Structural Optimization 4, 250–252 (1992)

Structural Optimization

Generalized shape optimization without homogenization*

G.I.N. Rozvany, M. Zhou and T. Birker

FB 10, Essen University, D–4300 Essen 1, Germany

Abstract Two types of solutions may be considered in generalized shape optimization. Absolute minimum weight solutions, which are rather unpractical, consist of solid, empty and porous regions. In more practical solutions of shape optimization, porous regions are suppressed and only solid and empty regions remain. This note discusses this second class of problems and shows that a solid, isotropic microstructure with an adjustable penalty for intermediate densities is efficient in generating optimal topologies.

1 Introduction

The aim of *generalized shape optimization* is the simultaneous optimization of both the *shape and topology* of the boundaries of two- or three-dimensional continua or of the interfaces between different materials in composites.

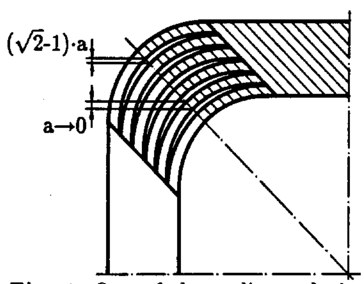


Fig. 1. One of the earliest solutions in generalized shape optimization (after Kohn and Strang 1983)

It was established by Kohn and Strang (1983) in the context of *plastic design* for torsion of a cross-section within a square area (Fig. 1) that generalized shape optimization may yield three types of regions, namely,

- *solid regions* (filled with material)
- *empty regions* (without material) and
- *porous regions* (some material, with cavities of inifinitesimal size).

Considering *elastic perforated plates* in plane stress or bending, it was found (e.g. Lurie and Cherkaev 1984; Murat and Tartar 1985; Kohn and Strang 1986) that one optimal microstructure for a *compliance constraint* consists of *rank-2 laminates* (ribs of first and second order infinitesimal widths in the two principal directions).

2 Solid-empty-porous (SEP) solutions

Analytical solutions based on rank-2 laminates for axisymmetric perforated plates by Rozvany, Olhoff, Bendsøe *et al.* (1987) and Ong, Rozvany and Szeto (1988) have shown that

- *a high proportion* of the available space in exact optimal designs consists of *porous regions*; and
- for low volume fractions the solution tends to that for *least-weight trusses* (Michell 1904) for plane stress and *least-weight grillages* (e.g. Prager and Rozvany 1977) for bending. This conclusion was also confirmed by Kohn and Allaire (1992).

Using both the correct microstructure (rank-2 laminates) and a "sub-optimal" microstructure (square or rectangular holes), near-optimal solutions were deterimined *numerically* by several investigators (e.g. Bendsøe and Kikuchi 1988; Suzuki and Kikuchi 1991; Bendsøe 1989; Diaz and Bendsøe 1992; Olhoff, Bendsøe and Rasmussen 1991).

Solutions with the correct *optimal microstructure* (rank-2 laminates) are rather *unpractical* for the following reasons:

- even an approximate, finite version of the rank-2 microstructure in porous regions, which are rather extensive, would require very *high manufacturing* costs;
- rank-2 laminates for perforated structures have zero shear stiffness in one direction, which makes these solutions completely *unstable* if the load direction is changed; and
- solutions are only available for a *single compliance or natural frequency constraint* which are *not realistic design problems.* This was also demonstrated recently by Sankaranarayanan, Haftka *et al.* (1992);
- owing to nonconvexity, these solutions may represent a *local optimum.*

However, solutions with rank-2 laminates are of *great theoretical value* because they represent an absolute limit on the structural weight, albeit for a somewhat artificial problem.

Sub-optimal microstructures tend to result in *more practical* solutions because they penalize and therefore suppress porous regions.

The term *homogenization* means that an inhomogeneous structural element, containing an infinite number of discontinuities in material or geometrical properties, is replaced by a homogeneous but anisotropic element, whose stiffness is direction but not location dependent within the element. The same idea was used in layout optimization already much earlier by others (e.g. Prager and Rozvany 1977) under terms like "grillage-like continua". This operation seems obvious to engineers, although it has no doubt interesting mathematical implications. Unfortunately, in the literature the term "shape optimization by homogenization" has become almost synonymous with "generalized shape optimization".

* Presented at the 18th Int. Congr. Theor. Appl. Mech., Haifa

3 Solid-empty (SE) solutions and solid, isotropic microstructure with penalty (SIMP) for intermediate densities

From an engineering point of view, it is more practical to aim at solutions in which porous regions are largely suppressed and then a second stage design procedure can produce a solution consisting of solid and empty regions only. This procedure was lucidly demonstrated by Olhoff *et al.* (1991) on an example involving a simply supported beam with a central point load (Figs. 2a and b).

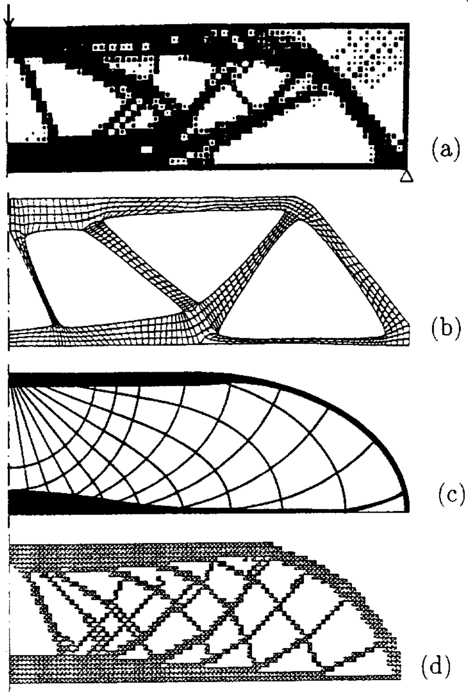


Fig. 2. (a) Topology for a simply supported beam (one half shown) using a sub-optimal microstructure, (b) design after a second stage process (after Olhoff *et al.* 1991), (c) the exact topology suggested by discretized truss solutions (Zhou and Rozvany 1991), (d) topology using a solid, isotropic microstructure with penalty for intermediate densities, SIMP (Zhou and Rozvany 1991)

It was suggested by the first author at a meeting in Karlsruhe in 1990 (see also Zhou and Rozvany 1991) that porous regions could be suppressed by adding to the material costs the *cost of manufacturing* of holes, thereby penalizing, and suppressing in the solution, porous regions. Once we decide that we want only solid and empty regions in the solution, *any* microstructure with an appropriate penalty for porous regions (or intermediate densities) can be assumed in the solution process. In selecting such a microstructure, the following objectives should be considered:

- simplicity of analysis and optimization;
- selective suppresion of porous regions by adjustable penalty; and
- capability of handling a variety of design conditions (e.g. combinations of deflection, stress, natural frequency and stability constraints for several load conditions).

It was demonstrated (Zhou and Rozvany 1991) that a *solid isotropic microstructure with penalty (SIMP)* for intermediate densities combined with new optimality criteria methods (COC, Rozvany and Zhou 1991), does result in very satisfactory SE-type topologies in generalized shape optimization. Figure 2c shows, for example, the "exact" topology for the same beam problem, suggested by discretized truss solutions (Zhou and Rozvany 1991, Figs. 15a-d). In the theoretical optimal solution, the number of intersecting members tends to infinity. The solution obtained with the SIMP model (Fig. 2d) seems to be much closer to this layout than the one obtained by using square cells (Fig. 2a).

An alternate optimal microstructure for plates with a compliance constraint was derived recently by Vigdergauz (1992). The latter may start with a Michell-structure or least-weight grillage at very low volume-fractions (Fig. 2c), then develops roundings at the corners (Fig. 3a), finishing up with elliptical holes at high volume fractions (Fig. 3b). In view of the above results, the solution obtained with SIMP in Fig. 2d seems to be a very good approximation of the exact solution at lower volume fractions.

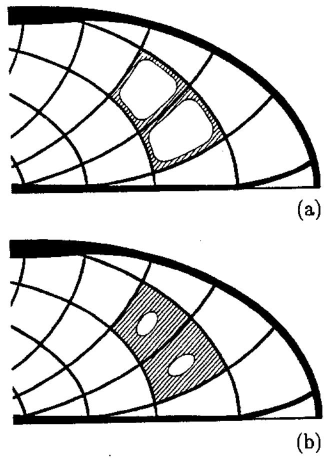
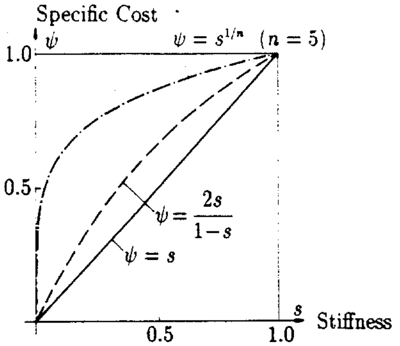


Fig. 3. Alternative optimal microstructures at higher volume fractions

In Fig. 4, three types of specific costs are compared for a plate in plane stress or bending. The straight line represents weight per unit area of a plate of variable thickness. The next curve shows the weight of a rank-2 laminate (perforated plate) with equal stiffness in two directions (Rozvany *et al.* 1987). Finally, the top curve represents the power-type cost function for a SIMP formulation.

The method employed herein was also proposed by Bendsøe (1989) under the term "direct approach" or "0-1 discrete optimization method with a suitable differentiable approximation" using an "artificial material". The authors of this paper do not find, however, that the results are highly mesh-dependent, nor that a physical interpretation of this model is "impossible".

Another test example concerns a cantilever beam, for which the analytical solution (Hemp 1973, pp. 97-99) is shown in Fig. 5a, a topology generated by using a microstructure with square holes (Suzuki and Kikuchi 1991) in Fig. 5b and SIMP solutions by M. Zhou (10800 constant strain triangular elements) and by T. Birker (1440 isoparametric square elements) in Figs. 5c and d. The resolving power of the latter method appears to be higher than that of traditional "homogenization", and net-dependence of the topology is restricted



—— solid isotropic microstructure
– – – rank-2 laminate
—·—· solid microstructure with penalty (SIMP)

Fig. 4. Specific cost functions for various microstructures

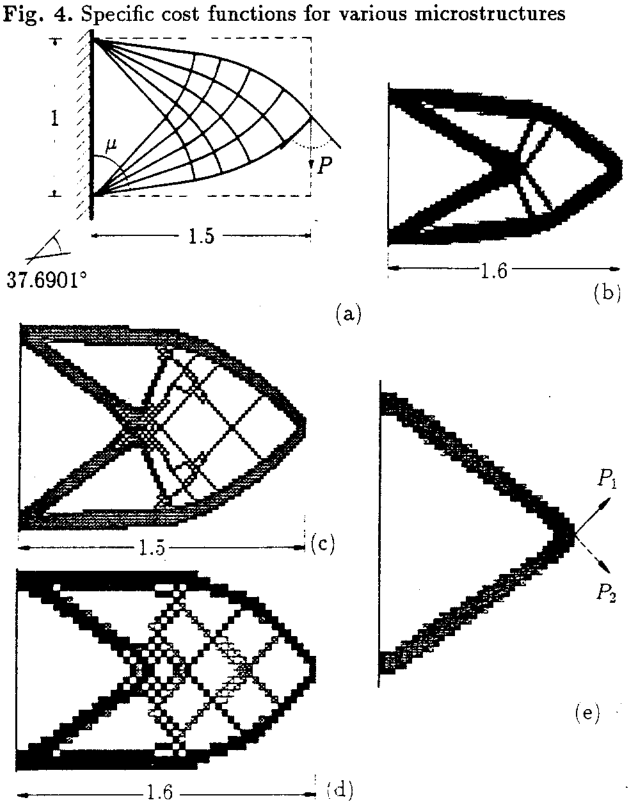


Fig. 5. Some results by an analytical method, by "homogenization" and by the SIMP procedure.

to a change in the number of "members". Finally, Fig. 5e shows a SIMP solution by T. Birker for *two alternate loads* (P_1 and P_2), which was also obtained analytically (Rozvany 1992) and by a discretized optimality criteria method (Zhou and Rozvany 1991). The SIMP procedure is also being used currently for three-dimensional continua subject to combinations of stress and deflection constraints, with the following *physical interpretation.* The permissible stress and Young's modulus of a material are proportional to the *density,* which varies between a maximum value and a very small minimum value, intermediate densities being penalized.

References

Bendsøe, M.P. 1989: Optimal shape design as a material distribution problem. *Struct. Optim.* **1**, 193-202

Bendsøe, M.P.; Kikuchi, N. 1988: Generating optimal topologies in structural design using a homogenization method. *Comp. Meth. Appl. Mech. Eng.* **71**, 197-224

Díaz, A.R.; Bendsøe, M.P. 1992: Shape optimization of multipurpose structures by a homogenization method. *Struct. Optim.* **4**, 17-22

Hemp, W.S. 1973: *Optimum structures.* Oxford: Clarendon

Kohn, R.V.; Allaire, G. 1992: Structural optimization using homogenization. *Proc. NATO ARW "Topology design of structures"* (held in Sesimbra, Portugal). Dordrecht: Kluwer

Kohn, R.V.; Strang, G. 1983: Optimal design for torsional rigidty. In: Atluri, S.N.; Gallagher, R.H.; Zienkiewicz, O.C. (eds.) *Hybrid and mixed finite element methods*, pp. 281-288. Chichester: Wiley & Sons

Kohn, R.V.; Strang, G. 1986: Optimal design and relaxation of variational problems, I, II, and III. *Comm. Pure Appl. Math.* **39**, 113-137, 139-182, 353-377

Lurie, K.A.; Cherkaev, A.V. 1984; G-closure for some particular set of admissible material characteristics for the problem of bending of thin elastic plates. *JOTA*, **42**, 305-316

Michell, A.G.M. 1904: The limits of economy of material in framestructures. *Phil. Mag.* **8**, 589-597

Murat, F.; Tartar, L. 1985: Calcul des variations et homogénéisation. In: *Les méthodes de l'homogénéisation: Théorie et applications en physique*, pp. 319-370. Eyrolles, Paris: Coll. de la Dir. des Etudes et Recherches de Elec. de France

Olhoff, N.; Bendsøe, M.P.; Rasmussen, J. 1991: On CAD-integrated structural topology and design optimization. *Comp. Meth. Appl. Mech. Eng.* **89**, 259-279

Ong, T.G.; Rozvany, G.I.N.; Szeto, W.T. 1988: Least-weight design of perforated elastic plates for given compliance: non-zero Poisson's ratio. *Comp. Meth. Appl. Mech. Eng.* **66**, 301-322

Prager, W.; Rozvany, G.I.N. 1977: Optimal layout of grillages. *J. Struct. Mech.* **5**, 1-18

Rozvany, G.I.N.; 1992: Optimal layout theory: analytical solutions for elastic structures with several deflection constraints and load conditions. *Struct. Optim.* **4**, 247-249

Rozvany, G.I.N.; Olhoff, N.; Bendsøe, M.P.; Ong, T.G.; Sandler, R.; Szeto, W.T. 1987: Least-weight design of perforated elastic plates I, II. *Int. J. Solids Struct.* **23**, 521-536, 537-550

Rozvany, G.I.N.; Zhou, M. 1991: The COC algorithm, Part I: cross-section optimization or sizing. *Comp. Meth. Appl. Mech. Eng.* **89**, 281-308

Sankaranarayanan, S.; Haftka, R.T.; Kapania, R.K. 1992: Truss topology optimization with simultaneous analysis and design. *Proc. 33rd AIAA/ASME/ASCE/AHS/ASC Struct. Dyn. Mat. Conf.* (held in Dallas), pp. 2576-2585. Washington DC: AIAA

Suzuki, K.; Kikuchi, N. 1991: A homogenization method for shape and topology optimization. *Comp. Meth. Appl. Mech. Eng.* **93**, 291-318

Vigdergauz, S. 1992: Two-dimensional grained composites of extreme rigidity. *18th Int. Congr. Theor. Appl. Mech.* (held in Haifa)

Zhou, M.; Rozvany, G.I.N. 1991: The COC algorithm, Part II: topological, geometrical and generalized shape optimization. *Comp. Meth. Appl. Mech. Eng.* **89**, 309-336

Received July 7, 1992