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A general optimization strategy for sound power minimization

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Abstract A general approach for minimizing radiated acoustic power of a baffled plate excited by broad band harmonic excitation is given. The steps involve a finite element discretization for expressing acoustic power and vibration analysis, analytical design sensitivity analysis, and the use of gradient-based optimization algorithms. Acoustic power expressions are derived from the Rayleigh integral for plates. A general methodology is developed for computing design sensitivities using analytical expressions. Results show that analytical sensitivity analysis is important from both computational time and accuracy considerations. Applications of the optimization strategy to rectangular plates and an engine cover plate are presented. Thicknesses are chosen as design variables.

1 Introduction

The design and control of structures that radiate minimum sound power is a subject of active research in structural acoustics. Such a strategy is typically approached by active vibration control. However, material tailoring and sizing using optimization techniques for complex structures is becoming more feasible due to advances in composites technology and in numerical methods for acoustics. The problem considered in this paper is the minimization of acoustic power radiated by plates which vibrate due to broad band harmonic excitation. The plate is modelled using finite elements and thicknesses of the finite elements are chosen as design variables. A general approach is presented involving the following steps.

1. The Rayleigh integral is discretized and acoustic power radiated by the plate is expressed as a quadratic form involving surface velocities and an impedance matrix.
2. The plate is modelled with finite elements and the eigenvalue problem is solved, followed by modal superposition, to obtain the velocities.
3. Analytical sensitivity coefficients of the acoustic power with respect to the design variables (plate thicknesses) is computed.
4. An optimization problem for power reduction due to broad band excitation is formulated and solved using a nonlinear programming technique.

The above approach may be viewed as a passive control strategy for minimizing noise radiation from vibrating structures. While noise reduction can also be achieved by active noise and/or vibration control, the extent of active control required can be greatly reduced if the design is first controlled passively. In the passive approach, sizes and shapes of the structural components and composite material tailoring are candidates for optimization. Early work on using optimization techniques and finite element models for noise reduction was done by Lang and Dym (1974) and Lalor (1979). Shape optimization has been shown to be effective by Bernard (1985) and Wilcox and Lalor (1987). More recent work in this area has been carried out by Lamancusa (1988), Sivakumar *et al.* (1991) and Naghshineh *et al.* (1992).

This paper presents a general approach for minimizing radiated power from vibrating structures due to single-frequency and broad band excitation. Emphasis is placed here on broad band excitation. A key feature of this work is the analytical calculation of design sensitivity coefficients for gradient-based optimization. Analytical sensitivity analysis dramatically reduces computation time and provides greater accuracy as compared to finite difference schemes, particularly in dynamic problems. Analytical expressions for power sensitivity for broad band requires eigenvector derivatives. They are computed using Nelson's method (Nelson 1976). Some of the earlier works on acoustic design sensitivity analysis was done by Smith and Bernhard (1989) and Cunefare and Koopmann (1992). These methods are based on the boundary element analysis and the approach is semi-analytical.

2 A finite element discretization for radiated power

The acoustic power radiated from flat plates in an infinite baffle is expressed in terms of the Rayleigh integral which uses the free field form of Green's function. Finite element discretization reduces the power expression to a quadratic form in terms of the nodal velocities of the plate. The development, a generalization of the approach applied to beams (Naghshineh *et al.* 1992), is described below.

Consider a baffled plate vibrating in out of plane motion and placed in a light fluid such as air. The pressure $p(\mathbf{r}'_s)$ at any observation point $\mathbf{r}'_s$ on the surface of the plate due to plate normal surface velocity $v(\mathbf{r}_s)$ at $\mathbf{r}_s$ can be written using the Rayleigh integral as

$$p(\mathbf{r}'_s) = \frac{i\omega\rho}{2\pi} \int_S v(\mathbf{r}_s) \frac{e^{-ikR}}{R} \, \mathrm{d}S \,, \tag{1}$$

where ρ is the density of air, ω is the frequency of plate vibration, c is the speed of sound in air, R is the distance between $\mathbf{r}'_s$ and $\mathbf{r}_s$, $k = \frac{\omega}{c}$ is the wave number, and $i = \sqrt{-1}$. The acoustic intensity at any point $\mathbf{r}'_s$ is written as

$$I(\mathbf{r}'_s) = \frac{1}{2} Re\{p(\mathbf{r}'_s) v^*(\mathbf{r}'_s)\} \,, \tag{2}$$

where v^* is the complex conjugate of v. Hence the acoustic power W radiated from the baffled plate can be obtained by integrating the intensity over the plate surface area as

$$W = \int_{S'} I(\mathbf{r}'_s)\, \mathrm{d}S' . \tag{3}$$

Using (1) and (2) in (3) and simplifying using the reciprocal property that $p(r_s)$ due to unit velocity at $\mathbf{r}'_s$ is equal to $p(\mathbf{r}'_s)$ due to unit velocity at $\mathbf{r}_s$, we have

$$W = \frac{1}{2} \int_{S'} \int_{S} v(\mathbf{r}_s) \frac{\omega\rho}{2\pi} \frac{\sin(kR)}{R} v^*(\mathbf{r}'_s)\, \mathrm{d}S\, \mathrm{d}S' . \tag{4}$$

Note that the singularity present in the pressure expression (1) at the point where source and receiver coincide has vanished in the power expression (4) since $\sin(kR)/R$ approaches k as R approaches zero. The power W is evaluated numerically using finite element discretization. In the present work, four noded quadrilateral elements are used in the mesh. The integral for the power is evaluated using Gaussian quadrature. The computational effort for this integral is proportional to the fourth power of the order of integration. Here, single point integration is used to reduce this effort. The accuracy of single point integration was compared with a method in which the order of integration was iteratively increased until convergence. The difference between the two integration results is about 1%. Thus, using single point integration, the following discretized power expression is obtained:

$$W = 4 \sum_{r=1}^{e} \sum_{s=1}^{e} v_r^T \frac{\omega\rho}{\pi} J_r J_s \frac{\sin(kR_{rs})}{R_{rs}} v_s^* , \tag{5}$$

where r and s indicate receiver and source elements, v_r and v_s are normal velocities and J_r, J_s are the values of the Jacobian at the element centre. Expressing v_r and v_s in terms of nodal values, we have

$$v_r(\xi, \eta) = \frac{1}{4} \sum_{i=1}^{4} N_i(\xi, \eta) V_i ,$$

where N_i are shape functions for the four noded quadrilateral element and V_i are the nodal values of the normal velocity. Hence

$$v_r(0,0) = \frac{1}{4} \sum_{I=1}^{4} V_I .$$

Thus power W can be written as

$$W = \frac{1}{2} \mathbf{V}^T \mathbf{B} \mathbf{V}^* , \tag{6}$$

where $\mathbf{V}$ is a global vector of dimension $N \times 1$, N = number of nodes. The matrix $\mathbf{B}$ is called the global impedance matrix and is a symmetric, positive definite real matrix. It is assembled from element matrices (for single point integration) as

$$\mathbf{b}_{rs} = \frac{\omega\rho}{2\pi} J_r J_s \frac{\sin(kR_{rs})}{R_{rs}} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} . \tag{7}$$

The rows of $\mathbf{b}_{rs}$ correspond to the node numbers of the receiver element r while the columns correspond to those of the source element s. This aspect needs to be considered in assembling the global matrix $\mathbf{B}$. The power in decibels is obtained from power in watts using the conversion formula,

$$W_{\mathrm{dB}} = 10.0 \log_{10} \frac{W_{\mathrm{watts}}}{10^{-12}} ,$$

where 10^{-12} watts is the reference.

3 Vibration analysis using finite elements

A four-node plate element (DKQ) has been implemented based on the discrete Kirchoff theory (Hughes 1987). Each node has three degrees of freedom - one out of plane translation and two rotations. The (12 × 12) element stiffness matrix $\mathbf{k}$ can be written as

$$\mathbf{k} = \frac{t^3}{12} \int_{-1}^{1} \int_{-1}^{1} \mathbf{B}^T \mathbf{D} \mathbf{B} \det \mathbf{J}\, \mathrm{d}\xi\, \mathrm{d}\eta , \tag{8}$$

where t = thickness of the element, $\mathbf{J}$ is the Jacobian matrix, $\mathbf{B}$ is a (3 × 12) strain-displacement matrix, $\mathbf{D}$ is a (3 × 3) material matrix, and ξ and η are natural coordinates. The integration of $\mathbf{k}$ is carried out numerically. The global stiffness matrix $\mathbf{K}$ is assembled from element $\mathbf{k}'s$ by taking into account the element connectivity. Similarly the global mass matrix $\mathbf{M}$ can be assembled from element mass matrices $\mathbf{m}$ given by

$$\mathbf{m} = \rho t \mathbf{m}_1 + \frac{\rho t^3}{12} \mathbf{m}_2 , \tag{9}$$

where $\mathbf{m}_1$ and $\mathbf{m}_2$ are (12 × 12) matrices which are independent of thickness.

Vibration analysis is carried out using modal superposition. Hysteritic proportional damping is assumed. The equations of motion for the structure is given by (Ewins 1984),

$$\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{K} + i\mathbf{H})\mathbf{x} = \mathbf{F} e^{i\overline{\omega} t} , \tag{10}$$

where $\mathbf{H} = \alpha\mathbf{K} + \beta\mathbf{M}$, $\overline{\omega}$ is the excitation frequency, $\mathbf{M}$ and $\mathbf{K}$ are the structural mass and stiffness matrices, α and β are the damping coefficients, x and $\ddot{x}$ are the displacement and acceleration vectors, and $\mathbf{F}$ is the vector representing the amplitude of the excitation force. The p-th component of $\mathbf{F}$ is denoted as A_p, which is a complex number. In (10) we first substitute for

$$\mathbf{x} = \mathbf{X} e^{i\overline{\omega} t} ,$$

where $\mathbf{X}$ is the amplitude of the response. This gives

$$(\mathbf{K} + i\mathbf{H} - \overline{\omega}^2 \mathbf{M})\mathbf{X} = \mathbf{F} . \tag{11}$$

Using modal superposition, we represent $\mathbf{X}$ as

$$\mathbf{X} = \sum_{j=1}^{m} y_j \mathbf{Q}^j , \tag{12}$$

where y_j is modal participation factor, m is the number of modes used and $\mathbf{Q}^j$ is the j-th eigenvector. The j-th resonant frequency ω_j, and mode shape $\mathbf{Q}^j$ are obtained from the eigenvalue problem $\mathbf{K}\mathbf{Q}^j = \omega_j^2 \mathbf{M}\mathbf{Q}^j$. Inverse iteration is used to solve the eigenvalue problem (Chandrupatla and Belegundu 1991). Substituting (12) in (11) gives

$$(\mathbf{K} + i\mathbf{H} - \overline{\omega}^2 \mathbf{M})\mathbf{Q}\mathbf{y} = \mathbf{F} .$$

Premultiplying the above equation by $\mathbf{Q}^T$ and using the property $\mathbf{Q}^T\mathbf{K}\mathbf{Q} = \mathrm{diag}(\omega_j^2)$ and $\mathbf{Q}^T\mathbf{M}\mathbf{Q} = \mathbf{I}$, we obtain the solution to the j-th equation as

$$y_j = \frac{\sum_{p=1}^{n} Q_{pj} A_p}{(1+i\alpha)\omega_j^2 + (i\beta - \overline{\omega}^2)}, \tag{13}$$

where n represents the degrees of freedom. The equations (12) and (13) are used to compute the displacement vector. The vector $\mathbf{X}$ is now introduced to represent the amplitude of the velocity vector and is given by

$$\dot{\mathbf{X}} = \sum_{j=1}^{m} i\overline{\omega} y_j \mathbf{Q}^j . \tag{14}$$

Components of the normal velocity vector $\mathbf{V}$ are extracted from (14) by ignoring the rotational velocity components since these produce motions that couple poorly with the bounding fluid. The vector $\mathbf{V}$ is then substituted into (4) to evaluate the radiated power. Above, the accuracy of the frequencies and mode shapes are dependent on the density of the finite element mesh. As a general rule, two elements per half-wavelength of the highest mode of interest is recommended.

4 Analytical sensitivity of power

A key step in solving the optimization problem is analytical design sensitivity analysis. Forward-difference schemes are prohibitively expensive owing to the large number of eigenvalue problems that must be solved for modal analysis. Also, the accuracy of finite difference formulae is poor because the eigenvalue analysis involves iteration. Analytical sensitivity coefficients $W_{,k} \equiv \mathrm{d}W/\mathrm{d}t_k$ = derivative of W with respect to the k-th design variable are obtained as follows. [For details the reader is referred to Salagame (1994), only basic concepts are summarized here for the sake of completeness.]

Differentiating (6) with respect to design variable b_k, we obtain

$$\frac{\mathrm{d}W}{\mathrm{d}b_k} = \mathbf{v}^T \mathbf{B} \frac{\partial \mathbf{v}^*}{\partial b_k} + \frac{1}{2} \mathbf{v}^T \frac{\partial \mathbf{B}}{\partial b_k} \mathbf{v}^* . \tag{15}$$

The power radiated from the structure depends on the excitation frequency. In most of the problems, the excitation frequency varies over a band including some of the natural frequencies of the structure. Hence the objective is to minimize the total power radiated from the structure over this band of frequencies.

Ideally, the total power in the band can be obtained by integrating the power versus frequency curve between frequencies of interest. However, the contribution to the total power is mainly from the power at resonant frequencies. Hence the total power in the band can be approximated as the sum of the power at each resonant frequency in the band. Specifically, let us consider a band consisting of three resonant frequencies. The total power in the band can be written as

$$W \approx W|_{\overline{\omega}=\omega_1} + W|_{\overline{\omega}=\omega_2} + W|_{\overline{\omega}=\omega_3} \equiv W_1 + W_2 + W_3 . \tag{16}$$

The task is to evaluate the sensitivity of W at a current design $\mathbf{b}^0$. Differentiating the above equation with respect to the k-th design variable, we obtain

$$\frac{\mathrm{d}W}{\mathrm{d}b_k} = \frac{\mathrm{d}W_1}{\mathrm{d}b_k} + \frac{\mathrm{d}W_2}{\mathrm{d}b_k} + \frac{\mathrm{d}W_3}{\mathrm{d}b_k}, \tag{17}$$

where

$$\left.\frac{\mathrm{d}W_i}{\mathrm{d}b_k}\right|_{b^0} = \lim_{\varepsilon \to 0} \frac{W|_{\overline{\omega}=\hat{\omega}_i} - W|_{\overline{\omega}=\omega_i(\mathbf{b}^0)}}{\varepsilon},$$

where $\hat{\omega}_i = \omega_i(b_1^0, b_2^0, \ldots, b_k^0 + \varepsilon, \ldots, b_N^0)$ and $\omega_i(\mathbf{b}^0)$ is the value of the first resonant frequency at the current design. We see that computing the sensitivity of the total power in the band requires computing the sensitivity $\frac{\mathrm{d}W_i}{\mathrm{d}b_k}$ = sensitivity of power at individual resonant frequencies, $\overline{\omega} = \omega_i$, $i = 1, 2, 3$ (or as many as in the band).

We are interested in computing $\frac{\mathrm{d}V}{\mathrm{d}b_k} = V_{,k}$, where V_j = velocity at node j at a known frequency $\overline{\omega}$. Consider (14), which expresses the velocity vector in terms of the mode shapes. Differentiating this equation with respect to design variable b_k yields

$$\dot{\mathbf{X}}_{,k} = \sum_{j=1}^{m} i\overline{\omega}(y_j \mathbf{Q}^j_{,k} + y_{j,k}\mathbf{Q}^j) + i\overline{\omega}_{,k} y_j \mathbf{Q}^j , \tag{18}$$

where $f_{,k} = \frac{\mathrm{d}f}{\mathrm{d}b_k}$. Since the sensitivities are evaluated at resonant frequencies for broad band excitation, $\overline{\omega}$ coincides with a resonant frequency. As the design changes, the resonant frequencies of the structure also change. Hence the term $\overline{\omega}_{,k}$ is not zero, but is the derivative of the eigenvalue with respect to thickness. The second term in the above equation involves the derivative of y_j, which can be obtained by differentiating (13). This derivative as well as the first term in the above equation involve derivatives of eigenvectors. Eigenvalue derivatives can be obtained from the generalized eigenvalue problem using Nelson's method (Nelson 1976).

The impedance matrix is a function of the excitation frequency. Hence, its derivative with respect to design variable b_k can be expressed as

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}b_k} = \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\overline{\omega}} \frac{\mathrm{d}\overline{\omega}}{\mathrm{d}b_k} . \tag{19}$$

The first derivative $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\overline{\omega}}$ can be computed by differentiating $\mathbf{b}_{rs}$ given by (7) with respect to $\overline{\omega}$. The element derivatives are then assembled to give $\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}\overline{\omega}}$. This assembly is similar to the assembly of $\mathbf{B}$ itself. However, it should be noted that as R approaches zero, $\sin(kR)/R$ approaches k and this fact should be used before differentiation. The second derivative is nothing but the eigenvalue derivative as discussed earlier.

4.1 Special case: fixed single frequency excitation

One special case of interest is when the structure is excited at a constant or fixed frequency, $\overline{\omega}$. There are two methods to evaluate $\frac{\mathrm{d}W}{\mathrm{d}b_k}$ at $\overline{\omega}$ in this special situation.

In the first method, we use the fact that $\overline{\omega}_{,k} = 0$. Hence the derivative of the impedance matrix with respect to a design variable is zero. Consequently, only the first term in (15) needs to be evaluated. The velocity derivative can be obtained by the method described above by substituting the value of the excitation frequency for $\overline{\omega}$. Another approach, which does not require eigenvector derivatives, is described by Salagame (1994).

5 Minimization of acoustic power

Acoustic power can be described in terms of the velocity amplitudes of the plate as discussed in Section 2. Hence the optimization problem is to

minimize $\quad W = \frac{1}{2} v^T B v^*$,

subject to:

weight $\leq W_0$, $\quad b_i^\ell \leq b_i \leq b_i^u$,

where b_i $i = 1, \ldots$, ndv are the ndv design variables. Our choice for design variables in this paper are element thicknesses. However, the formulation holds for other variables. The power W for the case of broad band is the total power in the band of frequencies which is approximated as the sum of powers at resonant frequencies, as discussed in detail in Section 4 [see (16)]. If the structure is excited at a single frequency, W is the power computed at the excitation frequency. We discuss the case of single frequency excitation first.

5.1 Single frequency harmonic excitation

The velocity v implicitly depends on thickness as described in (10)-(13) above. It is possible to associate groups of elements with a single design variable. However, the main difficulty is that the above problem has multiple local minima. The physical reason for this may be explained by the following argument. Let the resonance frequencies for a given design (or thickness distribution) be $(\omega_1, \omega_2, \omega_3, \ldots)$ Hz, and the excitation be at $\overline{\omega}$, $\omega_2 \leq \overline{\omega} \leq \omega_3$. Then, as the thickness of the plate is increased, the stiffness will increase and frequencies will consequently increase; the resonance frequencies ω_1 and ω_2 will "pass" by $\overline{\omega}$. At each pass, there will generally be a surge in radiated power. Thus, the graph of power vs thickness will have peaks and valleys, and then show a steady decline as the thickness continues to increase. This fact is illustrated for a rectangular plate in Fig. 1 where W is plotted vs t, the uniform thickness of the plate. The difficulty is that gradient-based optimization routines will only ensure a local minimum within the "valley" near the current design. This aspect has been discussed in greater length by Lamancusa (1993). In our work, analytical design sensitivity expressions have been derived for the derivatives of power with respect to the thicknesses. Optimization has been successfully applied to reduce power within the "valley". In the case when the force is applied at a single point on the plate, the optimized thickness distribution corresponds to a thickening of the plate at that location, as expected. The strategy for minimizing radiated power for broad band excitation is considered next.

5.2 Minimization of power within a band of frequencies

The challenge here is to find the optimum thickness distribution of a plate for minimum radiated acoustic power where the plate is excited over a band of frequencies. Firstly, if there are r resonance frequencies in a band, then the total power W within the band is assumed to be approximated by the sum of the powers at each frequency. To understand this, we must note that a variation in thickness will result in a variation in the resonance frequencies which will then result in a variation in the total power W. Unlike the single frequency excitation problem discussed above, the total power in (16) is a relatively smooth function of thickness. This can be explained as follows. A change in thickness will cause the resonance frequencies to shift. Thus, when viewing a graph of power vs thickness, there will be a shift in

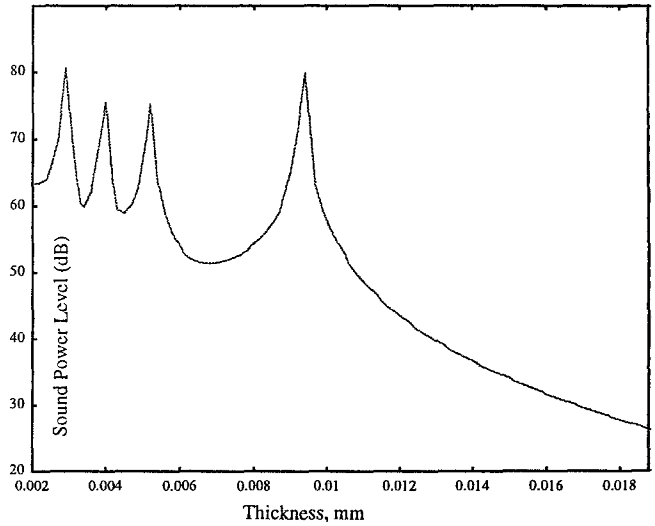

Fig. 1. Acoustic power as a function of thickness for fixed excitation frequency

the power peaks. However, since only the sum of the peaks in the power graph are being monitored, there is no sudden change in total power. Figure 2 illustrates this observation that W in (16) is a relatively smooth function of design for a rectangular plate, where $W = W_1 + W_2 + W_3$ is plotted vs t, t = thickness of the uniform plate. Compare this figure with Fig. 1 for single frequency excitation.

The sensitivities of the total power with respect to thickness variables are computed using the analytical expressions. These derivatives are then used in a nonlinear optimization program, which gives the optimum thickness distribution of the plate that minimizes W.

A final remark regarding the above problem is in order. The power W that is being minimized is defined as the power contributed from the first r resonance frequencies or modes. The values of these resonance frequencies, however, change during the thickness optimization. Thus, the band is not defined in terms of fixed frequency limits. Minimization of W within such fixed limits can be achieved although this requires a change in the formulation of the problem. Constraints on frequencies will have to be imposed and care must be taken since entering or exiting modes may cause non-differentiability of the power function. That is, the smoothness of the power vs thickness graph in Fig. 2 is not ensured if modes are allowed to enter or exit in the summation in (16). This is a topic that needs further study.

6 Numerical examples

6.1 Problem 1: point driven rectangular plate

The problem consists of a rectangular, isotropic, steel plate of dimensions 546 mm × 682 mm × 5 mm excited by a single force at its quarter point as shown in Fig. 3. The plate is clamped on all its edges and has damping assumed to be proportional with $\alpha = 0.1$ and $\beta = 0.0$. The finite element model consists of 81 nodes and 64 quadratic plate elements. A 1 N force is applied at the quarter-point of the plate, located at $(x, y) = (136.5, 160.5)$ mm. The amplitude of excitation force is set to 1 N. Finite element analysis gives surface normal

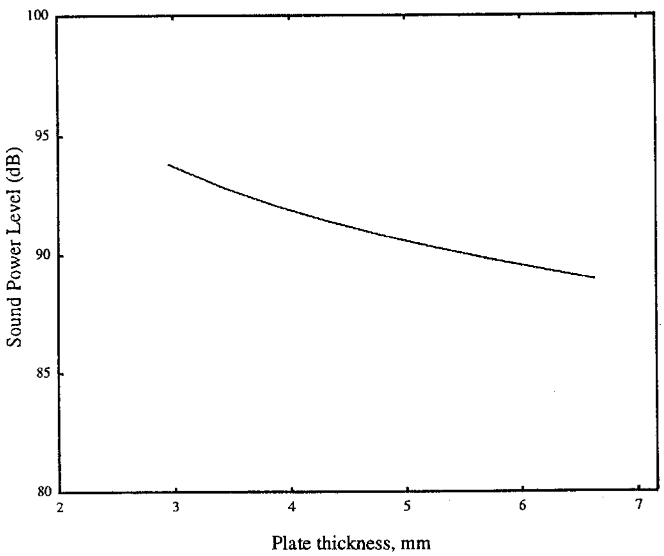

Fig. 2. Total power contributed from the first three modes as a function of thickness

velocities and acoustic power is determined using (7). The thickness of each element is the design variables. From finite element analysis, the first 3 natural frequencies of the plate are 123 Hz, 222 Hz and 293 Hz, respectively.

6.1.1 Case I: single frequency excitation

In this example, the element thicknesses have been grouped as indicated by the numbers in Fig. 3. Thus there is a total of 16 design variables. The plate is excited at $\overline{\omega} = 230$ Hz. The sound power at this frequency is equal to 90.2 dB. The optimum thickness distribution for the plate is shown in Fig. 4. It can be clearly observed that the mass is lumped around the point of excitation to reduce the sound power. The optimum sound power value is 84.5 dB.

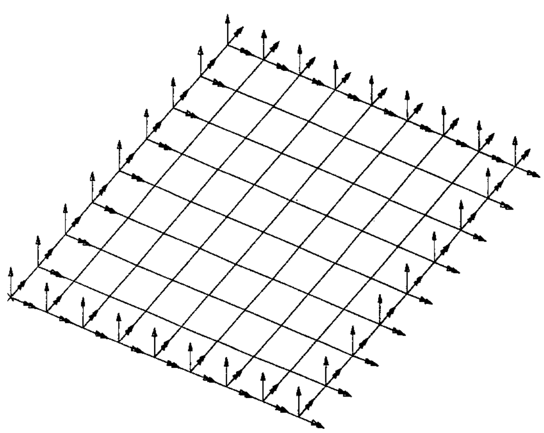

Fig. 3. Rectangular plate clamped on its edges

6.1.2 Case II: broad band excitation

For this problem, the power in the band up to the first three natural frequencies is considered. The sound power at these points are 87.6 dB, 83.7 dB, 84.1 dB, respectively. The total power $W = W_1 + W_2 + W_3$ is 90.3 dB. Note that the total power is obtained by adding the individual powers in watt units and then converting to decibels. The optimization problem is to minimize total power subject to a limit on

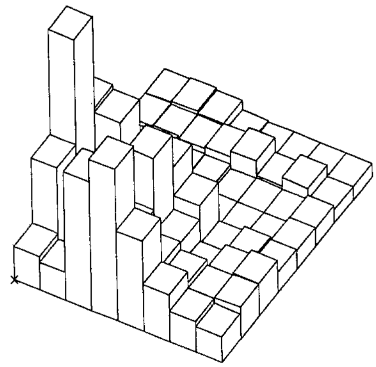

Fig. 4. Optimized rectangular plate for single frequency excitation (magnified)

maximum to minimum thickness ratio of 5.0.

The optimized thickness distribution is shown in Fig. 5. At optimum, the first three resonance frequencies are 176 Hz, 290 Hz and 387 Hz, with the power being 69.1 dB. Figure 6 shows the power versus frequency for three designs: the initial (uniform) design (weight = 14.6 kg), the optimized design (weight = 21.1 kg), and a uniform-thickness design for a plate having the same weight as the optimized design. We see that the optimized design is still far better than the uniform thickness design, implying that pure weight increase has had little to do with the power reduction.

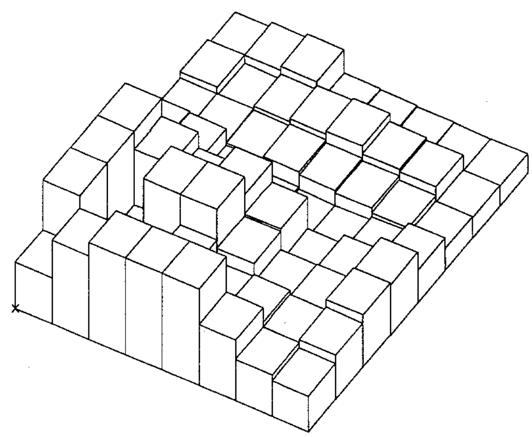

Fig. 5. Optimized rectangular plate for broad band (magnified)

6.2 Problem 2: broad band excitation of a Quad 4 engine timing chain cover plate

An engine cover plate made of steel is shown in Fig. 7. It is modelled using 80 quadrilateral finite elements. The damping coefficients are $\alpha = 0.01$, $\beta = 0$. Young's modulus = 2.068E11 N/m^2, density = 7820 kg/m^3. All boundary nodes of the cover plate are clamped with respect to rotations, but are supported on springs with stiffness equal to 105 N/m, i.e. very high stiffnesses such that the frequencies of the rigid body modes are well above the radiating plate modes of interest. The excitation forces correspond to one half of the boundary nodes subjected to a force of $F = 1\ N$ while the other half has $F = i\ N$, which is 90 deg. out of phase. The split is made along the Y-axis. The objective function is the total power radiated from the first three modes subject to a maximum-to-minimum thickness ratio of 3.0. For the initial

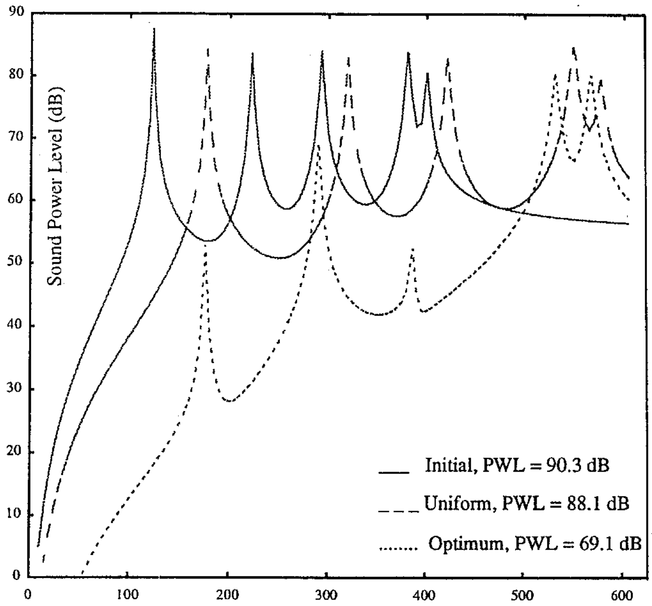

Fig. 6. Power vs frequency for rectangular plate

design, $t = 2$ mm (uniform), the first three resonance frequencies and associated sound power are 183 Hz (117.6 dB), 271 Hz (109.7 dB) and 315 Hz (112.8 dB), with total power = 119.3 dB. The thickness distribution for the optimized plate is shown in Table 1. In Table 1, the symbol "e1-5" means "elements 1 through 5"; the thicknesses for these five elements are given in the first row. All element numbers are indicated in Fig. 7. For the optimized design, the resonance frequencies and associated power are 128 Hz (104.2 dB), 221 Hz (102.7 dB) and 260 Hz (100.3 dB), and total power = 107.5 dB. The power versus frequency graph for the initial and optimized designs is shown in Fig. 8 and the optimum thickness distribution in Fig. 9. The results, of course, depend on the thickness bounds imposed during optimization. Furthermore, more than one starting design is recommended for nonlinear optimization problems. The weight of the initial and optimized plates are 1.77 kg and 1.20 kg, respectively.

7 Conclusions

A general gradient-based approach has been presented for minimizing sound power radiated from baffled plates. Analytical sensitivity analysis is essential in making this a viable approach. The optimization problem for fixed frequency excitation has multiple local minima and standard gradient approaches can only yield a local minimum. Here, the possibility of using certain global optimization methods needs to be investigated. For broad band excitation, the total power from a specified number of modes is a relatively smooth function of thickness and gradient methods are effective. The case of broad band excitation with fixed lower and upper limits on frequency needs further work to study the effect of modes entering or leaving the band. Numerical results for the engine cover plate show significant reduction in dB with just redistribution of the material in an optimal manner.

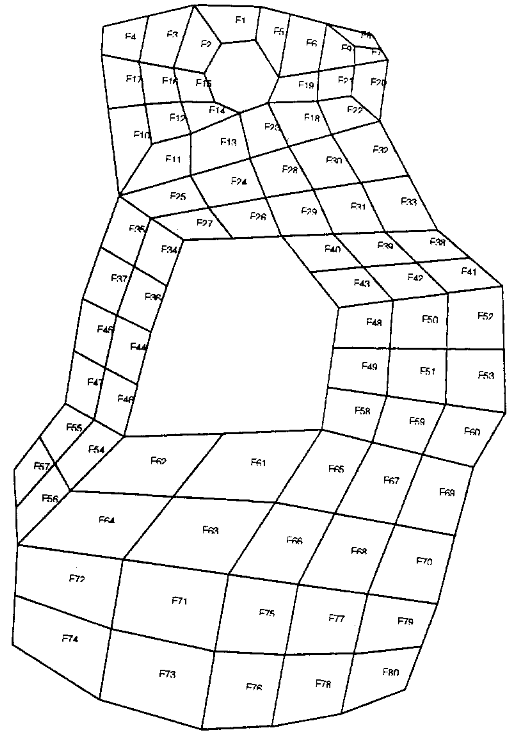

Fig. 7. Finite element model of the engine cover plate

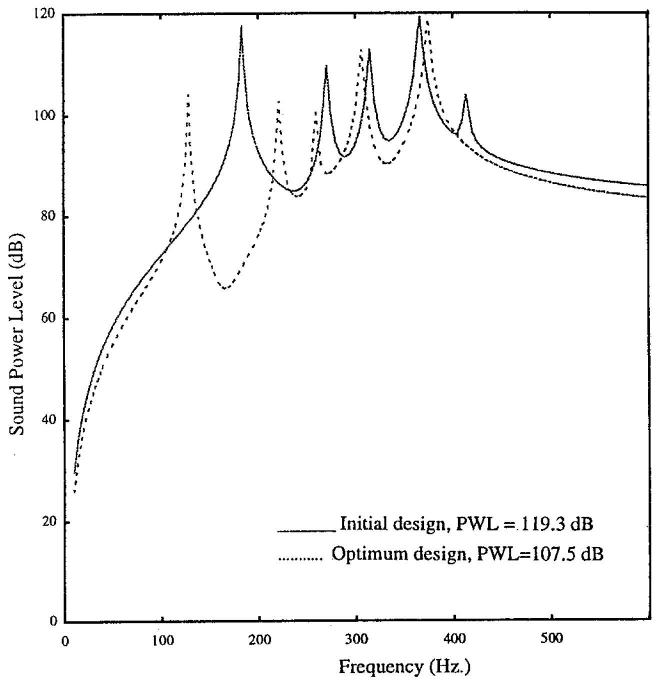

Fig. 8. Power vs frequency for engine cover plate

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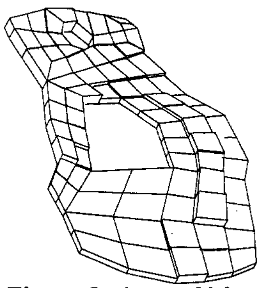

Fig. 9. Optimum thickness distribution of the engine cover plate

Table 1. Optimum element thicknesses for engine timing chain cover plate (mm)

e1–5	2.17	2.19	1.91	1.94	1.87
e6–10	1.95	2.01	2.02	2.00	1.79
e11–15	1.80	1.89	1.69	1.94	2.06
e16–20	1.94	1.98	1.79	2.02	2.06
e21–25	2.00	1.94	1.76	1.73	1.79
e26–30	1.04	1.88	1.72	1.23	1.61
e31–35	1.15	1.51	1.00	1.89	1.75
e36–40	1.90	1.80	1.00	1.59	1.00
e41–45	1.00	1.50	1.00	1.90	1.82
e46–50	1.36	2.11	1.00	1.00	1.51
e51–55	2.02	1.00	1.00	1.05	1.23
e56–60	1.00	1.18	1.00	1.80	1.00
e61–65	1.00	1.00	1.00	1.00	1.00
e66–70	1.00	2.01	2.50	1.00	1.00
e71–75	1.00	1.00	1.00	1.00	1.41
e76–80	1.62	1.26	1.00	1.00	1.00

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