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Chatter Stability Analysis for End Milling via Convolution Modelling

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This research discusses the methodology of developing a symbolic closed form solution that describes the dynamic stability of multiflute end milling. A solution of this nature facilitates machine tool design, machining parameter planning, process monitoring, diagnostics, and control. This study establishes a compliance feedback model that describes the dynamic behavior of regenerative chatter for multiflute tool–work interaction. The model formulates the machining dynamics based upon the interconnecting relationship of the tool geometry and the machining system compliance. The tool geometry characterises the cutting forces as a function of the process parameters and the material properties, while two independent vibratory modules, the milling tool and the workpiece, represent the machining system compliance. The compliance feedback model allows the development of a corresponding characteristic equation. By investigating the roots of the characteristic equation, this research symbolically expresses the stability of the system as a function of the cutting parameters, the tool geometry, the workpiece geometry, and the vibrational characteristics of the machine tool. Machining experimentation examining the fidelity of the regenerative chatter model is discussed. The dynamic cutting forces, cutting vibration, and surface finish of the machining process confirm the validity of the analytical prediction.

Keywords: Chatter; Convolution modelling; End milling

1. Introduction

The objective of this research is to describe the behaviour of regenerative chatter in multiflute end milling in explicit terms of the tool and the workpiece geometry, the material properties, the process parameters, and the vibrational characteristics of the machine tool. The closed form solution for chatter lends itself to the optimisation, monitoring, and control of machining processes.

The cyclic fluctuation of the cutting tool that results in the engagement and disengagement of the tool from the cutting surface represents the dynamic instability in milling. This periodic oscillation of the tool is frequently called chatter.

Machining operations try to avoid the development of chatter since it is highly undesirable. If not monitored and controlled, the vibrations that arise from chatter have both immediate and long term adverse effects: poor surface finish, inability of the machine tool to deliver dimensional accuracy, premature wear, chipping, and/or failure of the cutting tool, damage to the machine tool, and excessive noise. Therefore, it is advantageous to be able to predict the occurrence of chatter and to understand the relationship amongst the factors that generate and directly influence its progression.

When investigating the dynamic stability analysis for milling, it becomes apparent that there are two independent tasks that are critical for the success of the research. It is imperative that the work describes the complete and true dynamics of the milling process and that work employs appropriate techniques to characterise the machining stability in terms of a meaningful expression.

Previous study has demonstrated that the strongest sources of self-excited and regenerative vibrations are related to the structural dynamics of the machine tool and the feedback between subsequent cuts, and not the mechanics of chip formation [1]. This confirms the validity of modelling the chatter process in milling by a closed loop dynamic system. Presented first in [1], this theory has been accepted by many of the current research teams [1–5]. Essentially, vibrations within the system (either forced or self-excited) bias the cutting process and generate additional forces on the structure. These forces further excite the machine tool structure resulting in regenerative chatter. Figure 1 depicts Tlusty's [1] basic representation of chatter as a closed loop system. Tlusty's model is significant since it essentially states that chatter is only a function of the cutting process and the machine structure.

By simplifying the problem and restricting the analysis to a single cutting edge, Tlusty defines the function of the

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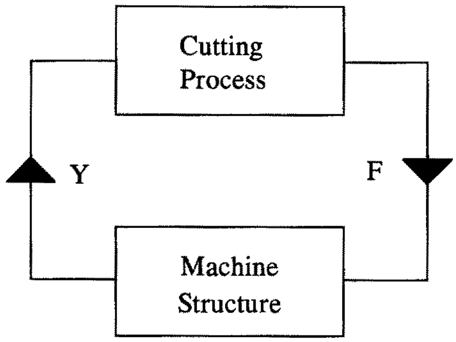

Fig. 1. Tlusty's closed loop representation of chatter. F, cutting force; Y, relative displacement.

structure as it relates to the cutting force. The equation describes the dynamics of turning and reveals that no damping (positive or negative) develops during the chip formation process since the cutting force is in phase with the chip thickness variation. Tlusty's work is important in the area of chatter since it proves that negative damping is not a necessary condition for the onset of chatter.

Minis *et al.* [4] suggest an alternative approach to calculate the closed loop system of the turning operation. It is based on a combination of the machine tool structure dynamics and the cutting process dynamics. The authors simplify the analysis of the complex vibratory system (the machine tool structure) by investigating the dynamics only at the cutting point.

The authors' final closed loop representation for the turning system results from the coupling of the machine tool structure dynamics and the cutting process dynamics. As compared to other work, this theory employs no *a priori* knowledge of the direction of the cutting force or the modal vibration directions of the structure. Consequently, no assumptions or simplifications with regard to the cutting force or tool orientation are necessary.

Wang et al. [6] express the instantaneous cutting force system as a function of the angular direction of the elemental cutting force function, the chip width density function, and the tooth sequencing function. This is dissimilar to any previous work. The authors based their model on a kinematic analysis of the end milling process. Nowhere in their examination of the dynamic system do the authors incorporate the compliance of the machine tool structure or the cutter/workpiece interface. However, the work of Wang et al. is unique and considerable since it defines the cutting forces as a closed form expression of the cutting parameters and the tool/workpiece geometry.

To arrive at their theoretical approach for the prediction of chatter, Minis and Yanushevsky [5] expand on Tlusty's simple closed loop model of chatter. The dynamic analysis is basically the same as the previous work of Minis concerning the prediction of chatter in turning [4]. However, the authors tailor the new model to incorporate the time-varying aspect of milling. As discussed previously, the final equation embodies both the machine tool dynamics and the cutting process dynamics.

In addition, this new method depicts the periodicity characteristic of the interrupted cutting process and the time delay intrinsic to regenerative chatter. The authors furnish the set of linear differential-difference equations in a general form which is applicable for both turning and milling.

After reviewing the key papers in the area of machine tool stability it is apparent that many critical issues still need to be addressed before applying these findings to the shop floor or to general machining practices. Most of the research does not easily lend itself to the practical monitoring and control of machine tools to avoid regenerative chatter. Generally, the work is not complete enough to provide a real-time analysis of the milling system dynamics. The objective of this research is to formulate a symbolic expression that depicts the behaviour of chatter in multiflute end milling as explicit variables of the tool and the workpiece geometry, the material properties, and the process parameters. The closed form solution of chatter will facilitate optimisation and control of the machining process.

2. Dynamic System Representation for Multiflute End Milling

When developing a closed form expression for end milling that completely describes the machining system dynamics, there are two major deficiencies in the available research. As compared to previous work, this research focuses on achieving two criteria: incorporating the total compliance of the machine tool system in the formulation of the characteristic equation and expressing the stability of the milling process as a function of the cutting parameters, the tool and the workpiece geometry, and the vibrational characteristics of the machine tool.

By expanding on the solution of Wang et al. and by investigating the compliance of the tool, the workpiece, and the machine tool, it is possible to achieve a characteristic equation for the dynamic stability analysis of end milling. Figure 2 depicts the proposed block diagram that describes the closed loop behaviour of chatter in multiflute end milling. The input to the system is an impulse train characterised by t_x, the feed per tooth in the feed direction. The X- and Y-directions are orthogonal with the feed defining the X-direction and the radial depth of cut denoting the Y-direction. The angular convolution model presented by Wang et al. [6] expresses the dynamic cutting process of end milling and relates the relative motion of the cutting edge to the corresponding forces realised during cutting. The design of the compliance feedback model illustrated in Fig. 2 fundamentally consists of the angular convolution model as the feed forward portion, while the total compliance of the machining system along with the impulse generator define the feedback portion. According to this configuration, the actual cutting forces excite disturbances in the vibratory model defined by the compliance of the machining system. This relative motion affects the engagement and disengagement of the cutter teeth and subsequently the feed per tooth. The impulse generator converts the output of the compliance elements, the movement of the tool, from a displacement to an impulse. Thereupon, the adjusted impulse alters the feed forward portion of the system through a summing junction.

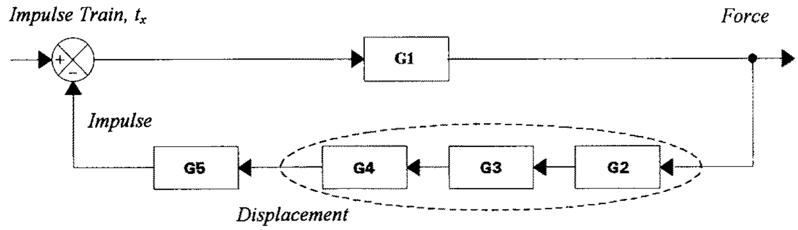

Fig. 2. Regenerative chatter model. G1, angular convolution model; G2, compliance of the machine tool; G3, compliance of the mill tool; G4, compliance of the workpiece; G5, impulse generator.

Owing to the closed form nature of the compliance feedback model, the relative motion of the tool as defined by the compliance in the system, modifies the impulse train to describe the dynamics of the end milling process more precisely. Essentially, the compliance feedback model for regenerative chatter in multiflute end milling consists of the cutting dynamics of end milling and the compliance effects of the machining system.

In closed form, the angular convolution model directly relates the instantaneous cutting forces to specific milling process parameters and tool/workpiece geometry. Equation (1) depicts the angular convolution model for the total forces for multiflute end milling as a function of the angular domain of the cutter.

$$\begin{bmatrix} \bar{F}_x(\theta) \\ \bar{F}_y(\theta) \end{bmatrix} = \begin{bmatrix} 1 & K_r \\ -K_r & 1 \end{bmatrix} \begin{bmatrix} P_1(\theta) \\ P_2(\theta) \end{bmatrix} CWD(\theta)\, TS(\theta)\, K_t \qquad (1)$$

The composition of the angular convolution model is three functions specifying disassociated elements of the cutting process. The instantaneous cutting forces yield, from the angular convolution of the element cutting force function ($P_1(\theta)$ and $P_2(\theta)$), the chip width density function ($CWD(\theta)$), and the tooth sequencing function ($TS(\theta)$). The mathematical form of equation (1) produces highly accurate estimations for the instantaneous cutting forces assuming ideal cutting conditions: constant feedrate, constant spindle speed, perfectly sharp tool, and complete machining stability. In addition, the milling machine exhibits no tilt of the major tool axis or cutter runout.

According to the regenerative chatter model, the compliance of the machining system serves as the primary feedback to the angular convolution model. Therefore, it is imperative to appropriately identify and mathematically describe the performance of the individual compliance elements that define the total machining system compliance. Owing to its complex nature, researchers quite often simplify the dynamic behaviour of machine tools. Investigators either decompose the machine tool into several smaller systems characterised by the primary structural elements or narrow the focus to an area describing the direct interaction of the cutter and the workpiece. By considering only the conditions at the cutting point, the total compliance of the milling machine simplifies to two primary vibratory modes: the machine tool and the workpiece. The machine tool incorporates the mechanical behaviour of the column, the overhang, the headstock, and the cutting tool, while the workpiece embodies the dynamic response of the workpiece, the fixturing device, the dynamometer, and the knee. The tip of the milling tool and the surface it projects onto the workpiece define the point of contact between the machine tool and the workpiece. In addition, a less complicated model yields, by assuming a second-order mass-spring-damper system, an adequate description of the two compliance elements.

The impulse generator is the last element in the compliance feedback loop of the regenerative chatter model. Its sole function is to produce an impulse function that corresponds to the actual displacement of the cutter at the tool tip. To summarise the function of the impulse generator, the input of the regenerative chatter model is an impulse train describing the ideal engagement of the cutter teeth. Before the compliance feedback signal is fed into the summing junction for comparison, the impulse generator converts the output, in terms of a physical deflection of the tool, into an equivalent sequence of impulses. The product of the summing junction is an adjusted impulse train that compensates for the dynamic behaviour of the physical system. Assuming a constant material removal rate, the delay in engagement of the succeeding tooth creates an increase in the magnitude of the impulse. When the tool does not deflect, the impulse generator has no function. Accordingly, the corrected impulse train is identical to the original reference input of the regenerative chatter model. The initial form of the impulse generator is the ratio of the response function and the driving function.

$$\frac{\text{Impulse}}{\Delta x} = \frac{s\left(-\exp\left[-\left(\frac{2\pi}{n}\right)s\right] + \left(1 + \frac{\Delta x}{x}\right)\exp\left[-\frac{2\pi}{n}\left(1 + \frac{\Delta x}{x}\right)s\right]\right)}{\Delta x} \qquad (2)$$

It is important to note that the resulting transfer function is mathematically intense and exhibits nonlinear behaviour. Regardless of this, the transfer function is valid and independent of the magnitude or the nature of the input.

3. Mathematical Procedure and Analysis to Characterise Stability

The plant that serves as the feed forward loop of the regenerative chatter model in the Laplace domain can be given as:

$$\frac{\begin{bmatrix} \bar{F}_x(s) \\ F_y(s) \end{bmatrix}}{\sum_{k=-\infty}^{\infty} \delta(-sj-nk)} = \begin{bmatrix} 1 & K_r \\ -K_r & 1 \end{bmatrix} \begin{bmatrix} P_1(s) \\ P_2(s) \end{bmatrix} CWD(s)\, K_t t_x n \qquad (3)$$

The oscillatory cutting forces estimated from equation (3) cause the machining system to vibrate. The interaction of both the machine tool and the workpiece compliance limit the degree of vibration of the machining system. The relative displacement of the cutter acts as the response function to the transfer function specifying the compliance. The plant denoting the compliance of the machining system can be derived directly from the Laplace transform of the second-order mass-spring-damper relating the compliance of the machine tool and the workpiece.

$$\Delta x = \frac{1}{(m_t + m_w)s^2 + (b_t + b_w)s + (k_t + k_w)} \begin{bmatrix} \bar{F}_x(s) \\ \bar{F}_y(s) \end{bmatrix} \quad (4)$$

The actual displacement of the cutter cannot be compared directly to the input of the system, owing to the inequivalency of the physical quantities. The impulse generator converts the displacement from the compliance into a corresponding impulse function. Mathematically, a step function portrays the dynamic behaviour of the cutter. The height of the step function depicts the relative motion of the mill tool. Equation (5) depicts the complete linearisation of the impulse generator in the Laplace domain.

$$\frac{\text{Impulse}}{\Delta x} = \left(\frac{1}{\left(1 + \frac{\pi s}{2}\right) t_x} - \frac{2\pi s}{n\left(1 + \frac{\pi s}{2}\right) t_x} \right) s \quad (5)$$

Equations (3) to (5) represent the Laplace transforms of the regenerative chatter modules. Unfortunately the embodiment of transcendental and nonlinear functions inhibit the direct utilisation of the transfer functions. Linearisation of these incompatible functions around zero, the critical region of stability, provides an approximation for the critical region of stability. However, linearisation techniques distort the magnitude of stability when the closed loop poles of the stability analysis deviate from the vicinity of the normal operating condition, zero.

The characteristic equation for stability is the direct application of Fig. 2 and the individual elements of the regenerative chatter model. The ensuing characteristic equation will not offer a precise boundary for stability, but an approximation. The characteristic equation closely represents the dynamic nature of the linearised model, not that of the exact regenerative chatter model. Equation (6) gives the mathematical construction of the final transfer function denoting the regenerative chatter model.

$$\frac{\begin{bmatrix} 1 & K_r \\ -K_r & 1 \end{bmatrix} \begin{bmatrix} P_1(s) \\ P_2(s) \end{bmatrix} CWD(s)\, K_t t_x n}{1 + \left(\begin{bmatrix} 1 & K_r \\ -K_r & 1 \end{bmatrix} \begin{bmatrix} P_1(s) \\ P_2(s) \end{bmatrix} CWD(s)\, K_t t_x n \right) \begin{bmatrix} C_x & 0 \\ 0 & C_y \end{bmatrix} \left(\frac{1}{\left(1 + \frac{\pi s}{2}\right) x} - \frac{2\pi s}{n\left(1 + \frac{\pi s}{2}\right) x} \right) s}$$

$$\begin{bmatrix} C_x & 0 \\ 0 & C_y \end{bmatrix} = \begin{bmatrix} \left(\frac{1}{(m_e)s^2 + (b_e)s + k_e} \right)_x & 0 \\ 0 & \left(\frac{1}{(m_e)s^2 + (b_e)s + k_e} \right)_y \end{bmatrix} \quad (6)$$

Table 1. Synopsis of vibration analysis.

	Spring constant (k)	Damping ratio (ζ)	Damped circular frequency of vibration (ω_d)
Mill tool			
X	5.04×10^6 (N/m)	0.086	17 423.27 (rad/s)
Y	5.39×10^6 (N/m)	0.138	15 337.26 (rad/s)
Workpiece			
X	1.00×10^9 (N/m)	0.0666	12 271.06 (rad/s)
Y-left	1.00×10^9 (N/m)	0.144	15 953.01 (rad/s)
Y-right	1.00×10^9 (N/m)	0.140	15 953.01 (rad/s)

Both the numerator and the denominator are rational expressions and require algebraic expansion to create the characteristic equations. The characteristic equations for regenerative chatter in the X- and Y-directions for multiflute end milling are dependent on the cutting parameters, the tool geometry, the workpiece geometry, and the vibration characteristics of the machine tool structure. Note that the respective characteristic equations predict only the onset or the presence of chatter. To accurately describe the kinematics of end milling, the derivation of the angular convolution model must remain in a stable cutting region. Once the cutter begins to vibrate, it is not feasible to effectively model the nonlinear behaviour of the cutter affected by regenerative chatter.

The regenerative chatter model provides a general form solution for describing the dynamic stability of multiflute end milling. This format provides a basis for the investigation of the relative influence and interaction of the symbolic terms. Stability is essentially a function of two sets of parameters: machine/workpiece characteristics and process variables. The process variables relate to the machining process parameters. Unlike the process variables, the machine/workpiece characteristics are time-invariant. These attributes are dependent on the geometry of the machine tool, the milling cutter, and the workpiece. The compliance of the machine tool and the workpiece and the average tangential (K_t), and the average radial (K_r), specific cutting pressure constants are the primary machine/workpiece characteristics. Standardised impact testing procedures provide the coefficients for the compliance terms (Tables 1 and 2) while a linear regression analysis of experimental machining data plotted in log–log format estimates the cutting pressure constants as $K_t = 802.64(\bar{t}_c)^{-.195}$ (N/mm^2) and $K_r = 0.1113(\bar{t}_c)^{-.382}$ for down milling of 7075-T6 aluminium by a $2\frac{1}{2}$-axis vertical end milling machining with a 0.5 (inch) diameter end mill with 4 flutes and a helix angle of 30°.

Table 2. Second-order mass-spring-damper approximations.

	Spring (k_e)	Damper (b_e)	Mass (m_e)
X	1 005 039 621.44 (N/m)	10 880.27 (kg/s)	6.63 (kg)
Y	1 005 391 607.11 (N/m)	17 474.79 (kg/s)	3.87 (kg)

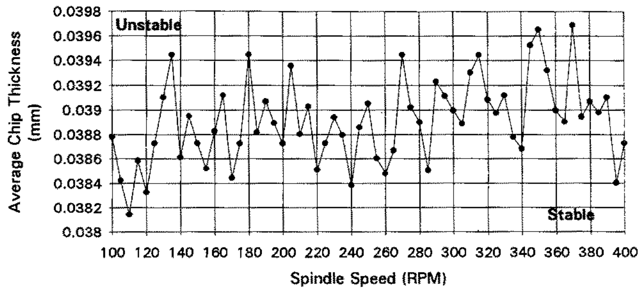

Fig. 3. Stability analysis: X-direction. (Down milling of 7075-T6 aluminium with a four-flute, 0.5 in. diameter, 30° helix angle end mill set at an RDOC of 0.0591 in. and an ADOC of 0.25 in.)

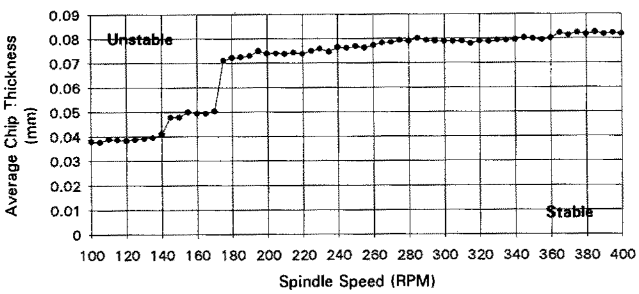

Fig. 4. Stability analysis: Y-direction. (Technical details as for Fig. 3.)

By incorporating the machine/workpiece constants (K_k, K_r, ζ, ω_d, k) into the characteristic equations, it is feasible to chart the stability of the end milling process as a function of the process variables. For both the X- and Y-directions, the stability analysis plots the spindle speed in revolutions per minute, against the average chip thickness in millimetres, based on the maximum feedrate that assures stable cutting (Figs 3 and 4). For complete stability of the machining process, both the X- and Y-directions must be stable. The stability analyses for total stability reveal that as the spindle speed increases the average chip thickness and the stability gradually increases and then suddenly decreases (Fig. 5). The interaction between the cutting dynamics and the machining system compliance accounts for this peculiar behaviour. At specific spindle speeds, the relative influence of the machining system compliance, as compared to the kinematics of cutting, dominates the regenerative chatter model and causes instability. In addition, a larger chip thickness generally occurs at higher spindle speeds. Therefore, faster spindle speeds allow for stable removal of more material and greater radial depths of cut.

Charting the stability is very beneficial to machinists. Either by selecting the spindle speed or the feedrate, machinists can estimate the chip thickness that corresponds to the maximum radial depth of cut or the maximum feed per tooth for chatter free cutting. This allows for machining optimisation and proper selection of the processing parameters for a given set of machine/workpiece characteristics.

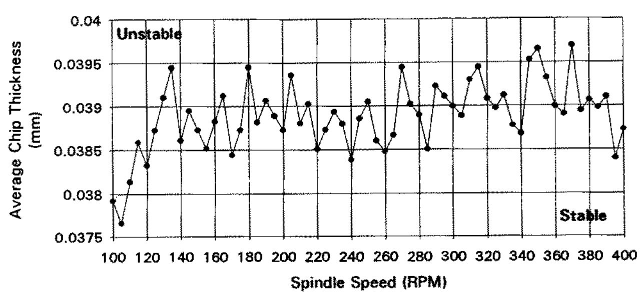

Fig. 5. Overall stability analysis. (Technical details as for Fig. 3.)

4. Model Verification

Four specifically designed experiments validate the regenerative chatter model and its assumptions:

1. Independence of the X- and Y-directions.
2. Step change in radial depth of cut.
3. Variations in the number of flutes on the cutter.
4. Augmentation of the feed per tooth.

Based on specific machining process parameters for each individual experiments, a combination of the stability analysis, the measured cutting forces, the machining vibrations, and the workpiece surface profile confirm the attributes and the analytical predictions of stable and unstable cutting. A statistical analysis on the first experiment confirms the independence and the decoupling of the primary cutting directions, the X- and Y-directions. Experiment 2 reveals that a smaller radial depth of cut provides a higher degree of stability. Experiment 3 indicates more stable machining with a larger number of flutes and cutting edges. As the feed per tooth increases, as seen in Experiment 4, the dynamic stability decreases (Fig. 6).

To provide a better understanding of the experimental procedure and methodology for the model verification, the results of Experiment 3 are briefly discussed. Experiment 3 examines how the total number of flutes on the end mill influences the dynamic stability of end milling. Experiment 3 machines similar test blocks of aluminium with both a four- and a three-flute end mill. A data acquisition system records the cutting forces and the vibrations during machining. In addition, a laser profilometer subsequently measures the surface roughness of the workpiece.

By analysing the recorded cutting forces, stable machining produces cutting forces that exhibit a periodic tooth signature

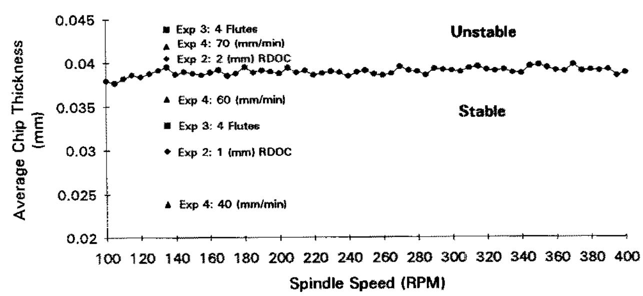

Fig. 6. Stability analysis: Experiments 1 to 4. (Technical details as for Fig. 3.)

and a constant tooth passing frequency. Instability affects the tooth signature and tends to generate an overall oscillation in the magnitude of the cutting forces proportional to the degree of instability. Dependent on the spindle speed and the number of cutting edges, the higher frequency tooth passing frequency often remains unaffected by the lower frequency vibrations induced by regenerative chatter. As charted by the accelerometers, stable cutting creates periodic impulses of constant amplitude and duration corresponding to the geometry of the individual cutting teeth. With dynamic instability, the accelerometers measure a higher degree of non-uniform disturbance with sporadic periods of relatively high and low vibrations. During chatter, the tool has a tendency to oscillate in and out of the workpiece. This relative movement influences the degree of vibration captured by the accelerometers. The inspection of a stable machined surface reveals a uniform removal of material and a characteristic surface profile. With a constant material removal rate and chip thickness, each individual tooth of a stable cutter removes a chip of comparable depth and width. Under unstable conditions, the period and depth of engagement of the cutter teeth fluctuate depending on the vibration of the cutter tip. As the cutter moves into or backs away from the workpiece, the cutter respectively removes a larger or smaller proportion of the workpiece.

Assuming identical machining parameters for both a three- and a four-flute cutter, the number of flutes on a cutter effects the average chip thickness by way of the feed per tooth. As the number of cutting edges increases, the feed per tooth reduces and yields a smaller average chip thickness. The average chip thickness for the four- and the three-flute cutters are 0.0328715 (mm) and 0.0439287 (mm), respectively. For this experiment, the addition of one flute reduces the average chip thickness by a third. It is evident that the total number of flutes possesses a strong interaction with the stability of the machining operation (Fig. 6).

Based on the chip thickness, the stability analysis predicts complete stability for the four-flute cutter and total instability for the three-flute cutter. By examining the cutting forces, the four-flute cutter demonstrates stable machining. The magnitude and the engagement of the individual cutter teeth repeat through the entire sampling period (Fig. 7). The tooth passing frequency and tooth signature are both constant. The cutting forces for the three-flute cutter reveal machine vibration (Fig. 8). The cutting forces are rather chaotic and of varying degrees of magnitude. The amplitude of the cutting forces is proportional to the displacement of the cutter tip. Large cutting forces result from vibrations that drive the cutter tip deeper into the workpiece. Shallow cuts require less energy and often occur when the cutter oscillates away from the workpieces. Regenerative chatter produces a fluctuation of the cutting forces that directly corresponds to the movement of the machine tool.

The machining vibrations are analogous to the cutting forces. For the four-flute cutter, as the spindle rotates, the teeth of the cutter engage the workpiece and generate a uniform series of pulsations (Fig. 9). With respect to the three-flute cutter, the vibration of the machine tool either magnifies or attenuates the vibration record (Fig. 10). Relative motion of the milling cutter can produce a lag in the impulse train and cancel any supplementary vibrations. However, synchronisation of the machine tool vibration and the cutter engagement augments the mechanical disturbance of the system.

The surface profile of the workpiece supports the forecast of chatter by the regenerative chatter model (Fig. 11). The

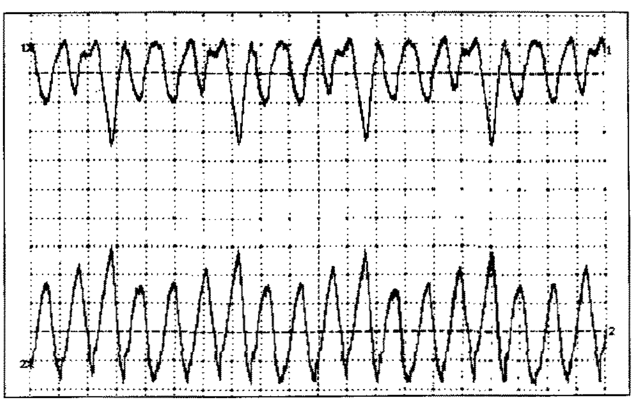

Fig. 7. Cutting forces: four-flute cutter. (Technical details as for Fig. 3; 135 r.p.m.; feed, 2.363 i.p.m.) 1, *X*-direction (100 ms/d); 2, *Y*-direction (1 V/d). 50 kHz sample rate.

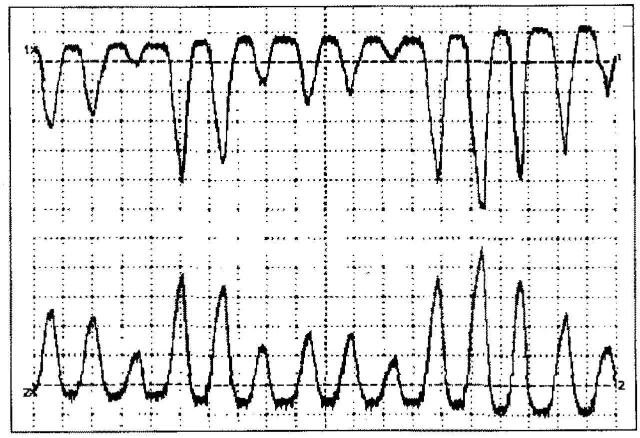

Fig. 8. Cutting forces: three-flute cutter. (Three-flute cutter, other technical details as for Fig. 7.)

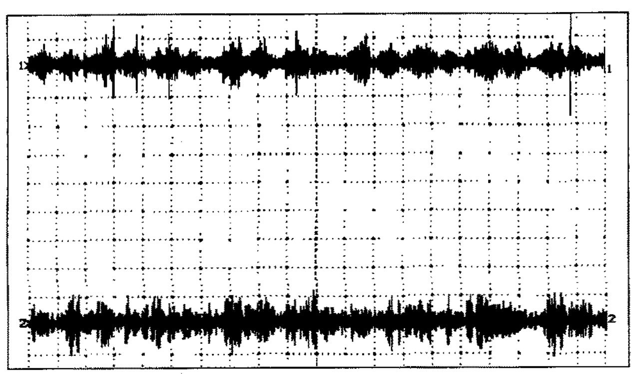

Fig. 9. Machining vibrations: four-flute cutter. (Technical details as for Fig. 7.)

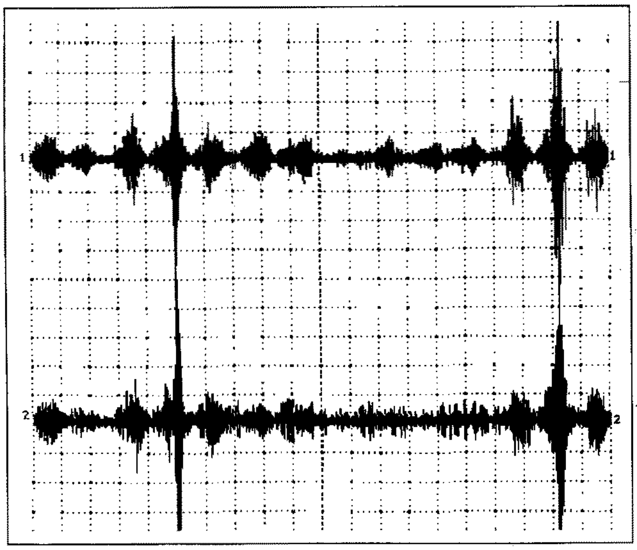

Fig. 10. Machining vibrations: Three-flute cutter. (Technical details as for Fig. 8.)

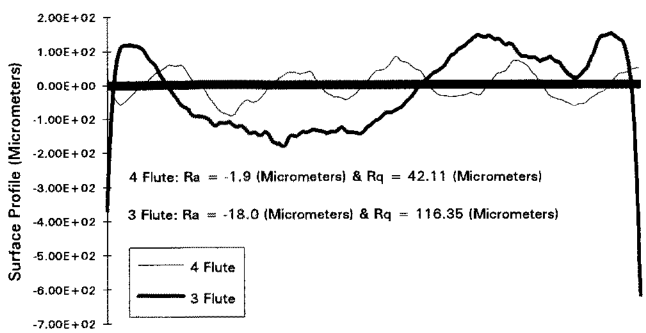

Fig. 11. Surface roughness: four- and three-flute cutter. 1000 point representation of 10 mm profile. (Down milling of 7075-T6 aluminium with a 0.5 in. diameter and 30° helix angle end mill set at an RDOC of 0.0591 in., an ADOC of 0.25 in., 135 r.p.m., and a feed of 2.363 i.p.m.)

surface finish machined by the four-flute cutter appears uniform in spacing and depth. Even allowing for minor discrepancies in geometry among the cutter teeth, the contour of the cut is highly repetitive. The surface profile shaped by the three-flute cutter depicts unstable cutting. Oscillation of the tool tip accounts for the waviness in the surface roughness. Considering a constant material removal rate, the milling cutter compensates for the machine vibration by adjusting the apparent depth of cut. Both delays in the engagement/disengagement of the cutter and variations in the depth of cut exist in the surface of the workpiece end milled with a three-flute cutter.

5. Applications to Control of Stability

The results of this research contribute to the area of totally unattended machining and real-time control. A solution of this nature facilitates machine tool design, machining parameter planning, and process control. In addition, the future application of design of experiments will expose the relationship and the significance of the model variables that activate and drive regenerative chatter. By utilising an intelligent CNC controller incorporating the regenerative chatter model, it becomes possible to pre-process the numerical machining instructions and predict whether chatter will arise. For example, Fig. 12 illustrates the tool path for the machining of an aluminium block. As the feed transverses the X-direction, the machining is stable for the given process parameters. However, for the same machining conditions, the cut becomes unstable as the cutter feeds in the Y-direction as a result of a larger radial depth of cut and a reduced stiffness of the machining system in the primary feed direction. Since the numerical code is susceptible to chatter, the controller can take corrective actions by altering the machining variables. For this example, a decrease in the feed ensures total stability in both the X- and Y-directions.

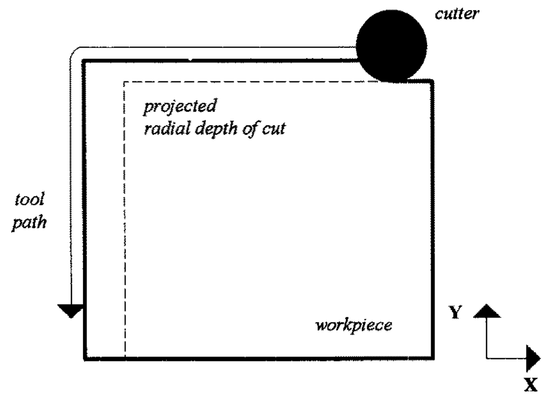

Fig. 12. CNC pre-processing example.

6. Conclusions

This research incorporates the total machining system compliance and the kinematics of multiflute end milling in the prediction and the identification of regenerative chatter. This work expresses the stability of the cutting system as a function of multiple variables in addition to the critical chip width. Regenerative chatter develops according to the machining process parameters, the tool geometry, the workpiece geometry, and the structure of both the machine tool and the workpiece. To confirm the validity of the regenerative chatter model, the experimental observations of the cutting forces, the machine vibration, and the surface profile of the four experiments support the analytical prediction of stability. Improving on earlier numerical, iterative, and integration techniques, the completed derivation provides a closed form symbolic equation for dynamic stability. A solution of this form explains the general behaviour of chatter while accomplishing an accurate stability analysis.

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Nomenclature

b	damping coefficient: mass-spring-damper representation
b_e	equivalent damping coefficient: mass-spring-damper representation
C	compliance element
CWD	chip with density function
D	diameter of cutter
d_a	axial depth of cut
d_r	radial depth of cut
$\bar{F}_{x,y}$	average total cutting force
K_r	radial specific cutting pressure constant
K_t	tangential specific cutting pressure constant
k	spring constant
k_e	equivalent spring constant
m	mass: mass-spring-damper representation
m_e	equivalent mass: mass-spring-damper representation
n	number of flutes on the cutter
$p_{x,y}$	elemental cutting forces
$P_{1,2}$	elemental cutting force functions
R	cutter radius
s	Laplace variable
TS	tooth sequencing function
$\underline{t}_c$	chip thickness
t_c	average chip thickness
t_x	feed per tooth
α	helix angle
Δx	actual displacement of cutter tip
δ	unit impulse function
ω_d	damped circular frequency of vibration
ζ	damping ratio
Ω	spindle speed