* $\mathfrak{B}(S)$.

Using the characteristic property of minimal prime ideals in a 0-distributive semilattice — Theorem 3 — we now prove

LEMMA 17. *Every prime annihilator ideal is minimal prime in a* 0*- distributive semilattice.*

PROOF. Let an annihilator ideal A^* in a 0-distributive semilattice S be prime. For any $x \in A^*$, we have $x \wedge a = 0$ for every $a \in A$. That is $A \subseteq \{x\}^*$ and hence $A \subseteq$ c$x\}^* - A^*$ proving that $\{x\}^* - A^* \neq \emptyset$. Thus, by Theorem 3, we get A^* to be a minimal prime ideal in S.

Now we state our main result in which we summerize the characterizations of minimal prime annihilator ideals.

THEOREM 18. *For any nonempty subset* A *of a* 0*-distributive semilattice* S *the following statements are equivalent*:

(1) A^* *is a dual atom in* $\mathfrak{B}(S)$.

(2) A^* *is a meet-prime element of* $\mathfrak{B}(S)$.

(3) A^* *is a minimal prime annihilator ideal.*

(4) A^* *is a prime annihilator ideal.*

We noted earlier that $\mathfrak{B}(S)$ is a Boolean lattice. If $\mathfrak{B}(S)$ satisfies the ascending chain condition then $\mathfrak{B}(S)$ is finite. Thus there will be only finite number of dual atoms in $\mathfrak{B}(S)$ when it satisfies the ACC. In accordance with this observation and Lemma 17, we are led to

LEMMA 19. *A* 0*-distributive semilattice* S *contains a finite family of minimal prime ideals with intersection* $\{0\}$ *when* $\mathfrak{B}(S)$ *satisfies ACC.*

PROOF. As $\mathfrak{B}(S)$ satisfies ACC, there will be only finite number of dual atoms, say $A_1^*, \ldots, A_n^*$ (n finite) in $\mathfrak{B}(S)$. As A_i^* is a dual atom, we get $A_i^* = \{a_i\}^*$ for some nonzero $a_i \in A_i$, $i = 1, \ldots, n$; see the proof of Lemma 15. Let $0 \neq x \in \bigcap_1^n A_i^*$. As $\mathfrak{B}(S)$ satisfies ACC, $\{x\}^*$ must be contained in some A_j^* for $j \leq n$; see CORNISH [2]. Since $x \in A_j^* = \{a_j\}^*$, we have $x \wedge a_j = 0$. $a_j \in \{x\}^* \subseteq A_j^*$ implies that $a_j = 0$ and this is impossible. Hence $\bigcap_1^n A_i^* = \{0\}$.

As A_i^* $(1 \leq i \leq n)$ are minimal prime ideals, the conclusion of the lemma follows immediately.

Use of Lemma 19 further leads to

THEOREM 20. *Let S be a 0-distributive semilattice. If $\mathfrak{B}(S)$ satisfies ACC then the set complement of union of dual atoms in $\mathfrak{B}(S)$ is the set of all dense elements of S.*

PROOF. Let $A_i^*, \ldots, A_n^*$ n finite, be the distinct dual atoms of $\mathfrak{B}(S)$ and let x be any element of $S - \bigcup_1^n A_i^*$. If $x \wedge y = 0$ for some $y \neq 0$, then as $\{y\}^* \neq S$, we have $\{y\}^* \subseteq A_j^*$ for some $j \leq n$. Thus $x \in \{y\}^* \subseteq A_j^*$ implies that $x \in \bigcup_1^n A_i^*$; a contradiction. Hence $x \wedge y = 0$ implies $y = 0$, i.e., $\{x\}^* = \{0\}$. Therefore $S - \bigcup_1^n A_i^* \subseteq D$ where D denotes the set of all dense elements of S. On the other hand, if $x \in \bigcup_1^n A_i^*$, then $x \in A_j^*$ for some $j \leq n$. Denote

$$A_j^* \wedge (\bigcap_{i \neq j} A_i^*) = \{x \wedge y : x \in A_j^*, y \in \bigcap_{i \neq j} A_i^*\}.$$

Clearly

$$A_j^* \wedge (\bigcap_{i \neq j} A_i^*) \subseteq \bigcap_1^n A_i^*.$$

But as $\bigcap_1^n A_i^* = \{0\}$, (see Lemma 19), we get

$$A_j^* \wedge \bigcap_{i \neq j} A_i^* = \{0\}.$$

Therefore, there must exist some $y \neq 0$ in $\bigcap_{i \neq j} A_i^*$ such that $y \wedge x = 0$. Hence $\{x\}^* \neq \{0\}$. Thus no element of $\bigcup_1^n A_i^*$ is dense. Therefore, $D \subseteq S - \bigcup_1^n A_i^*$ and we are through.

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REFERENCES

[1] D. H. ADAMS, Semigroup completions of lattices, *Proc. London Math. Soc.* **20** (1970), 659—668.[1] *MR* **42** # 5870

[2] W. H. CORNISH, Normal lattices, *J. Austral. Math. Soc.* **14** (1972), 200—215. *MR* **47** # 1703

[3] W. H. CORNISH and P. N. STEWART, Rings with no nilpotent elements and with the maximum condition on annihilators, *Canad. Math. Bull.* **17** (1974), 35—38.[2] *MR* **50** # 4654

[4] J. KIST, Minimal prime ideals in commutative semigroups, *Proc. London Math. Soc.* **13** (1963), 31—50. *MR* **26** # 1387

[5] Y. S. PAWAR and N. K. THAKARE, 0-distributive semilattices, *Canad. Math. Bull.* **21** (1978), 469—481. *MR* **80***f*: 06002

[6] T. P. SPEED, Spaces of ideals of distributive lattices, II: Minimal prime ideals, *J. Austral. Math. Soc.* **18** (1974), 54—72. *MR* **50** # 6954

[7] G. SZÁSZ, *Introduction to lattice theory*, Akadémiai Kiadó, Budapest, and Academic Press, New York, 1963. *MR* **29** # 3396

[8] P. V. VENKATANARASIMHAN, Ideals in semi-lattices, *J. Indian Math. Soc.* **30** (1966), 47—53. *MR* **35** # 1517

[9] P. V. VENKATANARASIMHAN, Stone's topology for pseudocomplemented and bicomplemented lattices, *Trans. Amer. Math. Soc.* **170** (1972), 57—70. *MR* **47** # 90

[10] P. V. VENKATANARASIMHAN, Ideals in pseudo-complemented lattices and semilattices, *Acta Sci. Math.* (Szeged) **36** (1974), 221—226. *MR* **49** # 10616

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DEPARTMENT OF MATHEMATICS
MARATHWADA UNIVERSITY
IND—431004 AURANGABAD
INDIA

Current address of N. K. Thakare:
JAI HIND COLLEGE OF ARTS, SCIENCE AND COMMERCE
IND—424002 DEOPUR, DHULE
INDIA

[1] Correction: ibidem, **21** (1970), 576.
[2] Correction: ibidem, **17** (1974), 425.