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Two-Person Bargaining: An Experimental Test of the Nash Axioms

By R. V. NYDEGGER[1]), and G. OWEN, Houston[2])

Abstract: Tests were carried out on thirty pairs of subjects, using three different sets of conditions, for the purpose of experimentally validating NASH's axioms (and also, incidentally, certain other hypotheses of two-person bargaining). Under validation, it was found that subjects' responses conformed to both the *symmetry* and *independence of irrelevant alternatives* axioms. On the other hand, the axiom of *invariance under linear transformations of utility* was constantly violated. This may be due to the fact that subjects, whenever possible, try to effect an interpersonal comparison of utility.

Many writers have explored various facets of bargaining situations, and a variety of "fair" or realistic solutions have been proposed [HARSANYI, 1956; ISBELL, 1959; NASH, 1950]. Interestingly, most treatments have been of a mathematical and/or economic slant, and there has been little demonstrative experimentation in this area. Optimally, solutions are devised, tested and tuned, and then actually put to the task of demonstrating their worth. The purpose of the set of studies reported in this paper was to experimentally evaluate some of the basic axioms of bargaining behavior in the hope of comparing optimal solutions with *in vivo* performance. Obviously, the logic of a mathematically derived axiom is not undermined by the irrational performance of a subject in an experiment, but in order to generalize from abstract models to "real life" behavior, care must be taken to fully appreciate the variety of variables that may enter into and affect bargaining behavior.

These studies began with the assumption that in an abstract (mathematical) setting, each outcome is represented by two numbers u and v, the respective utilities of players I and II. Thus, the set of all outcomes can be represented by a set S in the (u,v) plane. The problem is then entirely determined by the set S, and by two numbers, u_0 and v_0 – the utilities which players I and II would obtain in the absence of cooperation.

It is generally assumed that set S is compact (closed and bounded) or, at the very least, bounded above, in the sense that the subset S^+ of S, consisting of all (u,v) which satisfy $u \geq u_0$, $v \geq v_0$, is bounded. Further, S is non-empty containing at minimum the point (u_0, v_0). Finally, S is assumed to be convex (this convexity is to be obtained by means of joint randomizations if necessary).

[1]) Professor R. V. NYDEGGER, Department of Psychology, Rice University, Houston, Texas 77001.

[2]) Professor G. OWEN, Department of Mathematical Sciences, Rice University, Houston, Texas 77001.

NASH [1950] suggests that a "reasonable" decision rule should assign to each triple (S, u_0, v_0), a pair of numbers

$$F(S, u_0, v_0) = (u^*, v^*)$$

which would satisfy the following intuitive axioms:

A1. $(u^*, v^*) \in S$ (Feasibility)
A2. $u^* \geq u_0$; $v^* \geq v_0$ (Individual Rationality)
A3. If there is $(u, v) \in S$, with $u \geq u^*$, $v \geq v^*$, then $(u, v) = (u^*, v^*)$ (Pareto Optimality)
A4. If S is such that $(u, v) \in S$ whenever $(v, u) \in S$, and $u_0 = v_0$, then $u^* = v^*$ (Symmetry)
A5. If $F(S, u_0, v_0) = (u^*, v^*)$, and T is a subset of S such that $(u^*, v^*) \in T$, then $F(T, u_0, v_0) = (u^*, v^*)$ (Independence of Irrelevant Alternatives)
A6. If (S, u_0, v_0) is transformed into (T, w_0, z_0) by linear transformations of the two players' utility scales, then the solution (w^*, z^*) is obtained from (u^*, v^*) by the same transformation (Invariance under Linear Transformations of Utility).

There is one and only one solution rule which satisfies the above axioms, and this rule chooses the point (in S^+) which maximizes the product $(u - u_0)(v - v_0)$ of the increments in utility.

Before undertaking the experimental investigation of these axioms, further discussion should be undertaken. Axiom 1 is so basic that it really needs little clarification, as it asserts that the solution point must actually be available to the players. As such, it is really more fundamental than an axiom, and needs no further justification. If the solution point were not available, then the bargaining could not proceed, or at least would not result in a negotiated outcome.

Axiom 2 states simply that no player will accept less through cooperation than he could obtain by himself (without cooperating). Actually, in real life many examples might be found whereby this axiom would not hold. However, it must be asserted that this axiom is a reasonable rule in most bargaining settings where utility functions are reasonably obvious. Apart from this argument, it can be easily demonstrated that A2 is a logical consequence of the following axioms, and in that sense it is redundant.

Axiom 3 is best described as an efficiency axiom, in that it states that at the solution point neither player can increase his utility without decreasing the other player's utility. The obvious situations where this axiom does not hold are those that allow for inefficient results because of: (1) inadequate or distorted communication, or (2) lack of information or misinformation regarding the other player's utilities. However, in a situation where full communication is allowed and in which full and accurate information is available regarding utilities of both players, the efficiency axiom seems most reasonable.

Axiom 4 would be best described as the "fairness" axiom for it allows that if each player has the same possibilities, then they should get equal outcomes. In "actual" bargaining situations this axiom is often difficult to demonstrate – not because it is faulty, but rather because of the confounding effect of extraneous variables such as differences in bargaining ability, differences in the initial distribution of power, etc. Suffice it to say, that in an abstract sense (all other things being equal), the fairness axiom is difficult to quarrel with. Thus, as an axiomatic basis for bargaining behavior it seems not only reasonable, but even a necessary assumption.

Axiom 5 (Independence of Irrelevant Alternatives) requires a somewhat more thorough discussion as it is a bit more vulnerable to attack than the preceding four axioms. This axiom states the basic intransitivity of utility functions in a general sense. That is, if alternative (u_1, v_1) is jointly preferred to (u_2, v_2), then the introduction of a third alternative (u_3, v_3) should not affect the preference of (u_1, v_1) over (u_2, v_2).

For example, in a single-player situation a customer might enter a restaurant where no menu was available, and the waiter then might offer him a choice of roast beef or steak. After deciding that he wants steak, the customer is then told that a third dish, shrimp, is also available, but the waiter had forgotten to mention it. Now it would seem that two rational choices become obvious. The customer can insist on having his steak, or by valuing the shrimp higher than the steak he can select shrimp. It would, however, seem unlikely that after being offered the shrimp (which he did not want), the customer would change his preference from steak to roast beef.

As straightforward as this might seem, by varying the scenario but slightly, the situation becomes less clearcut. In this case let us assume two people (players I and II) enter the same restaurant, and are given the same initial choice, between steak and roast beef. However, because both players belong to a "gourmet club" they must both select the same meal in order to get the special price their club membership allows them. Then we find that player I prefers steak and player II prefers roast beef. Now, they must either negotiate some outcome, or use a decision rule like a coin toss to determine the result. Then, suppose that a third choice (shrimp) is made available after the solution is reached. Superficially this alternative seems irrelevant, but suppose player I prefers steak over roast beef and roast beef over shrimp, while player II's preferences are an exact reversal. Then, while steak might easily have been the first negotiated outcome, the availability of shrimp would make the roast beef the most likely solution to the dilemma. In the second (two-player) case, this axiom must be looked at carefully, for by adding an additional player, the situation which seems so intuitively obvious in the single-person case, becomes much more complicated.

Axiom 6 (Invariance under Linear Transformations of Utility) states, in effect, that the units by which utility is measured are really irrelevant to the outcome of the bargaining. While this seems reasonable, it is even stronger than it might appear, for it allows for the possibility that each player's utility will be measured in different units. This in turn devolves on the more difficult question of *interpersonal comparison of utilities.* This axiom implies that such is impossible, that if both players benefit from an agreement, it is impossible to say which one derives a greater (subjective) benefit from the outcome. For example, one dollar to a bum might mean much more than a million dollars to a billionaire. There have been arguments in the literature pro and con on the interpersonal comparison of utilities [BRAITHWAITE, 1955; ISBELL, 1959; LUCE and RAIFFA, 1957]. Consequently, this axiom lacks the general support and intuitive strength of the others, but is nonetheless still enough of an issue to warrant further testing.

In addition to these axioms advanced by NASH, an alternative bargaining model is suggested by SMORODINSKY and KALAI [1973]. Keeping A1, A2, A3, A4, and A6 intact, this model replaces A5 with a "monotonicity" axiom.

A5′: Let $S \subset T$, and suppose u_m, the maximum value of u for points (u,v) in S, is also the maximum value of u for (u,v) in T. Let (u^*, v^*) and $(\bar{u}, \bar{v})$ be the solutions of the situations (S, u_0, v_0) and (T, u_0, v_0) respectively. Then, $\bar{v} \geq v^*$.

Essentially, A5′ states that if the problem is changed in such a way that player I's best alternative is not improved, but player II's capabilities are not made worse, then II should do at least as well in the modified problem T as in the original problem S. In this light, A5′ seems quite reasonable, although it does of course conflict with NASH's Axiom 5. It should also be noted that a somewhat stronger version of the monotonicity axiom, which seems almost as reasonable, turns out to be inconsistent with axioms A1, A2, and A3 (see OWEN [1968], p. 142). In any case, it can be shown that axioms A1, A2, A3, A4, A5′ and A6 lead to a unique solution rule as follows: let u_m and v_m be the maximal values of u and v, respectively for points (u,v) in S^+. Then the solution is the maximal point (u^*, v^*) of the form

$$u^* = u_0 + t(u_m - u_0)$$
$$v^* = v_0 + t(v_m - v_0)$$

in the set S. In other words $u_m - u_0$, and $v_m - v_0$ are the "best hopes" of gain for players I and II respectively; the actual gains $u^* - u_0$ and $v^* - v_0$ should be divided in this same ratio.

In considering these axioms, the authors considered Axioms A5 (or A5′) and A6 most subject to objection, and felt that experimental substantiation would provide additional information which might prove helpful. Additionally, some question arose regarding Axiom A4 (Symmetry), not because of its general suitability, but rather because of the concern regarding variability in individual cases.

Thus, with these axioms in mind, and especially A5 (or A5′), A6, and A4, three sets of studies were devised to experimentally investigate these propositions. Specifically several hypotheses were offered and tested:

1) two players given one dollar to divide, will divide it equally (this confirms NASH's Axioms A1, A2, A3, and A4);
2) two players given one dollar to divide, when one player may not make more than 60¢ will view the added constraint as irrelevant, and will still divide the money equally (this prediction is consistent with NASH's Axiom A5 [Independence of Irrelevant Alternatives] and is in conflict with the SMORODINSKY-KALAI Model, Axiom A5′);
3) two players given 60 poker chips to divide when the rate of exchange is 2¢/chip for Player I and 1¢/chip for Player II, will divide the chips evenly, accepting playoffs of 60¢ for Player I and 30¢ for Player II (this is consistent with NASH's Axiom A6 – Invariance under Linear Transformations of Utility).

Method

Subjects: The *S*s for this study were 60 undergraduate male students from a large urban university. These *S*s were recruited voluntarily, and paid one dollar plus winnings for participating. The *S*s were randomly assigned to one of three conditions, and participated in pairs.

Procedure: The *S*s were brought into the experimental room where they were paid one dollar, seated at a table across from one another, and given the instructions for their particular condition (a complete set of instructions is available in the appendix of this paper). These instructions simply told them that they could have an additional dollar if they could agree as to how it should be divided. However, each experimental condition imposed different constraints on the bargaining.

In Condition A the *S*s were told to divide one dollar in any way they wished, as long as they both agreed on the solution. By Axioms A1, A2, A3, and A4 (without need for A5 or A6), it was expected that *S*s would divide the dollar evenly (50¢ each), with the possibility of slight variation for individual cases.

In Condition B, *S*s were also invited to divide one dollar, but with the additional constraint that Player I could not receive more than 60¢ while Player II could receive $1 if he could persuade his partner to agree. Using Condition A as a control, Condition B suggested two possibilities depending upon wheter A5 or A5′ were accepted. NASH (A5) would hold that the added constraint (60¢ maximum for Player I) is irrelevant information, and would not affect the bargaining; under A5′, however (SMORODINSKY-KALAI Model) Player I would have a maximum gain of $1 while Player II could only get 60¢ as his maximum gain. Thus, this model would predict a division of money which would be consistent with this 3 to 5 (60¢/$1) ratio, and thus we would expect a division of 37.5¢ for I and 62.5¢ for II. It would at least be expected that a deviation from the 50 – 50 split would result in favor of Player II.

In Condition C, players were asked to divide poker chips which could later be cashed in, at the rate of 2¢/chip for Player I, and 1¢/chip for Player II. The effect was to create a situation in which money (utility) could be transferred between two players at a rate of 2 to 1 (i.e., Player I must yield 2¢ to increase Player II's profits by 1¢). This rate of exchange yielded a maximum gain of $1.20 for Player I and 60¢ for Player II. Upon examination it may be seen that the relevant set S here, is obtained from that in Condition A by means of a linear transformation of the player's utility scales.

Thus, with Condition A providing the baseline for comparison by serving as a control group, Axiom A6 would lead us to predict an equal division of chips, leading to a monetary payoff of 60¢ for Player I and 30¢ for Player II.

Following the instructions in each condition the *S*s were allowed to bargain and arrive at a solution. After arriving at a solution they were paid, debriefed, thanked, and dismissed. It should also be mentioned here that side payments were not allowed. In fact, they were not even mentioned in the instructions lest the *S*s be given a suggestion they might not otherwise think of. In no case did any *S* even bring up the idea of side payments.

Results

The outcome of this study is quite impressive if for no other reason than the consistency of its results. All ten pairs of *S*s in Condition A divided the money equally (50¢ apiece) which tends to lend significant credence to NASH's Axioms A1, A2, A3, A4. This also provided a most reasonable comparison group for the other conditions.

In Condition B, again all *S*s divided the money equally with no deviation from the 50 – 50 solution. This is an important finding as it supports NASH's Axiom A5 over SMORODINSKY-KALAI's model (Axiom A5′).

Finally, the ten pairs of *S*s in Condition C also divided the money equally, but not the chips. While it was predicted that the players would divide the chips evenly yielding a 60¢ outcome for Player I and 30¢ for Player II, they actually divided the chips in such a way that Player I received 20 chips, yielding a 40¢ gain, and Player II received 40 chips yielding a 40¢ gain. This finding which calls Axiom A6 into question is quite important for two reasons. First, *all* *S*s split the money the same way – there was no variation. Second, the instructions were as neutral as possible, and did not suggest any particular solution to the problem.

Discussion

On the basis of the results of this study, Axioms A1 – A5 do seem to be valid representations of actual behavior in bargaining situations. It must be noted however, that both players were totally aware of each other's capabilities, and presumably, of each other's preferences (i.e., people would rather have more money than less). Further, full and open communication was allowed, which certainly allowed for the support of A1 and A2. It is doubtlessly true that these factors also contributed to the validation of Axiom A3 (Pareto Optimality), which might well fail to hold under other circumstances. The validation of Axiom A5 (Independence of Irrelevant Alternatives) was not particularly surprising, but because Condition B allowed a direct test of A5 (NASH) against A5′ (SMORODINSKY-KALAI) the finding is quite noteworthy.

The validation of the symmetry axiom (A4) was not surprising, and was likely due to both strategic and psychological considerations. The notions of equity [ADAMS, 1965] and distributive justice [HOMANS, 1961] must be taken into account here. In any social situation there are norms which outline the appropriate forms of behavior for a given person in a given setting. In the absence of explicit cues from the environment regarding appropriate behavior, people tend to draw heavily upon normative considerations when deciding how to behave. In an abstract experimental setting that is highly ambiguous to a naive *S*, one can be quite certain that both *S*s begin to define the situation in terms of mutually held norms. One of the most pervasive norms in our society (at least idealogically) is that of fairness and distributive justice. Two strangers who are negotiating in a situation where their capabilities are equal or quite similar are most likely going to decide upon a solution which allows for symmetry.

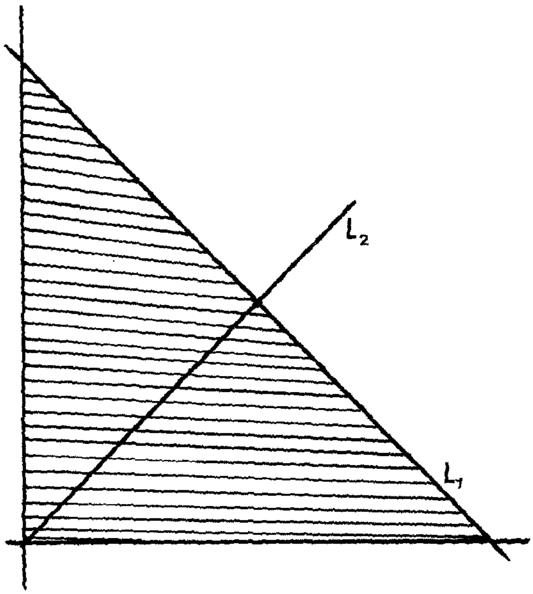
The somewhat surprising failure of Axiom A6 (Invariance under Linear Transformations of Utility) to hold up under the test of experimental validation, when coupled with the validation of A1 – A5 must be examined somewhat more closely here. The problem seems to lie with the assumption that interpersonal comparison of utility is meaningless. Upon examination we find that the norm of equity probably plays a part in Condition C just as it did in the other conditions.

Generally speaking, two principles seem to underlie most bargaining procedures: *efficiency* and *equity*. These can be defined as follows:

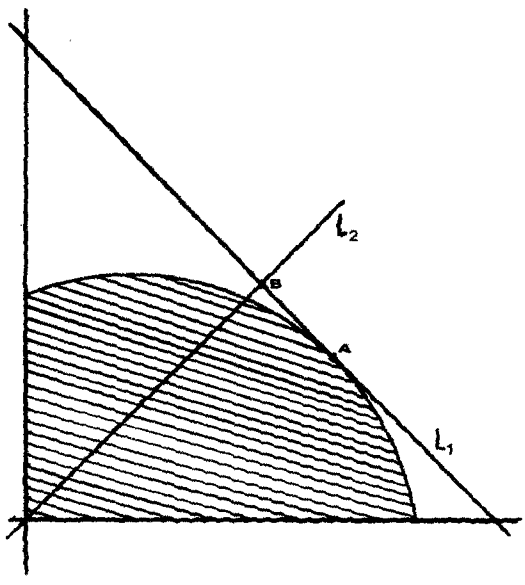
> *Efficiency:* An efficient solution is one where the solution point (u, v) maximizes the joint outcome $(u + v)$ of the two players.
>
> *Equity:* An equitable outcome is one where the solution point (u, v) is such that there is equal utility for both players; i.e., $u = v$.

Now in Condition A the *efficient set* consists of all points on the line $u + v = 100$, while the *equitable set* consists of all points on the line $u = v$. As can be seen in Figure 1 the intersection of these two lines at the point (50, 50) is a solution which is both equitable and efficient.

In Condition B, the same basic situation was presented, but with the constraint that Player I could not make more than 60¢. However, this information was in fact irrelevant because the same solution point (50,50) was still available, and allowed for the joint maximization of efficiency $(u + v = 100)$ and equity $(u = v)$. Thus, NASH's Axiom 5 seems to fit the general behavior of *S*s in an experimental setting.



Getting back to Condition C and the failure of A6 let us consider a more general case, such as that shown in Figure 2. Efficiency dictates that a solution point on line l_1 be reached that would maximize the sum $u + v$. This means that point A is the only feasible solution that satisfies the principle of efficiency. Equity, on the other hand, requires a solution point on line l_2, of which point B is clearly the "best" choice. Now, it is obvious that the two principles yield mutually exclusive solutions, and one of the two must yield.



If we might agree that there is no interpersonal comparison of utilities, then we must also agree that efficiency requires that we maximize, not $u + v$ but rather some linear form $\alpha u + \beta v$, where α and β are arbitrary positive numbers. Similarly, equity requires the maximization of $\alpha u = \beta v$ rather than $u = v$. Mathematically, it can be proven that there is only one choice of α and β which will allow the simultaneous satisfaction of both principles. This is accomplished by the NASH point (i.e., the point given by NASH's axioms). In effect, this causes the two points, A and B, of Figure 2 to coincide. Traditionally, mathematicians have resolved this dilemma in such a way.

On the other hand, if we allow for the possibility that people do attempt an interpersonal comparison of utility, then the resolution described above is not available, and one of the two principles must yield: an efficient result cannot be equitable, nor an equitable one efficient. The *S*s in the present study opted for equity at the expense of efficiency, which was obtained, at least in their opinions, by an equal division of money.

In summary, it should be pointed out that this research does not fully substantiate or negate any of the axioms tested, but rather gives some indication of their applicability in a descriptive sense. The move from an abstract game to "real life" behavior is best taken in carefully chosen steps which allow for the full appreciation of the complexity of the relevant issues. This study is but a small step, but hopefully one that will lead to further research on these problems.

Appendix

Bargaining Instructions: Condition I

(Give each *S* $ 1.00)

"This dollar is yours to keep; regardless of what else you do you may have this dollar for being a subject in this study."

(Place $ 1.00 on table between them)

"Now, here is another dollar which you may have, but only if you both agree how you will divide it. In otherwords, you may split this dollar in any way you wish, but in order to be paid you must both agree how the dollar will be divided up. If you do not agree, then neither of you will get additional money, and you may keep the original dollar and leave.

Do you have any questions?

You may begin."

(When finished, pay each *S* and have them sign a receipt.)

(Record data)

"Thank you for participating. The purpose of this study is to evaluate how people make compromises under various situations. If you would like to have a copy of the results of the complete study, please leave your name and mailing address, and we will mail the results to you around the end of the summer. Also, we would really appreciate it if you did not discuss the particulars of this study for one month. The reason is that if someone knew beforehand what the study was about, it would bias his performance.

Thank you."

Bargaining Instructions: Condition II

(Give each *S* $ 1.00)

"This dollar is yours to keep; regardless of what else you do, you may have this dollar for being a subject in this study."

(Place $ 1.00 on the table between them.)

"Now, here is another dollar which you may have, but only if you can decide how to divide it up. The only constraint is that player B can *not* get more than 60 ¢. Player A can get any amount, but player B can get only 60 ¢ or less. In order to get any money you must both agree as to how the money will be divided, otherwise neither of you gets any money besides the original dollar you were each given.

Do you have any questions?

You may begin."

(When finished, pay each *S*, have them sign a receipt, and record the data.)

"Thank you for participating. The purpose of this study is to evaluate how people make compromises under various situations. If you would like to have a copy of the complete study, please leave your name and mailing address, and we will mail the results to you around the end of the summer. Also, we would really appreciate it if you did not discuss the particulars of this study with anyone else for one month. The reason is that if someone knew beforehand what the study was about, it would bias his performance.

Thank you."

Bargaining Instructions: Condition III

(Give each *S* $ 1.00)

"This dollar is yours to keep regardless of what else you do, you may have this dollar for being a subject in this study."

(Place 60 poker chips on the table between them.)

"Now, here are 60 poker chips which we want you to divide between you. These chips can be cashed in for money, but only if you both agree as to how the split is to be made. The only constraint is that player B's chips will be worth 2 ¢ each, and player A's chips will be worth 1 ¢ each. Remember, to be paid any money you must both agree how the chips are to be divided. However you divide them, if you both agree, player B will be given 2 ¢ for each chip he has, and player A will be given 1 ¢ for each chip he has.

Do you have any questions?

You may begin."

(When finished, pay each *S* and have them sign a receipt, and record the data.)

"Thank you for participating. The purpose of this study is to evaluate how people make compromises under various situations. If you would like to have a copy of the results of the complete study, please leave your name and mailing address, and we will mail the results to you around the end of the summer. Also we would really appreciate it if you did not discuss the particulars of this study with anyone else for one month.The reason is that if someone knew beforehand what the study was about, it would bias his performance.

Thank you."

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