International Journal of Theoretical Physics, Vol. 10, No. 1 (1974), pp. 67-72

Measurement in Quantum Mechanics, and the EPR Paradox†

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Received: 17 *October* 1973

The problem of measurement in quantum mechanics was recognized very early to be qualitatively different from the corresponding problem in classical physics, Thus, von Neumann (von Neumann, 1932) devoted a whole chapter to this problem and in the forty odd years since, any number of authors have expressed their own views (D'Espagnat, 1971). It is probably reasonable to classify the various theories of measurement into four genera:

(1) The 'classical' or 'Copenhagen' approach (Bohr, 1934). This approach, espoused by Niels Bohr, has recently been reviewed in detail by Stapp, who connects this interpretation with the pragmatic philosophy of William James. This is, in fact, the orthodox view of quantum mechanics. The wave function 'describes the evolution of the probabilities of the actual things, not the evolution of the actual things themselves' (Stapp, 1971).

(2) The statistical interpretation (Ballentine, 1970). This approach rejects the quantum mechanical description of individual events, insisting that only the statistical average over many events has any meaning.

(3) The 'many worlds' theory (Everett, 1957). In this bizarre theory, the universe splits into a number of replicas of itself each time a measurement is performed. In each replica, the measured quantity takes on a different value. There are as many replicas as eigenvalues of the physical observable involved.

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† Based, in part, on an invited paper presented by the author before the annual meeting of the Virginia Academy of Sciences, Lexington, Virginia, May, 1972.

(4) The Wigner theory (Wigner, 1961, 1963, 1971). This theory is essentially a modification of the Copenhagen interpretation with the important distinction that a measurement is defined to have taken place only when an animate observer could in principle obtain the result. This same point of view seems also to be espoused by Feyman (Feyman, Leighton & Sands, 1964).

It is not my purpose in this note to describe any of these theories in detail; the reader is referred to the literature cited. I should point out, however, that the basic difficulty which has focused so much attention on the subject is the occurrence, in quantum mechanics, of coherent states and the fact that measurements destroy coherence. For calculation of cross sections, transition rates, or most 'practical' quantities, all of these theories give the same results. However, the question of coherent states seems to be so poorly understood that many physicists question the epistemology of the theory and much of the literature cited above describes attempts to study this.

The question 'Is the quantum mechanical description of nature complete?' was first raised in a famous paper (Einstein, Podolsky & Rosen, 1935). This 'EPR Paradox' has been discussed by many authors over the subsequent years (Bohr, 1935; Furry, 1936; Epstein, 1945; Bohm & Aharonov, 1957; Peres & Singer, 1960; Bohm, 1952; Bell, 1964; Breitenberger, 1965; Hooker, 1970, 1971; Krips, 1971; Sharp, 1961; Putnam, 1961), and is still the source of much controversy. (This paradox and two others, 'Wigner's Friend Paradox' and 'Schrödinger's Cat Paradox' are nicely reviewed (Jauch, 1968).) Our purpose in writing this note is to show that if one accepts the Wigner Theory of measurement, the EPR paradox can be avoided completely. Specifically, the Wigner Theory requires some sort of 'interaction' between the system being measured and the mind of the observer. This very interaction, as we shall see, can be used to resolve the EPR paradox in a non-paradoxical way. We are not advocating the existence of such an interaction, but rather are attempting to demonstrate how it does resolve the EPR paradox.

First, we briefly review the EPR paradox, the original version of which can be summarized as follows. Consider two systems, A and B, which interact for a certain finite length of time and then separate. (An example might be a scattering experiment involving short-range forces.) If we denote by $\Psi(x_A, x_B)$ the state of the system after the interaction; and if we denote by $\phi_i(x_a)$, $i = 1, 2, \ldots$ a complete set of state vectors for system A alone, then it is possible to expand

$$\Psi(x_A, x_B) = \sum_i \phi_i(x_A)\psi_i(x_B) \tag{1}$$

The state vectors $\phi_i(x_A)$ may be, in general, eigenstates of a linear self-adjoint operator Ω corresponding to some physical observable O:

$$\Omega\phi_i(x_A) = \omega_i\phi_i(x_A) \tag{2}$$

According to well-established principles of quantum mechanics (von Neumann, 1932), a measurement of Ω will yield one or the other of the eigenvalues ω_i

with a certain probability distribution. At the same time, the state vector representing system A will 'collapse' to the corresponding $\phi_i(x_A)$. This implies that $\psi(x_A, x_B)$ will collapse

$$\psi(x_A, x_B) \to \phi_i(x_A)\psi_i(x_B) \tag{3}$$

This result can be interpreted to mean that by performing a measurement on system A, a measurement on the totally separated system B has been performed, and that through some mysterious mechanism, the state vector representing system B has alos been 'collapsed' to $\psi_i(x_B)$.

It helps to give a concrete example (Bohm & Aharanov, 1957). They consider a molecule comprised of two spin 1/2 atoms in a ^1S state. They then suppose that the molecule can be adiabatically separated in a way which introduces no angular momentum into the system. Then the atoms are far apart, the state of the system will be represented according to equation (1) as (considering only spin variables)

$$\Psi(\sigma_A, \sigma_B) = \frac{1}{\sqrt{2}}\,[\Psi_{z+}(\sigma_A)\Psi_{z-}(\sigma_B) - \Psi_{z-}(\sigma_A)\Psi_{z+}(\sigma_B)] \tag{4}$$

where $\Psi_{z\pm}$ represents a state with z component of spin equal to $\pm\,\hbar/2$. We then pass atom A through, say, a Stern-Gerlach apparatus, and determine that its component along the z-axis, $\langle S_z\rangle_A$, is $+\,\hbar/2$. We thus conclude, without performing a measurement on B, that $\langle S_z\rangle_B = -\,\hbar/2$.

It is tempting to conclude that, if we refer to system B alone, it had to be represented *all along* (since we made no measurement on it) by the state vector $\Psi_{z-}(\sigma_B)$. However, the expansion, equation (4), is not unique. Let us expand $\Psi(\sigma_A, \sigma_B)$ in terms of eigenfunctions of S_x:

$$\Psi(\sigma_A, \sigma_B) = \frac{1}{\sqrt{2}}\,[\Psi_{x+}(\sigma_A)\Psi_{x-}(\sigma_B) - \Psi_{x-}(\sigma_A)\Psi_{x+}(\sigma_B)] \tag{5}$$

Now, by precisely the same argument as used in the preceding paragraph, we would conclude that system B had to be represented *all along* by the state vector $\Psi_{x-}(\sigma_B)$. But this contradicts the first statement, i.e., that it was represented by $\Psi_{z-}(\sigma_B)$ because S_x and S_z do not commute, and no physical system can simultaneously be in an eigenstate of both operators. This contradiction represents the essence of the paradox.

Since the experiment discussed above will probably never be performed, another experiment leading to the same 'paradox' was suggested (Bohm & Aharonov, 1957). The experiment involves the measurement of the polarization of X-rays emitted in the S-state annihilation of positronium. We recall that a photon has its spin either parallel (right circular polarization) or antiparallel (left circular polarization) to its direction of propagation. We describe these states by Ψ_R and Ψ_L, respectively.

A state of 'linear' polarization is simply a basic combination of Ψ_R and Ψ_L. Thus

$$\Psi_x = \frac{1}{\sqrt{2}}(\Psi_R + \Psi_L) \tag{6a}$$

$$\Psi_y = \frac{1}{\sqrt{2}}(\Psi_R - \Psi_L) \tag{6b}$$

where Ψ_x refers to x-polarized light, and Ψ_y to y-polarized. It is clear that a photon in a state of circular polarization is not in an eigenstate of linear polarization, and vice versa.

The use of polarized light to illustrate EPRP is as follows. Suppose positronium decays from a ^1S state into two photons. Conservation of linear momentum implies that the photons are emitted in opposite directions while conservation of angular momentum seems to imply that both photons have either right-circular polarization or both photons have left circular polarization. Thus a measurement of the circular polarization state of photon A tells us, without disturbing photon B, what its circular polarization state is. Thus, we might conclude that photon B had been in a certain state of circular polarization all along.

On the other hand, had we measured the axis of linear polarization of photon A, we could conclude that photon B had been in a pure state of linear polarization *all along*. Thus, photon B has *all along* been in two states which are mutually exclusive. The almost complete analog to the spin version of EPRP is obvious. This experiment was carried out, first by Wu and Shaknov (Wu & Shaknov, 1950) and more recently by Wu, Ullman and Kasday (Kasday, 1971); the first experiment measured only circular polarization but in the second linear polarization was measured (via Compton scattering). The results appear to be unequivocal: if photon A is in a state of right (resp. left) circular polarization, the photon B is always also in a state of right (resp. left) circular polarization; on the other hand, if the *linear* polarization of photon A is measured instead, then photon B is always in the same definite state of linear polarization. As Kasday puts it (Kasday, 1971), the photon 'decided in advance' the type of measurement which was to be performed on A, and adjusted its own state accordingly.

It is precisely this phenomenon which dismays so many physicists, and had led many authors to suggest explanation in terms of 'hidden variables' or the like (Bell, 1971). However, by now the experimental facts seem clear; hidden variable theories are *not* correct. One either believes that the second photon (or atom) 'decides in advance' the type of measurement to be performed the first, or concludes that all subsystems which have ever interacted in the part are forever correlated (as Feynman seems to believe) (Feynman, Leighton & Sands, 1964); or perhaps, that through unavoidable interaction with the rest of the universe, the correlation is preserved, as Sharp and Putnam indicate (Sharp & Putnam, 1961).

All of these resolutions of the EPR question seem unacceptable to us. It is the very basis of physics that experiments can be performed and results predicted, on 'isolated' systems; i.e., systems which are so far removed from interaction with other bodies—both spatially and temporally—that such interactions can be neglected. If Wigner's theory of measurement (item 4 above) is accepted, however, the paradox does not exist. Recall that in the Wigner theory the state vector collapses only after some human (or at least conscious) observer perceives, or could in principle perceive, the result of the measurement. A physicist might well express this theory in terms of an interaction potential between the measuring apparatus *and the mind of the observer.* It is in fact this potential which causes the state vector to 'collapse'. We might call this the 'Wigner potential'. (This suggests some sort of search for the quantum of this force field, or perhaps the association of some known particle with the quantum.)

At any rate, we are now in a position to understand EPR. After the experiment has been carried out on subsystem A, the observer must *still* be in a position to carry out the second experiment. This means the subsystem B must be in or near his apparatus, in a known position. If the observer has 'lost track' of this subsystem, for example, if it has passed out of his laboratory and merged with the rest of the world, he can never carry out the experiment. This means that the observer, throughout the entire time that he is carrying out the first experiment on A must be conscious of the trajectory of B. Thus, A and B are in fact interacting with each other, since each interacts, via the Wigner Potential, with the mind of the observer. This interaction causes the correlation which, experimentally, is known to exist. Incidentally, even if there are two separate observers making measurements on A and B, the above argument still holds, since there must be information passed between the observers to guarantee that they are measuring two subsystems of the original system, and not just a pair of random atoms.†

The task of expressing mathematically the evolution of the state vector still remains. One might proceed as follows. The state vector obeys the Schrödinger equation whenever one of the ordinary interactions (strong, electromagnetic, gravitational, weak) appears in the Hamiltonian. When the Wigner Potential appears, then the equation changes abruptly to the form $\Psi \to \Psi_i$, i.e., the state vector collapses to a particular eigenstate. It may even be possible to express *which* eigenstate, i.e., to reintroduce causality in the measuring process, through this potential, As mentioned above, it would be very nice if the quantum of the force could be identified.

In conclusion, we wish to reiterate that we are not suggesting that this description of Nature has any particular merit, although it is certainly no more bizarre than other theories which have been advocated, presumably seriously

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† Professor I. J. Good has suggested the folowing illustrative example. Suppose that there are two experimenters who have made prior arrangements to carry out appropriate experiments and suppose that the experiment involves two particles other than photons, so that they travel slower than light. After A has made his measurement, which he does quickly, he can transmit the result electromagnetically to B and thus B really might be able to predict the outcome of his part of the experiment with certainty. So the 'wave packet' is seen to have collapsed *before* B makes his measurement!

(Everett, 1957). We do wish to stress, however, that the Wigner theory of measurement does seem to do away with the EPR paradox, or better, makes that paradox merely a direct consequence of the paradoxical description of the measuring process itself.

Acknowledgements

The author wishes to acknowledge stimulating and, in many cases, heated discussion (among others) with P. Moldauer, D. Kaplan, F. Gürsey, J. Jacobs, E. Merzbacher, and J. McKnight. The author acknowledges the hospitality of the Rockefeller University, where some of this work was carried out.

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