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A Kinematic Model for Machine Tool Accuracy Characterisation

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A method for characterising machine tool accuracy is presented in this paper, using a compact quasi-static error model of a three-axis horizontal machining centre. Rigid-body kinematics and homogeneous coordinate transformation were used in this model. A metrology pallet was designed so that coefficents in the error model can be updated periodically. Ten measurement points were allocated on two perpendicular planes of the metrology pallet. One of the measurement planes was on the work table and the other was perpendicular to the table. Two different length touch trigger probes were required for error measurements. Establishing or updating the error model can be carried out in-process without disturbing the workpiece set-up. The proposed compact error model is an improvement over past work for a production environment because of its robustness and convenient calibration procedure. The error model was tested in two dimensions and a good error prediction capability was observed in warm machine states.

Keywords: Homogeneous coordinate transformation; Metrology pallet; Quasi-static error; Rigid-body kinematics

1. Introduction

The geometric error of a machined part is due to the machine tool structure and machining process. Error sources attributable to the machine tool structure can be classified as quasi-static errors and dynamic errors. Quasi-static errors are defined as "errors of relative position between the tool and workpiece that are varying slowly in time and are related to the structure of the machine tool itself" [1]. Quasi-static error sources include the geometric error of the machine tool structure, thermal effects of the machine tool, and static loading. It was reported that quasi-static error accounts for 70% of the total error attributable to the machine tool [2]. Lee [3] showed that temperature variation in a three-axis machining centre can be as much as 20°C and the resultant positioning error is up to 100 μm (0.004 in). Dynamic errors are caused by the spindle error, vibrations of the machine structure, and deflection under inertial forces. Dynamic errors can normally be controlled by proper damping, rigid structure, and adjustments of cutting parameters.

Machine tool structure accuracy can be improved by error avoidance or error compensation. In the error avoidance method, errors are reduced by eliminating the sources. This method is implemented by thoughtful design of the machine tool. Low expansion materials like invar or granite were proposed as machine tool structure material. Recently, polymer concrete was reported to exhibit better thermal stability than a cast iron structure. In other approaches, active heating [4], refrigeration cooling [5] and oil shower [1] were suggested. However, it is costly to implement these methods in a production environment. On the other hand, the error compensation method has a greater potential to improve machining accuracy at a reduced cost. In this approach, the quasi-static distortion of the machine tool workspace is calculated or measured for compensation purposes.

The error compensation method can be classified as a workspace approach and element approach. The workspace approach attempts to model the observed error as some deformation of an ideal workspace [6–11]. In this approach, however, the measurement of the distorted workspace is extremely time consuming because of a large amount of data collection. At the same time, adaptation to changing external or internal conditions is difficult. The element approach, on the other hand, addresses the distortion of individual machine tool elements. A number of attempts were made to relate the distortion of each element to the error in the workspace [12–16]. Recently, Ni and Wu [17] developed an on-line measurement system for machine tool geometric errors. Zhang et al. [18] presented a displacement method for machine geometry calibration. They also proposed a new scheme to calculate error components in a parametric form. Ferreira and Liu [19] proposed a scheme to correlate the element error to the workspace error and developed a method to measure the error. A metrology pallet and touch trigger probes were used for compensation. In this paper, Ferreira and Liu's model is further extended and a more rigorous error compensation scheme for a three-axis machining centre is presented [20].

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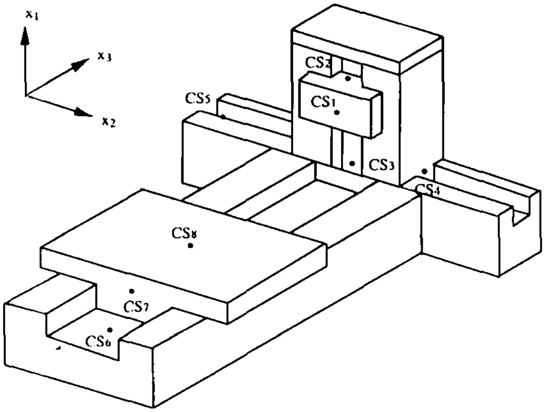

Fig. 1. Coordinate system of a three-axis machining centre.

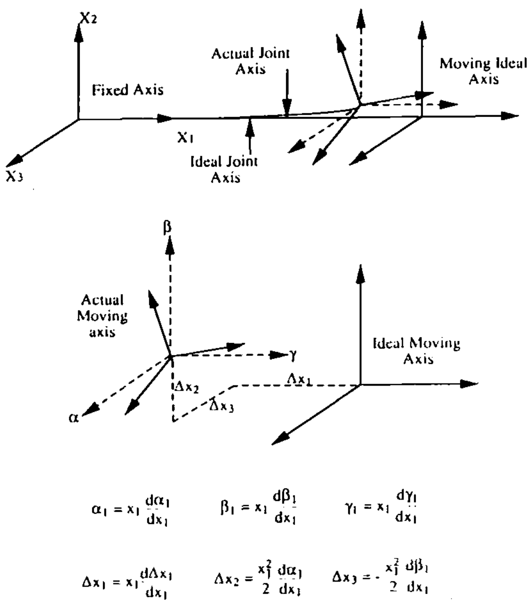

Fig. 2. Variation of angular and positioning errors of a joint.

2. Fundamentals of the Approach

A machining centre can be modelled as a kinematic chain with several rigid bodies (links) connected in series by prismatic joints [21]. One end of the chain is attached to the worktable, while the other is attached to the spindle. Fig. 1 shows a three-axis machining centre with eight different coordinate systems. There are three joints (CS2–CS3, CS4–CS5, and CS6–CS7) along the primary axes and four links (CS1–CS2, CS3–CS4, CS5–CS6, CS7–CS8). Using small angle approximations, the shape transformation of an inaccurate link, i, can be written as:

$$\mathbf{T}_i = \begin{bmatrix} 1 & -\alpha_i & \beta_i & a_i + \Delta a_i \\ \alpha_i & 1 & -\gamma_i & b_i + \Delta b_i \\ -\beta_i & \gamma_i & 1 & c_i + \Delta c_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where α_i, β_i, and γ_i are angular rotations characterising the angular (shape) error of the link; a_i, b_i and c_i are dimensions of the link in three axes; and Δa_i, Δb_i and Δc_i are dimensional errors along three axes.

In thisi work, variation of position, pitch, and yaw errors along an axis of a machining centre at different positions are approximated as a linear function of position x_1:

$$\alpha_1(x_1) = \alpha_1(0) + x_1 \frac{d\alpha_1}{dx_1}$$

$$\beta_1(x_1) = \beta_1(0) + x_1 \frac{d\beta_1}{dx_1}$$

$$\gamma_1(x_1) = \gamma_1(0) + x_1 \frac{d\gamma_1}{dx_1}$$

$$\Delta x_1 = x_1 \frac{d\Delta x_1}{dx_1}$$

where $d\alpha_1/dx_1$, $d\beta_1/dx_1$ and $d\gamma_1/dx_1$ are the constant rates of change of angular errors along the x_1-axis; and $d\Delta x_1/dx_1$ is the rate of accumulation of the positioning error (Fig. 2). When the coordinate frame is chosen to be tangential to the joint axis, then $\alpha_1(0) = 0$, $\beta_1(0) = 0$, and $\gamma_1(0) = 0$. Also, positioning errors in the x_2 and x_3 directions can be represented as

$$\Delta x_2 = \frac{x_1^2 d\alpha_1}{2\, dx_1}$$

$$\Delta x_3 = -\frac{x_1^2 d\beta_1}{2\, dx_1}$$

Then the following joint transformation is obtained for the x_1-axis:

$$\Phi_1 = \begin{bmatrix} 1 & -x_1 \frac{d\alpha_1}{dx_1} & x_1 \frac{d\beta_1}{dx_1} & x_1\left(1 + \frac{d\Delta x_1}{dx_1}\right) \\ x_1 \frac{d\alpha_1}{dx_1} & 1 & -x_1 \frac{d\gamma_1}{dx_1} & \frac{x_1^2 d\alpha_1}{2\, dx_1} \\ -x_1 \frac{d\beta_1}{dx_1} & x_1 \frac{d\gamma_1}{dx_1} & 1 & -\frac{x_1^2 d\beta_1}{2\, dx_1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, Φ_2 and Φ_3 can be expressed as

$$\Phi_2 = \begin{bmatrix} 1 & -x_2 \frac{d\alpha_2}{dx_2} & x_2 \frac{d\beta_2}{dx_2} & -\frac{x_2^2 d\alpha_2}{2\, dx_2} \\ x_2 \frac{d\alpha_2}{dx_2} & 1 & -x_2 \frac{d\gamma_2}{dx_2} & x_2\left(1 + \frac{d\Delta x_2}{dx_2}\right) \\ -x_2 \frac{d\beta_2}{dx_2} & x_2 \frac{d\gamma_2}{dx_2} & 1 & \frac{x_2^2 d\gamma_2}{2\, dx_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} 1 & -x_3\dfrac{d\alpha_3}{dx_3} & x_3\dfrac{d\beta_3}{dx_3} & \dfrac{x_3^2 d\beta_3}{2\,dx_3} \\ x_3\dfrac{d\alpha_3}{dx_3} & 1 & -x_3\dfrac{d\gamma_3}{dx_3} & -\dfrac{x_3^2 d\gamma_3}{2\,dx_3} \\ -x_3\dfrac{d\beta_3}{dx_3} & x_3\dfrac{d\gamma_3}{dx_3} & 1 & x_3\left(1+\dfrac{d\Delta x_3}{dx_3}\right) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By using the shape and joint transformation matrices described above, the relationship between the position vectors of a point in the spindle coordinate system and the worktable coordinate system of a 3-axis machining centre can be expressed as

$$[\bar{r}_{spindle}] = [T_1][\Phi_1][T_2][\Phi_2][T_3][\Phi_3][T_4][\bar{r}_{table}]$$

where $\bar{r}_{spindle}$ is the position vector of a point in the spindle coordinate system and $\bar{r}_{table}$ is the position vector of the same point in the table coordinate. T_i, $i = 1$ to 4, represents shape transformation from the spindle (i=1) to the work table (i=4), and Φ_j,$j = 1$ to 3, is the joint transformation of a joint, j, along x_j axis. By setting the origin of the spindle coordinate to be the point on the tool that is in contact with a workpiece ($\bar{r}_{spindle} = 0$) and closing the kinematic chain, the error vector at any point in the machine's workspace can be expressed as

$$-\bar{e} = \bar{p} + \mathbf{Q}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \mathbf{R}\begin{bmatrix} x_1 & x_2 \\ x_2 & x_3 \\ x_3 & x_1 \end{bmatrix} + \mathbf{S}\begin{bmatrix} x_1^2 \\ x_2^2 \\ x_3^2 \end{bmatrix} \qquad (1)$$

where

$$\bar{e} = \begin{bmatrix} e_{x_1} \\ e_{x_2} \\ e_{x_3} \end{bmatrix}$$

$$\bar{P} = \begin{bmatrix} \sum_{i=1}^{i=4} \Delta a_i \\ \sum_{i=1}^{i=4} \Delta b_i \\ \sum_{i=1}^{i=4} \Delta c_i \end{bmatrix} - \sum_{j=1}^{j=4} \begin{bmatrix} 0 & -\sum_{i=j}^{i=4} \alpha_i & \sum_{i=j}^{i=4} \beta_i \\ \sum_{i}^{i=4} \alpha_i & 0 & -\sum_{i=j}^{i=4} \gamma_i \\ -\sum_{i=j}^{i=4} \beta_i & \sum_{i=j}^{i=4} \gamma_i & 0 \end{bmatrix} \begin{bmatrix} a_j \\ b_j \\ c_j \end{bmatrix}$$

$$\mathbf{Q} = -\left[\begin{bmatrix} 0 & -\dfrac{d\alpha_i}{dx_i} & \dfrac{d\beta_i}{dx_i} \\ \dfrac{d\alpha_i}{dx_i} & 0 & -\dfrac{d\gamma_i}{dx_i} \\ -\dfrac{d\beta_i}{dx_i} & \dfrac{d\gamma_i}{dx_i} & 0 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{j=i} a_j \\ \sum_{j=1}^{j=i} b_j \\ \sum_{j=1}^{j=i} c_j \end{bmatrix}\right]_{i=1}^{i=3} - \begin{bmatrix} -\dfrac{d\Delta x_1}{dx_1} & -\sum_{i=3}^{i=4} \alpha_i & \beta_4 \\ \sum_{i=2}^{i=4} \alpha_i & -\dfrac{d\Delta x_2}{dx_2} & -\gamma_4 \\ -\sum_{i=2}^{i=4} \beta_i & \sum_{i=3}^{i=4} \gamma_i & -\dfrac{d\Delta x_3}{dx_3} \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 0 & \dfrac{d\alpha_3}{dx_3} & 0 \\ -\dfrac{d\alpha_2}{dx_2} & 0 & -\dfrac{d\alpha_3}{dx_3} \\ \dfrac{d\beta_2}{dx_2} & -\dfrac{d\gamma_3}{dx_3} & \dfrac{d\beta_3}{dx_3} \end{bmatrix}$$

$$\mathbf{S} = -\frac{1}{2}\begin{bmatrix} 0 & -\dfrac{d\alpha_2}{dx_2} & \dfrac{d\beta_3}{dx_3} \\ \dfrac{d\alpha_1}{dx_1} & 0 & -\dfrac{d\gamma_3}{dx_3} \\ -\dfrac{d\beta_1}{dx_1} & \dfrac{d\gamma_2}{dx_2} & 0 \end{bmatrix}$$

This is a quadratic equation which relates the error vector at any point in the workspace to its coordinates. The coefficient matrices/vectors consist of the machine constants (nominal dimension of the elements in the machine's kinematic chain) and the elemental errors (linear or angular error of individual elements).

3. Development of Compact Error Model

For the implementation of the model developed in the previous section, the following requirements must be satisfied:

1. The system should be robust enough to function on a shop-floor and all devices needed must be available on a shop-floor.
2. Since the values of the coefficients depend on the thermal state of the machine, frequent measurement cycles are required to update the model. The measurements must be made as unobtrusively as possible without wasting production time.

In order to develop a compact error model, 10 measuring points are selected at the periphery of the workspace of a horizontal machining centre (Fig. 3). Five points (P0, P5, P6, P7 and P8) are located on the worktable and the other 5 points are located on the plane perpendicular to the worktable. A metrology pallet consists of 6 rigid members connecting P0–P5, P0–P2, P2–P4, P4–P5, P5–P6 and P0–P7. A touch trigger probe is attached in the x_3-axis and approaching the measuring points from the positive x_3-direction. Then the metrology pallet can be mounted on the table without disturbing the workpiece. A short probe will measure the errors at 8 points (from P0 through P7) and a long probe will measure the errors at P8 and P9. ΔL in Fig. 3 is the difference in length of the probes. Constant error terms will

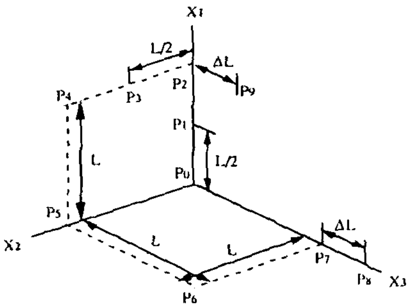

Fig. 3. Measurement positions in the workspace.

not affect the final accuracy of the workpiece because they represent *pure translation*. When P0 is used as a reference point, errors of the other 9 points can be calculated from equation (1):

Point 1

$$e_{x_1}^{(1)} = b_1 \frac{L}{2}\frac{d\alpha_1}{dx_1} - c_1 \frac{L}{2}\frac{d\beta_1}{dx_1} + \frac{L}{2}\frac{d\Delta x_1}{dx_1}$$

$$e_{x_2}^{(1)} = -\frac{L}{2}\alpha_2 - \frac{L}{2}(\alpha_3+\alpha_4) - \left[a_1\frac{L}{2} + \frac{L^2}{8}\right]\frac{d\alpha_1}{dx_1} + c_1\frac{L}{2}\frac{d\gamma_1}{dx_1}$$

$$e_{x_3}^{(1)} = \frac{L}{2}\beta_4 + \frac{L}{2}(\beta_2+\beta_3) + \left[a_1\frac{L}{2} + \frac{L^2}{8}\right]\frac{d\beta_1}{dx_1} - b_1\frac{L}{2}\frac{d\gamma_1}{dx_1}$$

Point 2

$$e_{x_1}^{(2)} = b_1 L\frac{d\alpha_1}{dx_1} - c_1 L\frac{d\beta_1}{dx_1} + L\frac{d\Delta x_1}{dx_1}$$

$$e_{x_2}^{(2)} = -L\alpha_2 - L(\alpha_3+\alpha_4) - \left[a_1 L + \frac{L^2}{2}\right]\frac{d\alpha_1}{dx_1} + c_1 L\frac{d\gamma_1}{dx_1}$$

$$e_{x_3}^{(2)} = L\beta_4 + L(\beta_2+\beta_3) + \left[a_1 L + \frac{L^2}{2}\right]\frac{d\beta_1}{dx_1} - b_1 L\frac{d\gamma_1}{dx_1}$$

Point 3

$$e_{x_1}^{(3)} = b_1 L\frac{d\alpha_1}{dx_1} - c_1 L\frac{d\beta_1}{dx_1} + L\frac{d\Delta x_1}{dx_1} + \frac{L}{2}(\alpha_3+\alpha_4) + \left[\left(\sum_1^2 b_i\right)\frac{L}{2} + \frac{L^2}{8}\right]\frac{d\alpha_2}{dx_2} - \left(\sum_1^2 c_i\right)\frac{L}{2}\frac{d\beta_2}{dx_2}$$

$$e_{x_2}^{(3)} = -L\alpha_2 - L(\alpha_3+\alpha_4) - \left[a_1 L + \frac{L^2}{2}\right]\frac{d\alpha_1}{dx_1} + c_1 L\frac{d\gamma_1}{dx_1} - \left(\sum_1^2 a_i\right)\frac{L}{2}\frac{d\alpha_2}{dx_2} + \left(\sum_1^2 c_i\right)\frac{L}{2}\frac{d\gamma_2}{dx_2} - \frac{L}{2}\frac{d\alpha_2}{dx_2} + \frac{L}{2}\frac{d\Delta x_2}{dx_2}$$

$$e_{x_3}^{(3)} = L\beta_4 + L(\beta_2+\beta_3) + \left[a_1 L + \frac{L^2}{2}\right]\frac{d\beta_1}{dx_1} - b_1 L\frac{d\gamma_1}{dx_1} - \frac{L}{2}\gamma_3 - \frac{L}{2}\gamma_4 - \left[\left(\sum_1^2 b_i\right)\frac{L}{2} + \frac{L^2}{8}\right]\frac{d\gamma_2}{dx_2} + \left[\left(\sum_1^2 a_i\right)\frac{L}{2} + \frac{L^2}{2}\right]\frac{d\beta_2}{dx_2}$$

Point 4

$$e_{x_1}^{(4)} = b_1 L\frac{d\alpha_1}{dx_1} - c_1 L\frac{d\beta_1}{dx_1} + L\frac{d\Delta x_1}{dx_1} + L(\alpha_3+\alpha_4) + \left[\left(\sum_1^2 b_i\right)L + \frac{L^2}{2}\right]\frac{d\alpha_2}{dx_2} - \left(\sum_1^2 c_i\right)L\frac{d\beta_2}{dx_2}$$

$$e_{x_2}^{(4)} = -L\alpha_2 - L(\alpha_3+\alpha_4) - \left[a_1 L + \frac{L^2}{2}\right]\frac{d\alpha_1}{dx_1} + c_1 L\frac{d\gamma_1}{dx_1} - \left(\sum_1^2 a_i\right)L\frac{d\alpha_2}{dx_2} + \left(\sum_1^2 c_i\right)L\frac{d\gamma_2}{dx_2} - L^2\frac{d\alpha_2}{dx_2} + L\frac{d\Delta x_2}{dx_2}$$

$$e_{x_3}^{(4)} = L\beta_4 + L(\beta_2+\beta_3) + \left[a_1 L + \frac{L^2}{2}\right]\frac{d\beta_1}{dx_1} - b_1 L\frac{d\gamma_1}{dx_1} - L\gamma_3 - L\gamma_4 - \left[\left(\sum_1^2 b_i\right)L + \frac{L^2}{2}\right]\frac{d\gamma_2}{dx_2} + \left(\sum_1^2 a_i\right)L\frac{d\beta_2}{dx_2} + L^2\frac{d\beta_2}{dx_2}$$

Point 5

$$e_{x_1}^{(5)} = L(\alpha_3+\alpha_4) + \left[\left(\sum_1^2 b_i\right)L + \frac{L^2}{2}\right]\frac{d\alpha_2}{dx_2} - \left(\sum_1^2 c_i\right)L\frac{d\beta_2}{dx_2}$$

$$e_{x_2}^{(5)} = -\left(\sum_1^2 a_i\right)L\frac{d\alpha_2}{dx_2} + \left(\sum_1^2 c_i\right)L\frac{d\gamma_2}{dx_2} + L\frac{d\Delta x_2}{dx_2}$$

$$e_{x_3}^{(5)} = -L\gamma_3 - L\gamma_4 - \left[\left(\sum_1^2 b_i\right)L + \frac{L^2}{2}\right]\frac{d\gamma_2}{dx_2} + \left(\sum_1^2 a_i\right)L\frac{d\beta_2}{dx_2}$$

Point 6

$$e_{x_1}^{(6)} = L(\alpha_3+\alpha_4) + \left[\left(\sum_1^2 b_i\right)L + \frac{L^2}{2}\right]\frac{d\alpha_2}{dx_2} - \left(\sum_1^2 c_i\right)L\frac{d\beta_2}{dx_2} - L\beta_4 - \left[\left(\sum_1^3 c_i\right)L + \frac{L^2}{2}\right]\frac{d\beta_3}{dx_3} + \left[\left(\sum_1^3 b_i\right)L + L^2\right]\frac{d\alpha_3}{dx_3}$$

$$e_{x_2}^{(6)} = -\left(\sum_1^2 a_i\right)L\frac{d\alpha_2}{dx_2} + \left(\sum_1^2 c_i\right)L\frac{d\gamma_2}{dx_2} + L\frac{d\Delta x_2}{dx_2} + L\gamma_4 - \left(\sum_1^3 a_i\right)L\frac{d\alpha_3}{dx_3} + \left[\left(\sum_1^3 c_i\right)L + \frac{L^2}{2}\right]\frac{d\gamma_3}{dx_3}$$

$$e_{x_3}^{(6)} = -L\gamma_3 - L\gamma_4 - \left[\left(\sum_1^2 b_i\right)L + \frac{L^2}{2}\right]\frac{d\gamma_2}{dx_2} + \left(\sum_1^2 a_i\right)L\frac{d\beta_2}{dx_2} + \left(\sum_1^3 a_i\right)L\frac{d\beta_3}{dx_3}\left[\left(\sum_1^3 b_i\right)L + L^2\right]\frac{d\gamma_3}{dx_3} + L\frac{d\Delta x_3}{dx_3}$$

Point 7

$$e_{x_1}^{(7)} = -L\beta_4\left[\left(\sum_1^3 c_i\right)L + \frac{L^2}{2}\right]\frac{d\beta_3}{dx_3} + \left(\sum_1^3 b_i\right)L\frac{d\alpha_3}{dx_3}$$

$$e_{x_2}^{(7)} = L\gamma_4 + \left[\left(\sum_1^3 c_i\right)L + \frac{L^2}{2}\right]\frac{d\gamma_3}{dx_3} - \left(\sum_1^3 a_i\right)L\frac{d\alpha_3}{dx_3}$$

$$e_{x_3}^{(7)} = \left(\sum_1^3 a_i\right) L \frac{d\beta_3}{dx_3} - \left(\sum_1^3 b_i\right) L \frac{d\gamma_3}{dx_3} + L \frac{d\Delta x_3}{dx_3}$$

When the length of probe is changed by ΔL, following expressions are obtained:

Point 8

$$e_{x_1}^{(8)} = -L\beta_4 - \left[(c_1 - \Delta L) L + \left(\sum_2^3 c_i\right) L + \frac{L^2}{2}\right] \frac{d\beta_3}{dx_3} + \left(\sum_1^3 b_i\right) L \frac{d\alpha_3}{dx_3}$$

Point 9

$$e_{x_2}^{(9)} = -L\alpha_2 - L(\alpha_3 + \alpha_4) - \left[a_1 L + \frac{L^2}{2}\right] \frac{d\alpha_1}{dx_1} + (c_1 - \Delta L) L \frac{d\gamma_1}{dx_1}$$

From these equations, the coefficients of equation (1) can be found:

$$\frac{d\alpha_1}{dx_1} = \frac{8e_{x_2}^{(1)} - 4e_{x_2}^{(2)}}{L^2} \quad (2)$$

$$\frac{d\alpha_2}{dx_2} = \frac{(e_{x_2}^{(2)} + e_{x_2}^{(5)}) - e_{x_2}^{(4)}}{L^2} \quad (3)$$

$$\frac{d\alpha_3}{dx_3} = \frac{-(e_{x_1}^{(5)} + e_{x_1}^{(7)}) + e_{x_1}^{(6)}}{L^2} \quad (4)$$

$$\frac{d\beta_1}{dx_1} = \frac{4e_{x_3}^{(2)} - 8e_{x_3}^{(1)}}{L^2} \quad (5)$$

$$\frac{d\beta_2}{dx_2} = \frac{-(e_{x_3}^{(2)} + e_{x_3}^{(5)}) + e_{x_3}^{(4)}}{L^2} \quad (6)$$

$$\frac{d\beta_3}{dx_3} = \frac{e_{x_1}^{(8)} - e_{x_1}^{(7)}}{L\,\Delta L} \quad (7)$$

$$\frac{d\gamma_1}{dx_1} = \frac{e_{x_2}^{(2)} - e_{x_2}^{(9)}}{L\,\Delta L} \quad (8)$$

$$\frac{d\gamma_3}{dx_3} = \frac{(e_{x_3}^{(5)} + e_{x_3}^{(7)}) - e_{x_3}^{(6)}}{L^2} \quad (9)$$

$$\frac{d\gamma_2}{dx_2} = \frac{8}{L^2}\left[e_{x_3}^{(3)} - \frac{e_{x_3}^{(5)}}{2} - e_{x_3}^{(2)} - \frac{L^2}{2}\left[\frac{-(e_{x_3}^{(2)} + e_{x_3}^{(5)}) + e_{x_3}^{(4)}}{L^2}\right]\right] \quad (10)$$

$$\alpha_3 + \alpha_4 = \left(\sum_1^2 c_i\right)\left[\frac{-(e_{x_3}^{(5)} + e_{x_3}^{(2)}) + e_{x_3}^{(4)}}{L^2}\right] - \left(\sum_1^2 b_i + \frac{L}{2}\right)\left[\frac{(e_{x_2}^{(2)} + e_{x_2}^{(5)}) - e_{x_2}^{(4)}}{L^2}\right] + \frac{e_{x_1}^{(5)}}{L} \quad (11)$$

$$\alpha_2 = -\left(\sum_1^2 c_i\right)\left[\frac{-(e_{x_3}^{(5)} + e_{x_3}^{(2)}) + e_{x_3}^{(4)}}{L^2}\right] - \left[a_1 + \frac{L}{2}\right]\left[\frac{8e_{x_2}^{(1)} - 4e_{x_2}^{(2)}}{L^2}\right] - \frac{e_{x_2}^{(2)}}{L} - \frac{e_{x_1}^{(5)}}{L} + c_1\left[\frac{e_{x_2}^{(2)} - e_{x_2}^{(9)}}{L\,\Delta L}\right] + \left(\sum_1^2 b_i + \frac{L}{2}\right)\left[\frac{(e_{x_2}^{(5)} + e_{x_2}^{(2)}) - e_{x_2}^{(4)}}{L^2}\right] \quad (12)$$

$$\beta_4 = -\frac{e_{x_1}^{(7)}}{L} + \left(\sum_1^3 b_i\right)\left[\frac{-(e_{x_1}^{(5)} + e_{x_1}^{(7)}) + e_{x_1}^{(6)}}{L^2}\right] - \left(\sum_1^3 c_i + \frac{L}{2}\right)\left[\frac{e_{x_1}^{(8)} - e_{x_1}^{(7)}}{L\,\Delta L}\right] \quad (13)$$

$$\beta_2 + \beta_3 = \frac{e_{x_3}^{(2)}}{L} + \frac{e_{x_1}^{(7)}}{L} - \left[a_1 + \frac{L}{2}\right]\left[\frac{4e_{x_3}^{(2)} - 8e_{x_3}^{(1)}}{L^2}\right] + b_1\left[\frac{e_{x_2}^{(2)} - e_{x_2}^{(9)}}{L\,\Delta L}\right] - \left(\sum_1^3 b_i\right)\left[\frac{-(e_{x_1}^{(7)} + e_{x_1}^{(5)}) + e_{x_1}^{(6)}}{L^2}\right] + \left(\sum_1^3 c_i + \frac{L}{2}\right)\left[\frac{e_{x_1}^{(8)} - e_{x_1}^{(7)}}{L\,\Delta L}\right] \quad (14)$$

$$\gamma_4 = \frac{e_{x_2}^{(7)}}{L} - \left(\sum_1^3 c_i + \frac{L}{2}\right)\left[\frac{(e_{x_3}^{(5)} + e_{x_3}^{(7)}) - e_{x_3}^{(6)}}{L^2}\right] + \left(\sum_1^3 a_i\right)\left[\frac{-(e_{x_1}^{(5)} + e_{x_1}^{(7)}) + e_{x_1}^{(6)}}{L^2}\right] \quad (15)$$

$$\gamma_3 = -\left[\left(\sum_1^2 b_i\right) + \frac{L}{2}\right]\frac{8}{L^2}\left[e_{x_3}^{(3)} - \frac{e_{x_3}^{(5)}}{2} - e_{x_3}^{(2)} + \frac{L^2}{2}\left[\frac{-(e_{x_3}^{(2)} + e_{x_3}^{(5)}) - e_{x_3}^{(4)}}{L^2}\right]\right] - \left(\sum_1^3 a_i\right)\left[\frac{-(e_{x_1}^{(5)} + e_{x_1}^{(7)}) + e_{x_1}^{(6)}}{L^2}\right] + \left(\sum_1^3 c_i + \frac{L}{2}\right)\left[\frac{(e_{x_3}^{(5)} + e_{x_3}^{(7)}) - e_{x_3}^{(6)}}{L^2}\right] - \frac{e_{x_3}^{(5)}}{L} - \frac{e_{x_2}^{(7)}}{L} + \left(\sum_1^2 a_i\right)\left[\frac{-(e_{x_3}^{(2)} + e_{x_3}^{(5)}) - e_{x_3}^{(4)}}{L^2}\right] \quad (16)$$

$$\frac{d\Delta x_1}{dx_1} = \frac{e_{x_1}^{(2)}}{L} - b_1\left[\frac{8\,e_{x_2}^{(1)} - 4\,e_{x_2}^{(2)}}{L^2}\right] + c_1\left[\frac{4\,e_{x_3}^{(2)} - 8\,e_{x_3}^{(1)}}{L^2}\right] \quad (17)$$

$$\frac{d\Delta x_2}{dx_2} = -\left(\sum_1^2 c_i\right)\frac{8}{L^2}\left[e_{x_3}^{(3)} - \frac{e_{x_3}^{(5)}}{2} - e_{x_3}^{(2)} - \frac{L^2}{2}\left(\frac{-(e_{x_3}^{(2)} + e_{x_3}^{(5)}) - e_{x_3}^{(4)}}{L^2}\right)\right] + \frac{e_{x_2}^{(5)}}{L} + \left(\sum_1^2 a_i\right)\left[\frac{(e_{x_2}^{(5)} + e_{x_2}^{(2)}) - e_{x_2}^{(4)}}{L^2}\right] \quad (18)$$

$$\frac{d\Delta x_3}{dx_3} = \frac{e_{x_3}^{(7)}}{L} - \left(\sum_1^3 a_i\right)\left[\frac{e_{x_1}^{(8)} - e_{x_1}^{(7)}}{L\,\Delta L}\right] + \left(\sum_1^3 b_i\right)\left[\frac{(e_{x_3}^{(7)} + e_{x_3}^{(5)}) - e_{x_3}^{(6)}}{L^2}\right] \qquad (19)$$

The shape transformation coefficients $\alpha_3 + \alpha_4$ and $\beta_2 + \beta_3$ are grouped together because they always appear at the same time. Also, it is not necessary to evaluate the shape transformation coefficients α_1, β_1, γ_1 and γ_2 because they contribute only to a constant term (pure translation) and all translational contributions are incorporated in the P0 calibration.

From the above equations, all 18 coefficients of the error model (equation (1)) can be calculated by measuring three coordinate measurements of the calibration point P0 and 19 error components of 7 points (from P1 to P7) using a short touch trigger probe, and 2 error components of 2 points (P8 and P9) using a long probe.

4. Experimental Works and Discussion

The error model developed in the previous section was tested in the two-dimensional case. A three-axis horizontal machining centre (Cincinnati Milacron T-10) was used for the experimental work. The workspace is approximately a cube of 660 mm per side. The machine is rated at 10 hp at the spindle. The axes are controlled by individual stepper motors and the position feedback is obtained through a resolver. The resolution of the machine is rated as 2.5 μm (0.0001 in), and repeatability at 7.6 μm (0.0003 in) "under certain conditions". Two touch-trigger probes (100 mm length and 150 mm length Renishaw MP-3) were used for coordinate measurements. Repeatability of the long probe was 11 μm (0.00043 in) while that of the short probe was 6 μm (0.00024 in) [11].

A two-dimensional metrology pallet was made as shown in Fig. 4. A steel plate (12 in × 12 in × 1 in) was ground on all sides. Then twenty-five carbide inserts were mounted on the pallet using epoxy glue. Nominal dimensions of measurement points (A_1–A_{25}) are shown in Fig. 4. Actual coordinates of these points on the pallet were measured using a coordinate measuring machine (Brown & Sharpe Micro Validator Series). The resolution of the coordinate measuring machine (CMM) was rated as 2.5 μm (0.0001 in), and the repeatability as 3.8 μm (0.00015 in). Actual coordinates of the measurement points deviate from nominal coordinates within ± 1 mm (0.04 in). Alignment error of the carbide inserts were within ± 1°. Hence, additional positioning error due to misalignment of the carbide inserts is less than 3% of the positioning error itself.

The metrology pallet was mounted in the x_1x_2-plane (perpendicular to the worktable). Point A_5 was used as a reference (P0). The 100 mm length probe was used to measure machine coordinates for P0 (A_5), P1 (A_{15}), P2 (A_{25}), P3 (A_{23}), P4 (A_{21}), and P5 (A_1) while the 150 mm length probe was used for measuring P9 (A_{25}). The differences between the current probe reading and CMM reading of the measurement points on the pallet are current errors. Then coefficients of

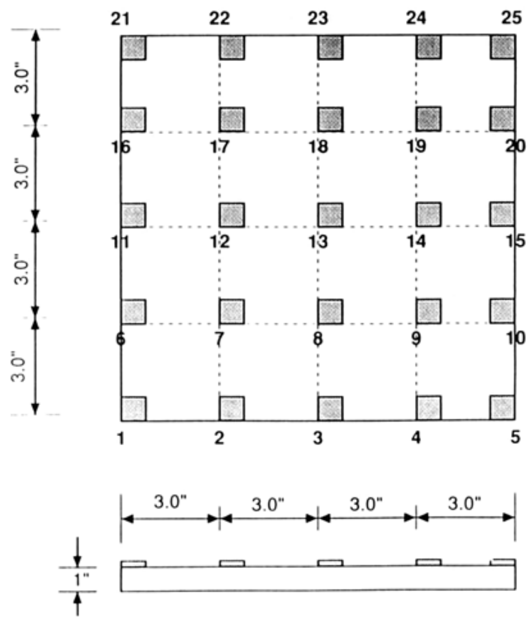

Fig. 4. Metrology pallet configuration.

the model can be calculated from equations (2)–(19). Errors at the intermediate points were also measured and compared with the predicted errors according to equation (1).

The previously described procedure was carried out twice for a cold machine. Then the machine was warmed up for 4 hours by moving all three axes continuously at 50 in/min and rotating the spindle at 4000 r.p.m. Error measurements were carried out again in the warm condition. Fig. 5 shows the measured and calculated errors along the diagonal of the metrology pallet (A_1, A_7, A_{13}, A_{19}, and A_{25}) in the cold machine condition. It can be seen that the repeatability of the whole system is approximately 25 μm (0.001 in). The inadequate error prediction in the cold state is partly due to insufficient accuracy of the metrology pallet and probe system and due to quadratic approximation of the error. However, it can be seen from Fig. 6 that thermal drift of the warm machine is over 100 μm (0.004 in). Using the current method, errors can be predicted within 25 μm (0.001 in). It can be seen that error terms higher than a quadratic function are relatively unimportant in quasi-static distortion of a machine tool.

Ferreira and Liu [19] have previously demonstrated the error prediction capability of the elemental approach. In their work, the positioning error, Δx_i, was assumed to be controllable and was eliminated from the error model. In the current work, however, a new metrology pallet was designed and the positioning error, Δx_i, was included in the coefficient calculation for equation (1). Furthermore, the new pallet was on two perpendicular planes (x_1x_2- and x_2x_3-plane). Hence, the metrology pallet can be mounted on the table without disturbing machining and the equipment required in the current work is easily available in the production environment. It is believed that the thermal drift prediction demonstrated

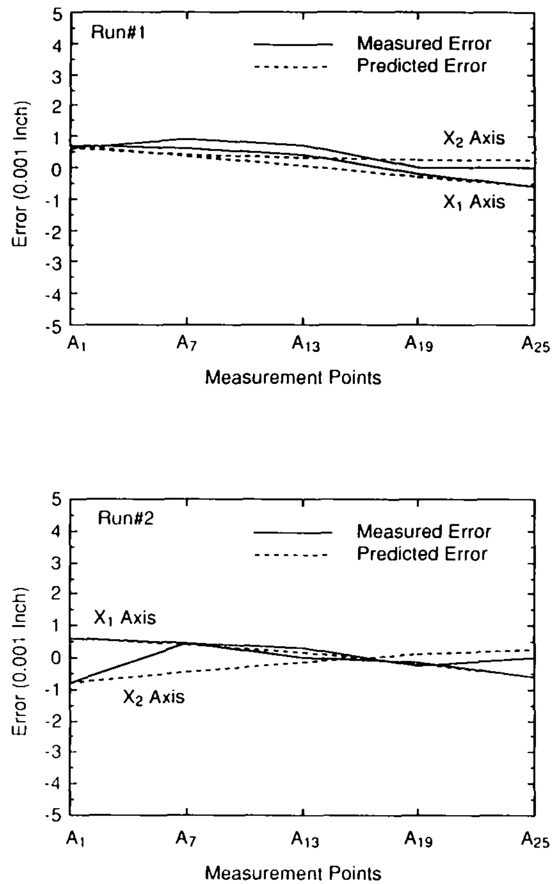

Fig. 5. Measured and predicted errors along a diagonal of a cold machine.

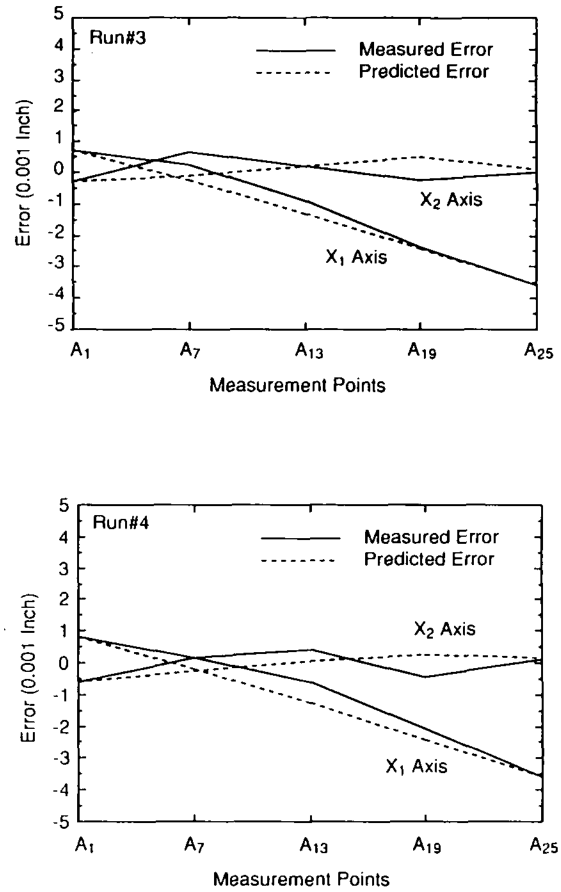

Fig. 6. Measured and predicted errors along a diagonal of a warm machine.

in this work can also be applied to machine tool distortion due to static load.

5. Conclusions

This paper presents a method for developing an error model which expresses the observed error in a 3-axis machining centre workspace as a function of its elemental errors. The contribution of each elemental error (i.e. coefficient of the error model) can be calculated by measuring 10 points of a pre-calibrated metrology pallet mounted on the work table. This methodology is robust and easy to update as thermal drift progresses. Hence, it is believed that the proposed method is applicable for the production environment. Also, the proposed error modelling methodology can provide valuable information for error avoidance because the contribution of each of the elemental errors to the workspace distortion can be estimated.

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